Chapter 5

A higher dimensional cosmological model for the search of dark energy source

5.1 Introduction

The enigmatic dark energy (DE) has gained the reputation of being one of the most discussed topics of paramount importance in cosmology since its profound discovery (Riess et al. 1998; Perlmutter et al. 1999). Its property with huge negative pressure with repulsive gravitation causing the universe to expand at an expedited rate still remains a mystery. It is believed to uniformly permeate throughout the space and vary slowly or almost consistent with time (Chan 2015b; Carroll 2001a, 2001b; Peebles & Ratra 2003). Cosmologists around the world have invested tremendous scientific efforts with a strong focus to hunt its origin and are still scrabbling for a perfect answer. Different authors have put forward their own versions of the answer with convincing evidences in support. Some worth mentioning studies on the search of the root of DE which have not escaped our attention in the past few years are briefly discussed below.

Singh & Kar (2019) assert that emergent D-instanton might lead us to the root of DE. Huterer & Shafer (2018) analyse the twenty years old history of DE and the current status. Wang et al. (2018) investigate the evolution of the DE using a non-parametric Bayesian approach in the light of the latest observation. According to Capolupo (2018), vacuum condensate can indicate us the origin of DE. A study on neutrino mixing as the origin of DE is presented by Capolupo et al. (2007). The explanation for DE with pure quantum mechanical method is presented by Dikshit (2019). According to Alexander et al. (2010), DE emerges due to condensation of fermions formed during the early evolution. The explanation of a physical mechanism as a source of DE can be seen in the work of Gontijo (2012). A cosmological model involving an antineutrino star is proposed by Neiser (2020) in an attempt to find the origin of DE. Josset et al. (2017) obtain DE from the violation

of energy conservation. A unified dark fluid is obtained as a source of DE by Josset et al. (2017). Lastly, we can witness a claim by Tripathy et al. (2015) that the presence of particle with imaginary energy density can lead us to the source of DE.

From literatures and observations (Agrawal et al. 2018; Araujo 2005; Carroll 2001a; Law 2020; Ray et al. 2013; Singh & Singh 2019b; Straumann 2007; Wu & Yu 2005), it is obvious that the massive universe is dominated by the mystic DE with negative pressure and positive energy density. This qualifies DE a completely irony of nature as the dominating component is also the least explored. So, there a lot more hidden physics behind this dark entity yet to be discovered. Contradicting to the condition of positive energy density, it is surprising that many authors has come up with the notion of negative energy density (NED) with convincing and fascinating arguments in support. Carroll (2001a) predicts a condition in which NED is possible only if the DE is in the form of vacuum energy. Besides defying the energy conditions of GR, NED also disobeys the second law of thermodynamics (Hawking & Ellis 1973). However, the condition should be solely obeyed on a large scale or on a mean calculation, thereby neglecting the probable violation on a small scale or for a short duration, in relativity (Epstein 1965; Fewster 2012; Ford & Roman 1996; Graham & Olum 2003; Helfer 1998a, 1998b; Pfenning & Ford 1998; Roman 1986; Visser & Barcelo 2000). Hence, in the initial epoch, if there were circumstance of defiance for a short duration measured against the present age which is estimated to be 13.825 ± 0.037 Gyr by the latest Planck 2018 result (Collaboration et al. 2020), it will remain as an important part in the course of evolution. According to Nemiroff et al. (2015), under certain conditions, a repelling negative gravitational pressure can be seen with NED. It further mention of a repelling negative phantom energy with NED. Energy density assuming negative value with equation of state parameter (EoS) $\omega < -1$ is predicted by Macorra & German (2004). Fay (2014) asserts that the universe evolves by inflation when the coupled fluid has NED in the initial epoch. Wong et al. (2019) discuss negative vacuum energy density in Rainbow Gravity. According to Parker & Fulling (1973), the introduction of quantized matter field with NED to energy momentum tensor might by pass cosmological singularity. Huang (1990) investigates models which evolved with NED in the infinite past. Ijjas & Steinhardt (2019) discuss NED asserting that their models evolve with a bounce. The authors continued that there might be bounces in the future too. Lastly, an accelerating universe with NED is studied by Sawicki & Vikman (2013).

The sessions of the Prussian Academy of Science during the four Thursdays of the month of November 1915 can be marked as the most memorable moments in the life of the great Einstein. On 4th, 11th, 18th and 25th of the month, he presented four of his no-

table communications (Einstein 1915a, 1915b, 1915c, 1915d) at the sessions, which led to the foundation of GR. Since then, a number of authors have been exploring gravitation in different settings. In Weyl's work of 1918 (Weyl 1918), we can witness the first trial to extend GR with the aim of bringing together gravitation and electromagnetism geometrically. Similar to that of Weyl, Lyra's modification (Lyra 1951) by proposing a gauge function into the structureless manifold provides one of the well appreciated alternate or modified theories of gravitation. The static model with finite density in Lyra's modified Riemannian geometry is similar to the static Einstein model (Singh & Singh 1991). The scalar-tensor treatment based on Lyra's geometry yields the same effects, as GR, under certain limits (Yadav 2020). Lyra's work are further extended by other well known authors (Halford 1970, 1972; Sen 1957, 1960; Sen & Dunn 1971). Cosmologists choose to opt alternate or modified theories of gravitation in order to precisely understand the underlying mechanism of the late time expedited expansion of the universe. Many other authors too have succeed in developing fascinating and worth appreciating modified theories which have served the purpose of explaining the expanding paradigm in a quite convincing way (Barker 1978; Bekenstein 2004; Brans & Dicke 1961; Chamseddine & Mukhanov 2013; Nojiri & Odintsov 2003, 2014; Nordtvedt 1970; Saez & Ballester 1986; Sotiriou & Faraoni 2010).

During the past few years, there has been an increasing interest among cosmologists to study the ambiguous DE paired with Lyra Manifold (LM). Recently, Bhardwaj & Rana (2020) investigate the existence of Lyra's cosmology with interaction of normal matter and DE. In the work of Hova (2013), we can witness a DE model in a LM which proves that the expansion paradigm can be illustrated in the absence of a negative pressure energy component. It is further mentioned that DE is naturally of geometrical origin. The investigation of a two component DE model in LM can be seen in the publication of Khurshudyan et al. (2014). Ram et al. (2020) predict a cosmological model of anisotropic DE paired with LM in consonant with the present observation. The study of of a magnetized DE model in Lyra setting is presented by Pawar et al. (2014). A discussion on the effect of DE on model with linear varying deceleration parameter in LM can be found in the work of Patra et al. (2019). Katore & Hatkar (2015) present the isotropization of DE distribution in LM. A Kaluza-Klein DE model is studied in LM thereby obtaining an exponentially expanding universe by Aditya et al. (2019). From the research of Singh & Sharma (2014a), we can find a DE model in LM where constant deceleration parameter is assumed. Brans–Dicke scalar field as a DE candidate in LM is illustrated by Zia & Maurya (2018). A DE model with quadratic equation of state is presented in the framework of LM by Mollah et al. (2018). Lastly, Yadav & Bhardwaj (2018) search for the existence of Lyra's cosmology with minimal interaction between DE and normal matter and obtain that the time varying

displacement $\beta(t)$ co-relates with the nature of cosmological constant $\Lambda(t)$.

The chance of space-time having more than 4D has captivated many authors. This has ignited a spark of interest among cosmologists and theorological physicists so that, in the past few decades, there has been a trend among authors opting to choose higher dimensional space-time to study cosmology. Higher dimensional model was introduced by Kaluza (1921) and Klein (1926) in an attempt to unify gravity with electromagnetism. Higher dimensional model can be regarded as a tool to illustrate the late time expedited expanding paradigm (Banik & Bhuyan 2017). Investigation of higher dimensional space-time can be regarded as a task of paramount importance as the universe might have come across a higher dimensional era during the initial epoch (Singh et al. 2004). From the investigations of Alvax & Gavela (1983) and Guth (1981), it can be found that extra dimensions generate huge amount of entropy which gives possible solution to flatness and horizon problem. Marciano (1984) asserts that the detection of a time varying fundamental constants can possibly show us the proof for extra dimensions. Since we are living in a 4D space-time, the hidden extra dimension in 5D is highly likely to be associated with the invisible DM and DE (Chakraborty & Debnath 2010). Astefanesei et al. (2020), Bahrehbakhsh (2018), Demirel (2019), Ghaffarnejad et al. (2020), Montefalcone et al. (2020), Oli (2014), Saha & Ghose (2020), Samanta et al. (2014), Singh & Desikan (1997), Shinkai & Torii (2015) and Singh & Singh (2019b) are some of the authors who have worked on higher dimensional space-time during the last few years.

Keeping in mind the above notable works by different authors, we have analysed a spherically symmetric (SS) metric in 5D setting within the framework of LM, with the aim of predicting a possible source of DE. Here, we observe the field equations with due consideration of reasonable cosmological assumptions within the limit of the present cosmological scenario. The chapter is structured into sections. In Sect. 5.2, in addition to obtaining the solutions of the field equations, the cosmological parameters are also solved. In Sect. 5.3, the physical and kinematical aspects of our model are discussed with graphs. Considering everything, a closing remark is presented in Sect. 5.4.

5.2 Formulation of problem with solutions

The five-dimensional SS metric (Samanta & Dhal 2013) is given by

$$ds^2 = dt^2 - e^\mu \left(dr^2 + r^2 d\Theta^2 + r^2 \sin^2 \Theta d\phi^2 \right) - e^\delta dy^2 \qquad (5.2.1)$$

where $\mu = \mu(t)$ and $\delta = \delta(t)$ are cosmic scale factors.

The modified Eintein's field equations in Lyra geometry appear in the form

$$R_{ij} - \frac{1}{2}g_{ij}R + \frac{3}{2}\varphi_i\varphi_j - \frac{3}{4}g_{ij}\varphi_k\varphi^k = -T_{ij} \tag{5.2.2}$$

where φ_i is the displacement vector and other symbols have their usual meaning as in Riemannian geometry. The displacement vector φ_i takes the time dependent form

$$\varphi_i = (\beta(t),\ 0,\ 0,\ 0,\ 0) \tag{5.2.3}$$

The assumption that φ_i is time independent i.e. constant is vague as there is no specific mathematical or physical explanation showing that a constant displacement vector contributes to the late time acceleration of the universe (Yadav 2020). Above all, assuming displacement vector field as a constant is just for convenience sake without any scientific reason (Singh & Desikan 1997).

The energy momentum tensor T_{ij}, considered as a perfect fluid, in the co-moving coordinates is given by

$$T_{ij} = (\rho + p)\, u_i u_j - pg_{ij} \tag{5.2.4}$$

where ρ and p respectively represent the energy density and isotropic pressure of the matter source. The five velocity vector u^i satisfies

$$u^i u_i = 1,\ u^i u_j = 0 \tag{5.2.5}$$

Now, the surviving field equations are obtained as follows

$$\frac{3}{4}\left(\dot{\mu}^2 + \dot{\mu}\dot{\delta} - \beta^2\right) = \rho \tag{5.2.6}$$

$$\ddot{\mu} + \frac{3}{4}\dot{\mu}^2 + \frac{\ddot{\delta}}{2} + \frac{\dot{\delta}^2}{4} + \frac{\dot{\mu}\dot{\delta}}{2} + \frac{3}{4}\beta^2 = -p \tag{5.2.7}$$

$$\frac{3}{2}\left(\ddot{\mu} + \dot{\mu}^2\right) + \frac{3}{4}\beta^2 = -p \tag{5.2.8}$$

where an overhead dot represents differentiation w.r.t. t.

From continuity equation, we have

$$\dot{\rho} + \frac{3}{2}\dot{\beta}\beta + 3H\left(\rho + p + \frac{3}{2}\beta^2\right) = 0 \tag{5.2.9}$$

According to Bahrehbakhsh (2018), assuming that β and ρ are independent without any interaction, Eq. (5.2.9) can be separately written as

$$\dot{\rho} + 3H\left(\rho + p\right) = 0 \tag{5.2.10}$$

$$\dot{\beta}\beta + 3H\beta^2 = 0 \tag{5.2.11}$$

where H is the Hubble parameter.

From Eqs. (5.2.7) and (5.2.8), the expression for cosmic scale factors are obtained as

$$\mu = l - \log\left(k - t\right)^{\frac{2}{3}} \tag{5.2.12}$$

$$\delta = m - \log(k - t)^{\frac{2}{3}} \tag{5.2.13}$$

where l, m, k are arbitrary constants.

Now, we obtain the expression for the cosmological parameters as follows.

Spatial volume:
$$V = e^{\frac{3\mu + \delta}{2}} = e^{\frac{3l + m}{2}}\left(k - t\right)^{-\frac{4}{3}} \tag{5.2.14}$$

Scale factor:
$$a(t) = V^{\frac{1}{4}} = e^{\frac{3l + m}{8}}\left(k - t\right)^{-\frac{1}{3}} \tag{5.2.15}$$

Scalar expansion:
$$\theta = u^i_{;j} = \frac{3\dot{\mu}}{2} + \frac{\dot{\delta}}{2} = \frac{4}{3}\left(k - t\right)^{-1} \tag{5.2.16}$$

Hubble parameter:
$$H = \frac{\Theta}{4} = \frac{1}{3}\left(k - t\right)^{-1} \tag{5.2.17}$$

Deceleration parameter:
$$q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 = -4 \tag{5.2.18}$$

With $\Delta H_i = H_i - H, (i = 1, 2, 3, 4)$ representing the directional Hubble's parameters,

anisotropic parameter A_h is defined as

$$A_h = \frac{1}{4} \sum_{i=1}^{4} \left(\frac{\Delta H_i}{H} \right)^2 = 0 \tag{5.2.19}$$

Shear Scalar:

$$\sigma^2 = \frac{1}{2} \sigma_{ij} \sigma^{ij} = \frac{1}{2} \sum_{i=1}^{4} \left(H_i^2 - 4H \right) = \frac{2}{9} \left(1 - \frac{1}{k-t} \right)^2 \tag{5.2.20}$$

From Eqs. (5.2.11) and (5.2.17), the expression for displacement vector is obtained as

$$\beta = d\,(t-k) \tag{5.2.21}$$

where d is an arbitrary constant.

From Eqs. (5.2.6), (5.2.12), (5.2.13) and (5.2.21), the expression for energy density is obtained as

$$\rho = \frac{3}{4} \left(\frac{8 - \left(3d\,(k-t)^2 \right)^2}{9\,(k-t)^2} \right) \tag{5.2.22}$$

From Eqs. (5.2.8), (5.2.12) and (5.2.13), the expression of pressure is obtained as

$$p = -\frac{8\left(1 + (k-t)^2\right) + \left(3d\,(k-t)^3\right)^2}{12(k-t)^4} \tag{5.2.23}$$

5.3 Discussion

For convenience sake and to obtain realistic results, specific values of the constants are chosen i.e., $l = m = 1, d = -1, k = 3.45497$ and the variations of some of the physical parameters with cosmic time t are provided as figures in this section. A scale of 1 Unit $=$ 4 Gyr is taken along the time axis of each graph so that the point $t = 3.45$ corresponds to 13.8 Gyr which align with 13.825 ± 0.037 Gyr, the present age of the universe estimated by the latest Planck 2018 result (Collaboration et al. 2020).
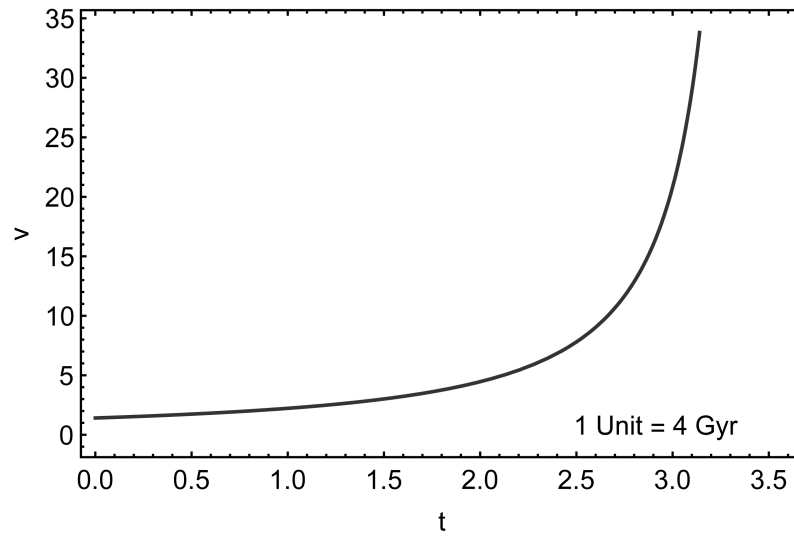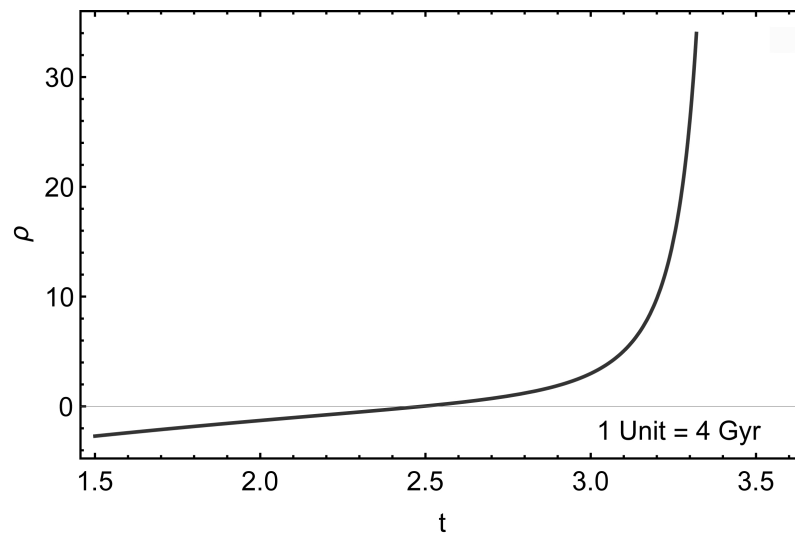
Figure 5.1: Variation of spatial volume V with time t when $l = m = 1, k = 3.45497$ showing its increasing nature throughout the evolution.
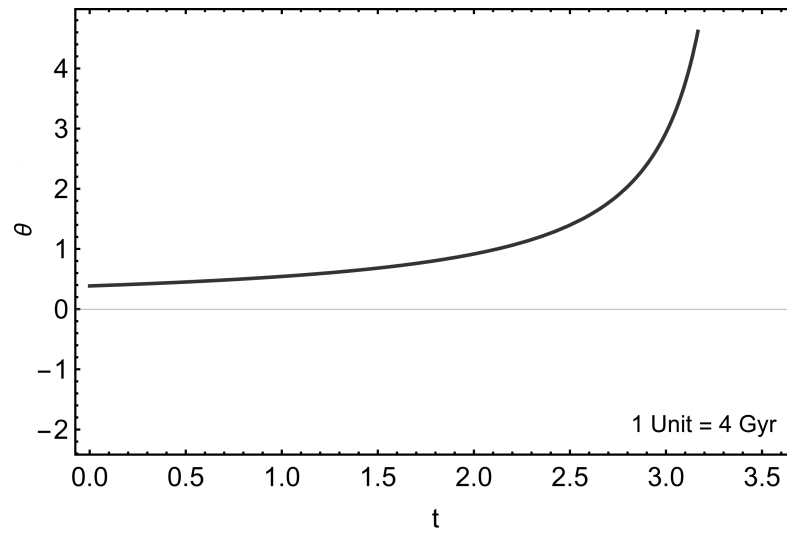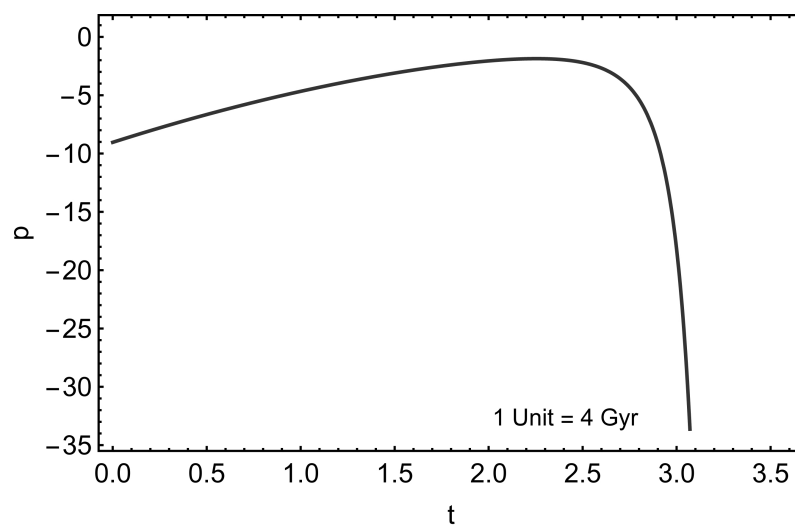

Figure 5.2: Variation of scalar expansion θ with time t when $l = m = 1, k = 3.45497$ showing its increasing nature throughout the evolution.
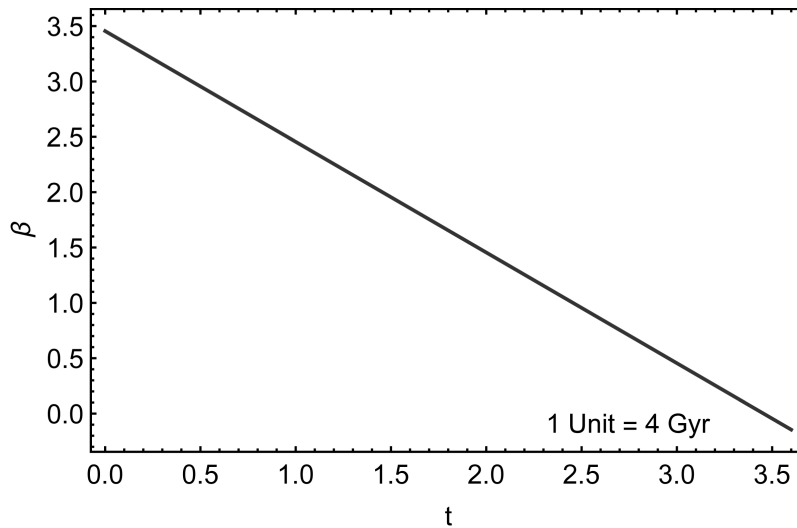
Figure 5.3: Variation of energy density ρ with time t when $l = m = 1, k = 3.45497$ showing its transition from being negative to positive during evolution.


Figure 5.4: Variation of pressure p with time t when $l = m = 1, k = 3.45497$ showing its negative nature all through.

Figure 5.5: Variation of displacement vector β with time t when $d = -1, k = 3.45497$ showing its decreasing nature.

Figs. 5.1 and 5.2 can be regarded as the perfect evidences of the present spacial expansion of the universe at an expedited rate. From Figs. 5.3 and 5.4, we can witness a transition of the energy density ρ of the model universe from being negative to positive during the course of evolution whereas the pressure p of the model is negative all through. In short, the universe expands at an expedited rate with ρ and p both negative. This negative p can be regarded as the indication of the presence of DE. In this scenario, we can predict that DE in the form of vacuum energy (VE) or cosmological constant (CC) is dominating the model, as mentioned in the work of Carroll (2001a), NED is possible only if the DE is in the form of VE. When $t \to \infty$, both V and $\theta \to 0$ showing that, in the far future, the expanding phenomenon will cease, the universe will be dominated by gravity, resulting to collapse and ultimately ending at the big crunch singularity. This may be supported by the fact that DE density may decrease faster than matter leading DE to vanish at $t \to \infty$ (Peebles & Ratra 2003). Additionally, when $t \to \infty$, ρ again starts to become negative. In this condition, due to the presence of NED, according to Ijjas & Steinhardt (2019), we can assume that the model universe represents an oscillating model, each cycle evolving with a big bang and ending at a big crunch, undergoing a series of bounces. Additionally, from Fig. 5.5, it is clear that the displacement vector β is a decreasing function of time. Here, we can assert that β acts as the time dependent cosmological constant (Agarwal et al. 2011; Halford 1970; Perlmutter et al. 1999; Riess et al. 1998). Hence, it is fascinating to observe that LM itself can be regarded as a DE model.
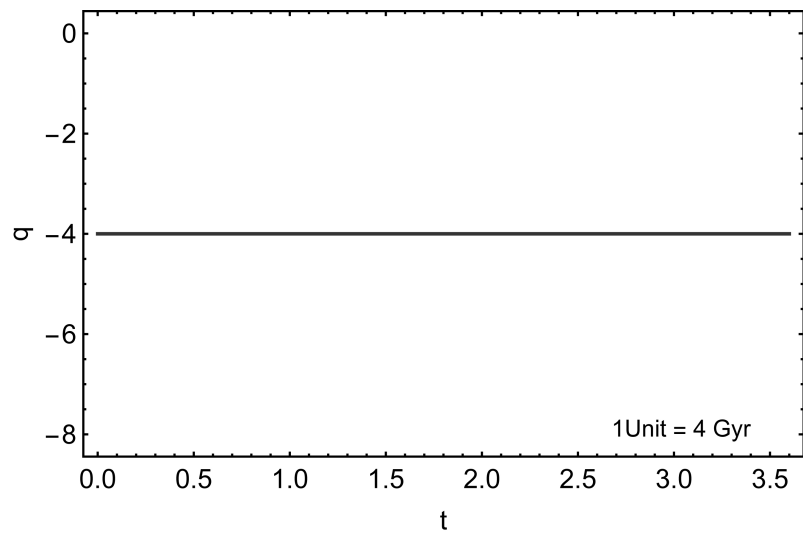
Figure 5.6: Variation of deceleration parameter q with time t when $l = m = 1, k = 3.45497$.
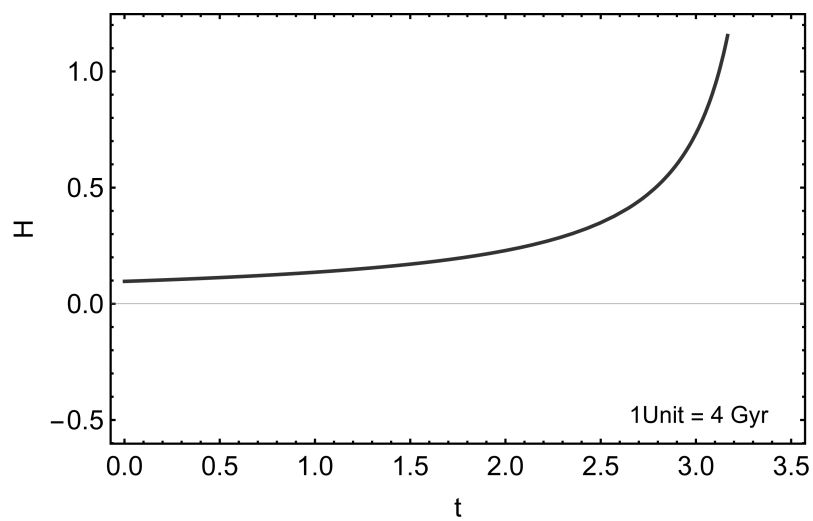
Figure 5.7: Variation of Hubble parameter H with time t when $k = 3.45497$.
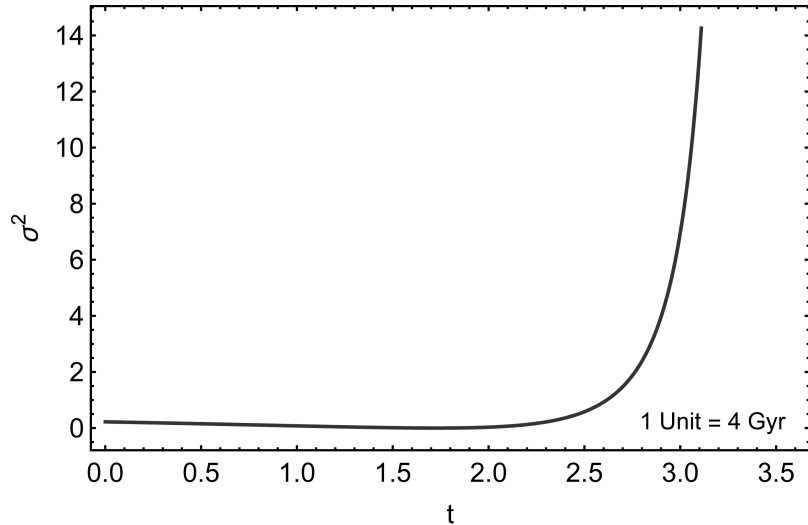
Figure 5.8: Variation of shear scalar σ^2 with time t when $l = m = 1, k = 3.45497$.

Accelerated expansion can be attained when $-1 < q < 0$ whereas $q < -1$ causes super-exponential expansion (Singh & Bishi 2017). Fig. 5.6 shows that the deceleration parameter q is a negative constant -4 indicating that the model universe undergoes super-exponential expansion. It may be noted that in a higher dimensional theory with CC, super-exponential inflation (expansion) can be attained if H increases with t (Polock 1988; Shaft & Wetterich 1985; Wenerich 1985). In our case, H is increasing as shown in Fig. 5.7. Shear scalar σ^2 provides us the rate of deformation of the matter flow within the massive cosmos (Ellis & Elst 1999). From Fig. 5.8, we can see that σ^2 evolves almost constantly, then diverges after some finite time. From Eq. (5.2.19), the anisotropic parameter $A_h = 0$. From these, we can sum up that initially, the isotropic universe expands with a slow and uniform change of shape, but after some finite time, the change becomes faster.

Lastly, with a scale of 1 Unit = 4 Gyr, the point $t = 3.45$ corresponds to 13.8 Gyr which align with 13.825 ± 0.037 Gyr, the present age of the universe estimated by the latest Planck 2018 result (Collaboration et al. 2020). At the point $t = 3.45$ and assuming $k = 3.45497$, from Eq. (5.2.17), the numeric value of the Hubble parameter is measured to be $H = 67.0691$ which is very close to $H_0 = 67.36 \pm 0.54 \text{ kms}^{-1} \text{ Mpc}^{-1}$, the value estimated by the latest Planck 2018 result (Collaboration et al. 2020).

5.4 Conclusions

In this chapter, with due consideration of reasonable cosmological assumptions within the limit of the present cosmological scenario, we have analysed a SS metric in 5D setting within the framework of LM. The model universe is predicted to be a DE model, dominated by VE

or CC. The displacement vector also acts as the time dependent DE. The model represents an oscillating model, each cycle evolving with a big bang and ending at a big crunch, undergoing a series of bounces. Our universe undergoes super-exponential expansion. It may be noted that in a higher dimensional theory with CC, super-exponential inflation (expansion) can be attained if H increases with t (Polock 1988; Shaft & Wetterich 1985; Wenerich 1985). In our case, H is increasing. Initially, the isotropic universe expands with a slow and uniform change of shape, but after some finite time, the change becomes faster. Then, the change slows down and tends to become uniform after expanding without any deformation of the matter flow for a finite time period. Lastly, the Hubble parameter is measured to be $H = 67.0691$ which is very close to $H_0 = 67.36 \pm 0.54$ kms^{-1} Mpc^{-1}, the value estimated by the latest Planck 2018 result (Collaboration et al. 2020). We have constructed a model in LM appearing as a DE model; nonetheless, the work we have put forward is just a toy model. The model needs further deep study considering all the observational findings, which will be our upcoming work.