Chapter 6

$f(R,T)$ gravity model behaving as a dark energy source

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6.1 Introduction

The ambiguous dark energy (DE) has been regarded as one of the most tantalizing topics in cosmology since its profound discovery in 1998 (Riess 1998; Perlmutter 1999). It is considered to be the reason behind the late time expanding universe at an expedited rate due to its huge negative pressure with repulsive gravitation. It is uniformly permeated throughout the space and vary slowly or almost consistent with time (Carroll 2001a, 2001b; Chan 2015b; Peebles & Ratra 2003). Cosmologists all over the map have conducted a series of studies with the aim of hunting its origin and are still scrabbling for a perfect answer. Some worth mentioning such studies that have not escaped our notice in the recent years are briefly discussed below.

Singh & Kar (2019) assert that emergent D-instanton might indicate us a hint to the root of DE. A cosmological model associated with an antineutrino star is constructed by Neiser (2020) in order to search the origin of DE. Dikshit (2019) presents an explanation for DE with pure quantum mechanical method. Huterer & Shafer (2018) investigate the twenty years old history of DE and the current status. The authors in Wang et al. (2018) study the evolution of the DE using a non-parametric Bayesian approach in the light of the latest observation. Capolupo (2018) claims that vacuum condensate can provide us the origin of DE. According to Josset et al. (2017), DE is originated from the violation of energy conservation. A unified dark fluid is obtained as a source of DE by Tripathy et al. (2015). The presence of particle with imaginary energy density can lead us to the source of DE (Chan 2015a). The explanation of a physical mechanism as a source of DE is presented by Gontijo (2012). Lastly, according to the work of Alexander et al. (2010), DE evolves as a result of the condensation of fermions formed during the early evolution.

It is an obvious fact that the universe is dominated by the cryptic DE with negative pressure and positive energy density (Araujo 2005; Agrawal et al. 2018; Carroll 2001a; Law 2020; Ray et al. 2013; Singh & Singh 2019b; Straumann 2007; Wu & Yu 2005). This qualifies DE a completely irony of nature as the dominating component is also the least explored. As against the positive energy density condition, it is fascinating to see many authors introducing the concept of possibility of negative energy density (NED) with convincing arguments in support. Ijjas & Steinhardt (2019) discuss NED where models evolve with a bounce. The authors continued that there might be bounces in the future too. The discussion of negative vacuum energy (VE) density in Rainbow Gravity can be seen in the work of Wong et al. (2019). According to Nemiroff et al. (2015), under certain conditions, a repelling negative gravitational pressure with NED. Further, we can find a repelling negative phantom energy with NED. Fay (2014) claims that the universe evolves by inflation when the coupled fluid has NED in the initial epoch. An accelerating universe with NED is studied by Sawicki & Vikman (2013). Macorra & German (2004) present an explanation of energy density with negative value with equation of state parameter (EoS) $\omega < -1$. Carroll (2001a) predicts that NED is possible only if the DE is in the form of VE. Huang (1990) investigates models which evolved with NED in the infinite past. According to Parker & Fulling (1973), the introduction of quantized matter field with NED to energy momentum tensor might by pass cosmological singularity. Besides defying the energy conditions of GR, NED also disobeys the second law of thermodynamics (Hawking & Ellis 1973). However, the condition should be solely obeyed on a large scale or on a mean calculation, thereby neglecting the probable violation on a small scale or for a short duration, in relativity (Epstein et al. 1965; Fewster 2012; Ford & Roman 1996; Graham & Olum 2003; Helfer 1998a, 1998b; Pfenning & Ford 1998; Roman 1986; Visser & Barcelo 2000). Hence, in the initial epoch, if there were circumstance of defiance for a short duration measured against the present age of 13.830 ± 0.037 Gyr estimated by the latest Planck 2018 result (Collaboration et al. 2020), it will remain as an important part in the course of evolution.

In the present cosmology, authors prefer to opt alternate or modified theories of gravitation in order to precisely understand the underlying mechanism of the late time expedited expansion of the universe. One such well appreciated modified theory is the $f(R,T)$ gravity introduced by Harko et al. (2011) in which the gravitational Lagrangian is represented by an arbitrary function of the Ricci scalar R and the trace T of the energy-momentum tensor. In the past few years, this theory has captivated many cosmologists and theoretical physicists as it presents natural gravitational substitutes to DE (Chirde & Shekh 2019). Recently, Myrzakulov (2020) studies the theory and predicts the conditions to obtain expanding universe in the absence of any dark component. Sahoo et al. (2020) investigate a mixture of barotropic fluid and DE in $f(R,T)$ gravity where the model evolves from the Einstein static era and approaches ΛCDM. Pawar et al. (2019) study a modified holographic Ricci DE model in the theory obtaining a singularity free model. Zia et al. (2018) investigate $f(R,T)$ gravity discussing future singularities in DE dominated universe. In the research of Srivastava & Singh (2018), we can find a discussion of new holographic DE model in $f(R,T)$ gravity thereby obtaining ΛCDM in the late times. Fayaz et al. (2016) examine ghost DE model within the theory, predicting model behaving as phantom or quintessence like nature. The investigation of cosmological models within the theory without DE is observed in the work of Sun & Huang (2016). Mishra et al. (2016b) and Singh & Kumar (2016) study the relation of the theory with DE. Houndjo and Piattella (2012) present a reconstruction of the theory from holographic DE. The study cosmological model in $f(R,T)$ gravity obtaining DE induced cosmic acceleration is presented by Mishra et al. (2016a). Zubair et al. (2016) discuss Bianchi space-time within the theory with time-dependent deceleration parameter. Ahmed et al. (2016) investigate model in which the cosmological constant is considered as a function of T. Rao & Rao (2015) discuss a higher dimensional anisotropic DE model within the theory obtaining the EoS parameter $\omega = -1$. Jamil et al. (2012) construct models within the theory asserting that dust fluid leads to ΛCDM. Houndjo (2012) predicts a model in $f(R,T)$ gravity that transit from matter dominated to accelerating phase. From these worth appreciating studies, it won't be a wrong guess to sum up that there must be some sort of hidden correspondence between the pair of DE and $f(R,T)$ gravity. Consequently, in this work, we will try to find out if $f(R,T)$ itself behaves as a DE source.

The possibility of space-time possessing with more than 4D has fascinated many authors. In the recent years, there has been a trend of preferring higher dimensional space-time to study cosmology. Higher dimensional model, in GR, was introduced by Kaluza (1921) and Klein (1926) in an effort to unify gravity with electromagnetism. Higher dimensional model can be regarded as a tool to illustrate the late time expedited expanding paradigm (Banik & Bhuyan 2017). Investigation of higher dimensional space-time can be regarded as a task of paramount importance as the universe might have come across a higher dimensional era during the initial epoch (Singh et al. 2004). Marciano (1984) asserts that the detection of a time varying fundamental constants can possibly show us the proof for extra dimensions. According to Alvarez & Gavela (1983) and Guth (1981), extra dimensions generate huge amount of entropy which gives possible solution to flatness and horizon problem. Since we are living in a 4D space-time, the hidden extra dimension in 5D is highly likely to be associated with the invisible DM and DE (Chakraborty & Debnath

2010).

Keeping in mind the above notable works by different authors, we have analysed a spherically symmetric (SS) metric in 5D setting within the framework of $f(R,T)$ gravity with focus to predict a possible source of DE. Here, we observe the field equations with due consideration of reasonable cosmological assumptions within the limit of the present cosmological scenario. The chapter is divided into sections. After introduction, in Sect. 6.2, the field equations of $f(R,T)$ gravity theory are discussed. In Sect. 6.3, in addition to obtaining the solutions of the field equations, the cosmological parameters are also solved. In Sect. 6.4, the physical and kinematical aspects of our model are discussed with graphs. Considering everything, a closing remark is presented in Sect. 6.5.

6.2 The field equations of $f(R,T)$ gravity theory

The action of $f(R,T)$ gravity theory is given by

$$S=\int\left(\frac{1}{16\pi}f(R,T)+\mathcal{L}_m\right)\sqrt{-g}d^4x \tag{6.2.1}$$

where $g\equiv det(g_{ij})$, f is an arbitrary function of the Ricci scalar $R=R(g)$ and the trace $T=g^{ij}T_{ij}$ of the energy-momentum tensor of matter T_{ij} defined by Koivisto (2006) as

$$T_{ij}=-\frac{2}{\sqrt{-g}}\frac{\delta\left(\sqrt{-g}\mathcal{L}_m\right)}{\delta\,g^{ij}} \tag{6.2.2}$$

Here, the matter Lagrangian density $\mathcal{L}_m$ is assumed to rely solely on g_{ij} so that we obtain

$$T_{ij}=g_{ij}\mathcal{L}_m-2\frac{\partial\mathcal{L}_m}{\partial g^{ij}} \tag{6.2.3}$$

The action S is varied w.r.t. the metric tensor g^{ij} and hence, the field equations of $f(R,T)$ gravity is given by

$$f_R\left(R,T\right)R_{ij}-\frac{1}{2}f\left(R,T\right)g_{ij}+\left(g_{ij}\Box-\nabla_i\nabla_j\right)f_R\left(R,T\right)=8\pi T_{ij}-f_T\left(R,T\right)T_{ij}-f_T\left(R,T\right)\theta_{ij} \tag{6.2.4}$$

where

$$\theta_{ij}=-2\,T_{ij}+g_{ij}\mathcal{L}_m-2g^{lk}\frac{\partial^2\mathcal{L}_m}{\partial g^{ij}\partial g^{lk}} \tag{6.2.5}$$

Here, the subscripts appearing in f represent the partial derivative w.r.t. R or T and $\Box\equiv\nabla^i\nabla_i$, ∇_i being the covariant derivative.

With ρ and p respectively representing the energy density and pressure such that the five velocity u^i satisfies $u^i u_i = 1$ and $u^i \nabla_j u_i = 0$, we opt to use the perfect fluid energy-momentum tensor of the form

$$T_{ij} = (p+\rho)\, u_i u_j - p g_{ij} \tag{6.2.6}$$

We assume that $\mathcal{L}_m = -p$ so that Eq. (6.2.5) is reduced to

$$\theta_{ij} = -2T_{ij} - p g_{ij} \tag{6.2.7}$$

In general, the field equations of $f(R,T)$ gravity also rely on the physical aspect of the matter field and consequently, there exists three classes of field equations as follows

$$f(R,T) = \begin{cases} R + 2f(T) \\ f_1(R) + f_2(T) \\ f_1(R) + f_2(R) f_3(T) \end{cases} \tag{6.2.8}$$

Our study will be dealing with the class $f(R,T) = R + 2f(T)$, where $f(T)$ represents an arbitrary function so that the field equations of the modified theory is be reduced to

$$R_{ij} - \frac{1}{2} R g_{ij} = 8\pi\, T_{ij} + 2f'(T)\, T_{ij} + \left\{\, 2p\, f'(T) + f(T) \,\right\}\, g_{ij} \tag{6.2.9}$$

where the prime indicates differentiation w.r.t. T and we assume that $f(T) = \lambda T$, where λ is an arbitrary constant.

6.3 Formulation of the problem and solutions

The five-dimensional SS metric is given by (Samanta & Dhal 2013)

$$ds^2 = dt^2 - e^{\mu}\left(dr^2 + r^2 d\Theta^2 + r^2 \sin^2\Theta d\phi^2\right) - e^{\delta} dy^2 \tag{6.3.1}$$

where $\mu = \mu(t)$ and $\delta = \delta(t)$ are cosmic scale factors.

Now, using co-moving co-ordinates, the surviving field equations are obtained as follows

$$-\frac{3}{4}\left(\dot{\mu}^2 + \dot{\mu}\dot{\delta}\right) = (8\pi + 3\lambda)\,\rho - 2p\lambda \tag{6.3.2}$$

$$\ddot{\mu} + \frac{3}{4}\dot{\mu}^2 + \frac{\ddot{\delta}}{2} + \frac{\dot{\delta}^2}{4} + \frac{\dot{\mu}\dot{\delta}}{2} = (8\pi + 4\lambda)\, p - \lambda\rho \tag{6.3.3}$$

$$\frac{3}{2}\left(\ddot{\mu}+\dot{\mu}^2\right)=(8\pi+4\lambda)\,p-\lambda\rho \tag{6.3.4}$$

where an overhead dot indicates differentiation w.r.t. t.

From Eqs. (6.3.12) and (6.3.13), the expressions for the cosmic scale factors are obtained as

$$\mu=a-3\log\left(2\left(k-3t\right)\right) \tag{6.3.5}$$

$$\delta=b-3\log\left(2\left(k-3t\right)\right) \tag{6.3.6}$$

where a, b, k are arbitrary constants.

Now, the expressions for spatial volume v, scalar expansion θ, Hubble parameter H, deceleration parameter q, shear scalar σ^2 and anisotropic parameter A_h are obtained as follows.

$$V=e^{\frac{3a+b}{2}}\left(2\left(k-3t\right)\right)^{-6} \tag{6.3.7}$$

$$\theta=18\left(k-3t\right)^{-1} \tag{6.3.8}$$

$$H=\frac{9}{2}\left(k-3t\right)^{-1} \tag{6.3.9}$$

$$q=-1.7 \tag{6.3.10}$$

$$\sigma^2=\left(\frac{27-2\left(k-3t\right)}{18\left(k-3t\right)}\right)^2 \tag{6.3.11}$$

$$A_h=0 \tag{6.3.12}$$

From Eqs. (6.3.11) and (6.3.13), the expressions for the pressure p and the energy density ρ of the model universe are respectively obtained as

$$p=\frac{243\lambda-324\left(8\pi+3\lambda\right)}{4\left(k-3t\right)^2\left(-5\lambda^2-32\pi^2-28\pi\lambda\right)} \tag{6.3.13}$$

$$\rho=\frac{(8\pi+4\lambda)}{\lambda}\left(\frac{243\lambda-324\left(8\pi+3\lambda\right)}{4\left(k-3t\right)^2\left(-5\lambda^2-32\pi^2-28\pi\lambda\right)}\right)-\frac{162}{\lambda\left(k-3t\right)^2} \tag{6.3.14}$$

The expression for the scalar curvature R is obtained as

$$R = \frac{513}{(k-3t)^2} \tag{6.3.15}$$

6.4 Discussions

For convenience sake and to obtain realistic results, specific values of the arbitrary constants involved are chosen i.e., $a = b = 1, k = 15$ and $\lambda = -5.06911$ and -12.5856. The graphs of the cosmological parameters w.r.t. cosmic time t are presented with the detailed discussion in view of the latest observations.

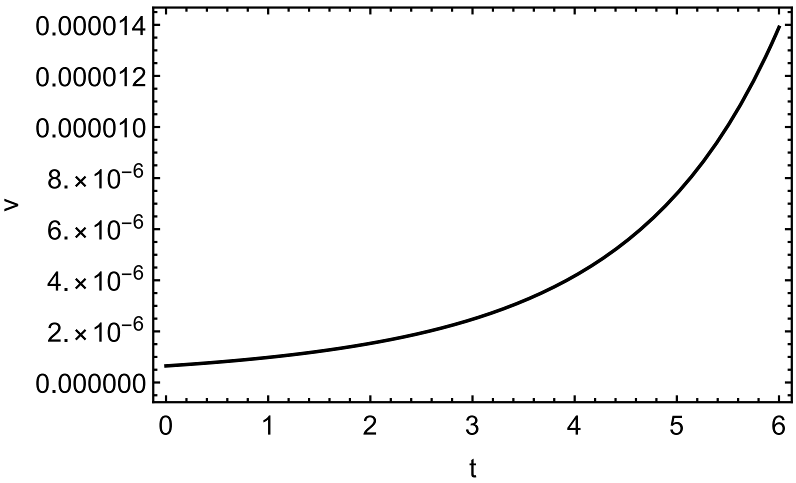

Figure 6.1: Variation of the spatial volume V with t when $a = b = 1, k = 15$.

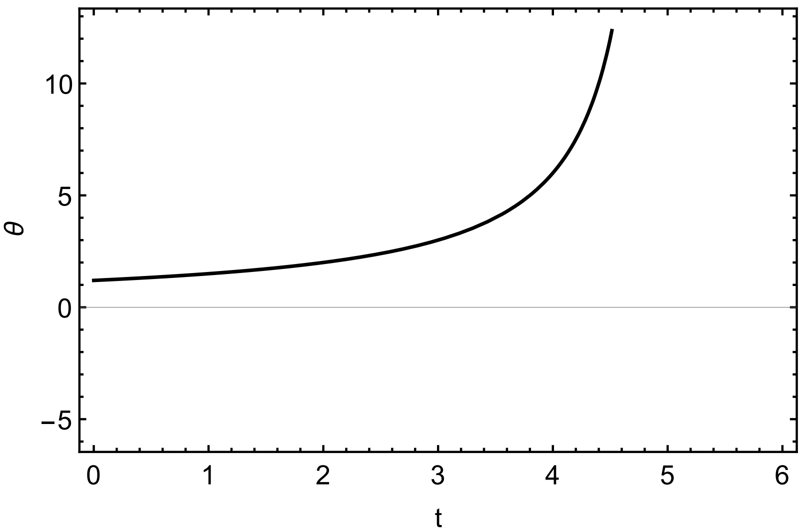

Figure 6.2: Variation of the expansion scalar θ with t when $a = b = 1, k = 15$.

Figs. 6.1 and 6.2 can be regarded as the perfect evidences for the present spatial expansion at an expedited rate. When $t \rightarrow 0$, V and other related parameters are constants ($\neq 0$), implying that the model universe doesn't evolve from an initial singularity.

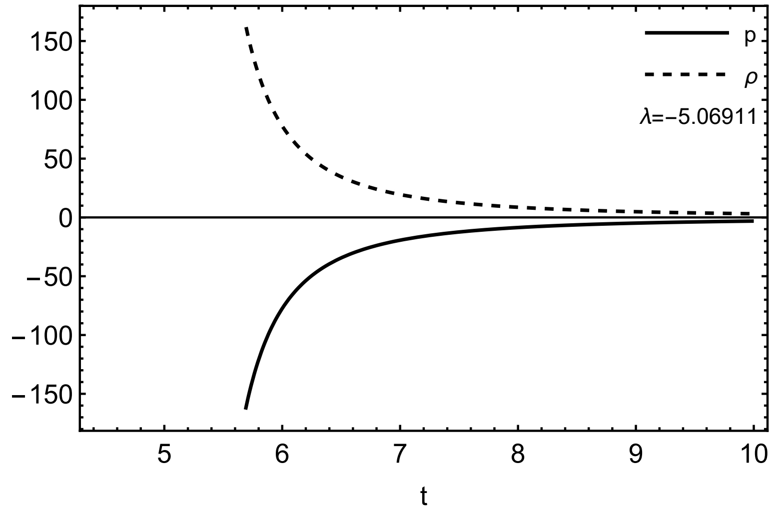

Figure 6.3: Variation of the pressure p and energy density ρ with t when $\lambda = -5.06911$.

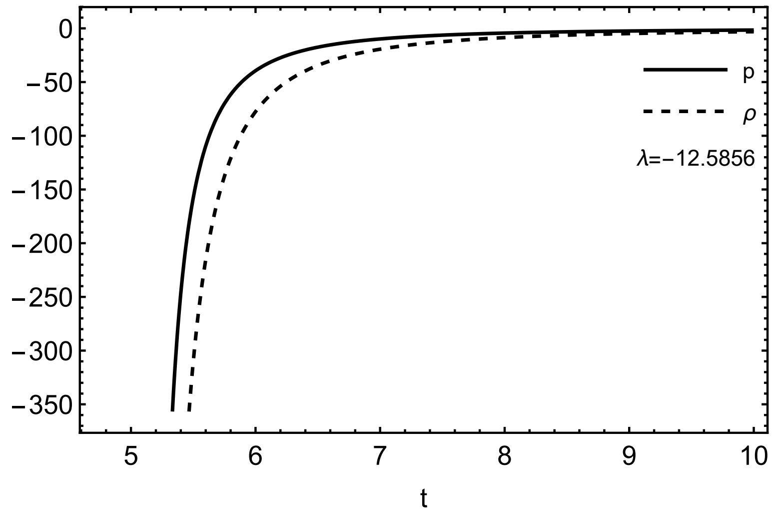

Figure 6.4: Variation of the pressure p and energy density ρ with t when $\lambda = -12.5856$.

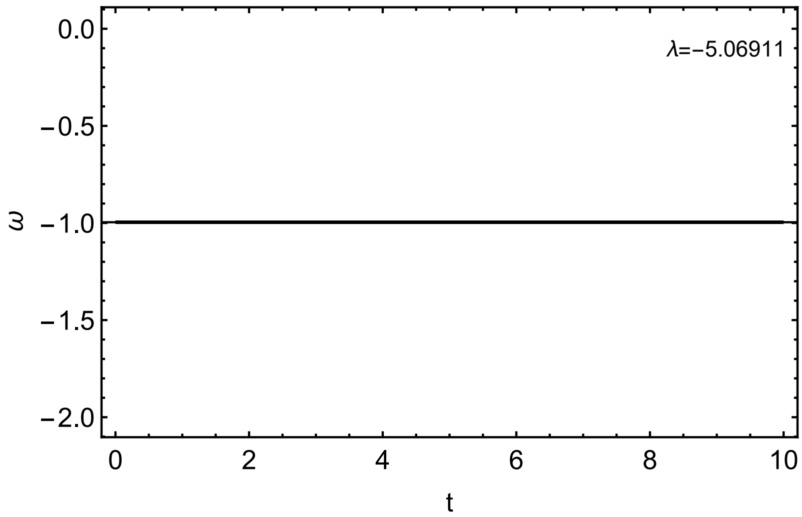

Figure 6.5: Variation of the DE EoS ω with t when $a = b = 1, k = 15, \lambda = -5.06911$.

Fig. 6.3 shows the variation of the pressure p and energy density ρ when $a = b = 1, k = 15, \lambda = -5.06911$. From the graph, it is obvious that the model is experiencing accelerated expansion with negative p and positive ρ. Here, the model evolves with a large ρ and it converges to become constant at late times. This phenomenon is a clear indication of the presence of DE as the present cosmology believes that the late time accelerating universe is due to the dominant and slowly varying or constant DE with negative pressure and positive energy density (Araujo 2005; Agrawal et al. 2018; Carroll 2001a; Law 2020; Ray et al. 2013; Singh & Singh 2019b; Straumann 2007; Wu & Yu 2005). In order to predict the nature, the graph of $\omega = \frac{p}{\rho}$ which is the DE EoS parameter is plotted in Fig. 6.5 which shows that $\omega = -1$. Hence, we can sum up that the $f(R,T)$ gravity model we have constructed turns out to be a DE model, DE in the form of VE or the CC. Fig. 6.4 shows the variation of the pressure p and energy density ρ when $a = b = 1, k = 15, \lambda = -12.5856$. In this case, the model undergoes expansion at an expedited rate with p and ρ both negative. This negative p can be regarded as the indication of the presence of DE. In this scenario too, we can predict that DE in the form of VE is dominating the model, as predicted by Carroll (2001a), NED is possible only if the DE is in the form of VE. Hence, in both the cases, it is fascinating to see that the constructed $f(R,T)$ gravity theory model behaves as a DE (vacuum energy) model. We have not considered the case when $\lambda > 0$ as it yields positive pressure which is not reliable in the present scenario.

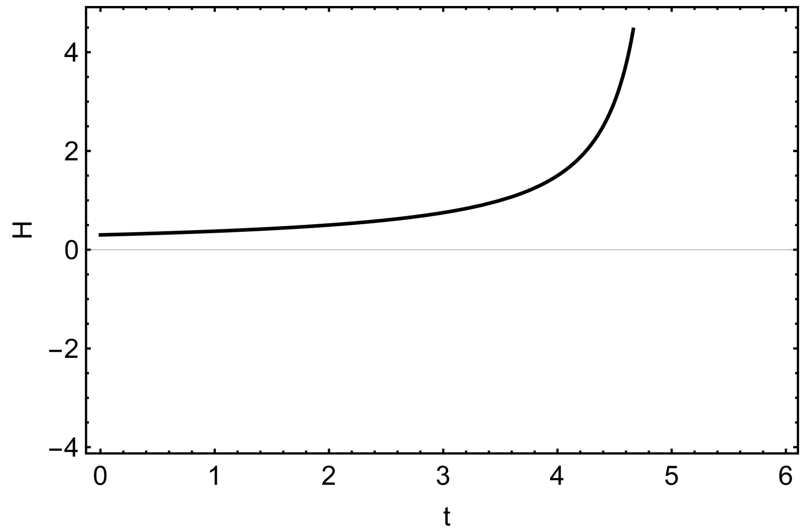

Figure 6.6: Variation of the Hubble parameter H with t when $k = 15$.

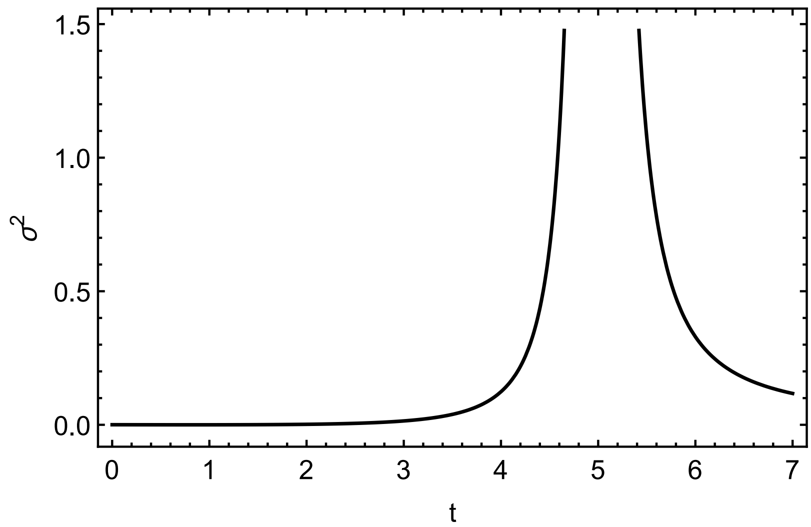

Figure 6.7: Variation of the scalar scalar σ^2 with t when $k = 15$.

Fig. 6.6 shows that the Hubble parameter H appears to remain almost constant in the early evolution so that our universe was in the inflationary epoch experiencing rapid exponential expansion (Crevecoeur 2016). Accelerated expansion can be attained when $-1 < q < 0$ whereas $q < -1$ causes super-exponential expansion (Singh & Bishi 2017). Eq. (6.3.10) shows that $q = -1.7$ indicating that the model universe undergoes super-exponential expansion. It may be noted that in a higher dimensional theory with CC or VE, super-exponential inflation (expansion) can be attained if H increases with t (Polock 1988; Shaft & Wetterich 1985; Wenerich 1985). In our case, H increases after the initial epoch as shown in Fig. 6.6. Shear scalar σ^2 provides us the rate of deformation of the matter flow within the massive cosmos (Ellis & Elst 1999). From Fig. 6.7, we can see that

σ^2 evolves constantly, then diverges after some finite time and again converges to become constant after vanishing for a finite period. From Eq. (6.3.21), the anisotropic parameter $A_h = 0$. From these, we can sum up that initially, the isotropic universe expands with a slow and uniform change of shape, but after some finite time, the change becomes faster. Then, the change slows down and tends to become uniform after expanding without any deformation of the matter flow for a finite time period.

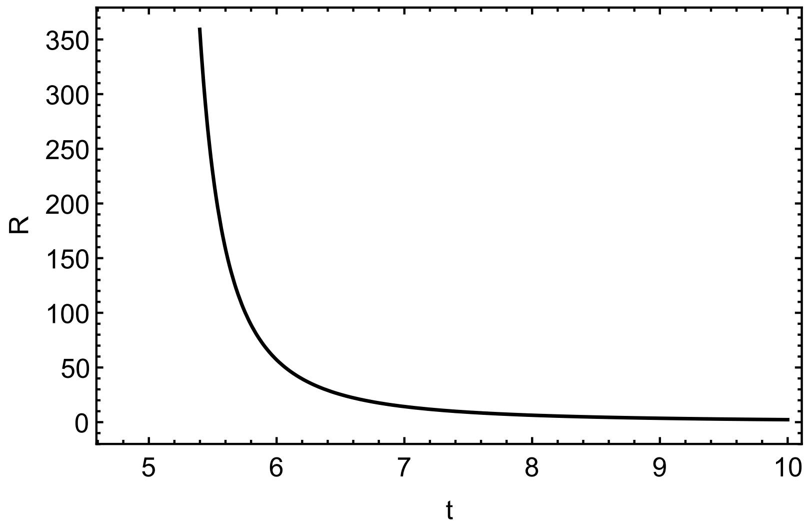

Figure 6.8: Variation of the scalar curvature R with t when $k = 15$.

Fig. 6.8 shows the decreasing nature of the scalar curvature R with cosmic time t. Similar observation can also be seen in the recent studies (Pavlovic & Sossich 2017; Pashitskii & Pentegov 2016). It tends to become constant in the future. At $t = 13.8$ Gyr which align with 13.830 ± 0.037 Gyr, the approximate present age of the universe estimated by the latest Planck 2018 result (Collaboration et al. 2020), the scalar curvature is obtained to approach a constant $R = 0.72$. $R = 0$ corresponds to an exactly flat expanding universe (Bevelacqua 2006; Gueorguiev & Maeder 2020; Kleban & Senatore 2016). However, in the recent years, arguments against the notion of exactly flat universe have been put forwarded by many authors (Javed et al. 2020; Khodadi et al. 2015; Nashed & Hanafy 2014; Valentino et al. 2020). In the present scenario, the universe is assumed to be close to or nearly flat, but not exactly flat (Adler & Overduin 2005; Levin & Freese 1994; Nashed & Hanafy 2014). Additionally, the latest Planck 2018 results (Collaboration et al. 2020) estimating the value of overall density parameter Ω ranging close to unity can also regarded as an evidence for nearly flat universe, as for an exactly flat universe, $\Omega = 1$ (Holman 2018; Khodadi et al. 2015; Levin & Freese 1994). Hence, our model obtaining a small and constant $R = 0.73$ is justified. Kim et al. (2002) and Tiwari (2016), in their studies, assert that R is constant for de-Sitter phase. So, the reason for R approaching a constant can be regarded as an indication for the model approaching the de-Sitter phase dominated by VE or CC in the finite time future avoiding singularity. According to Falls et al. (2018), accelerated expansion will lead R to approach a nearly constant value so that the universe behaves in the same manner as a de-Sitter universe in the future. Many other authors have also asserted that the expanding universe will end at the de-Sitter phase dominated by VE, avoiding singularity (Basilakos et al. 2018; Carneiro 2006; Dymnikova 2019; Dyson et al. 2002; Krauss and Starkman 2000; Markkanen 2018; Nojiri and Odintsov (2004); Sakharov 1966; Starobinsky 2000; Zilioti et al. 2018).

6.5 Conclusions

In this chapter, within the framework of $f(R,T)$ gravity theory, we have analysed a SS space-time in 5D setting. The variation of the pressure p and energy density ρ with cosmic time t are analysed when $\lambda = -5.06911$ and -12.5856. In both the cases, it is fascinating to see that our $f(R,T)$ gravity theory model behaves as a DE (vacuum energy) model. The model is isotropic and free from an initial singularity. The model expands with a slow and uniform change of shape, but after some finite time, the change becomes faster. Then, the change slows down and tends to become uniform after expanding without any deformation of the matter flow for a finite time period. The scalar curvature R is decreasing with time which is consistent with the recent studies. The model is predicted to approach the de-Sitter phase dominated by VE or CC in the finite time future avoiding singularity. We have constructed a model where $f(R,T)$ gravity theory itself behaves as a DE (VE) model; nonetheless, the work we have put forward is just a toy model. The model needs further deep study considering all the observational findings, which will be our upcoming work.