Chapter 7

Scale covariant theory as a dark energy model

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7.1 Introduction

Since its profound discovery in 1998, dark energy (DE) (Riess et al. 1998; Perlmutter et al. 1999) has become a topic of paramount importance in the field cosmology. This ambiguous dark entity is the leading factor in the present expansion of the universe at an increasing rate. In fact, DE has earned the reputation of being one of the most mysterious components of the universe. Cosmologists all over the map have put tremendous scientific efforts into figuring out its root and enigmatic nature, but are still in the quest for the right answers. Few of such noteworthy recent efforts are mentioned below.

An $f(R,T)$ gravity model is proposed as a DE source in the study of Singh & Singh (2021b). They further study a 5D cosmological model to find a source of DE (Singh & Singh 2021a). A cosmological model is proposed by Neiser (2020) so as to find the root of DE. Paul & Sengupta (2020) discuss the generalized phenomenological models of DE. Capolupo (2018) predicts that vacuum condensate may lead the way to the source. Wang et al. (2018) present the evolution of this dark entity considering the recent findings. Collaboration *et al.* (2016) present the future of DE beyond the bound of cosmological aspects. Chan (2015a) claims that particles with imaginary energy density might give us a clue to the origin of the dark component. Lastly, a physical mechanism is presented as one of the origins by Gontijo (2012).

Cosmologists all over the globe have performed numerous theoretical and practical attempts to obtain hints as to exactly predict the hidden physics behind the late time expansion. Two well appreciated theoretical approaches have been brought to light to serve the purpose of illustrating the phenomenon. First, viable candidates of DE are developed. Second, modifying ETG. Aside from these two, several other cosmologists have subsequently proposed additional intriguing theories that explain the expanding phenomena satisfactorily. Narain & Li (2018) assert that an Ultraviolet Complete Theory causes the expanding phenomenon. Berezhiani (2017) explains the expansion by dark matter-baryon interactions in the absence of DE. Lastly, Gorji (2016) explains the mystic phenomenon by the infra red corrections.

To classify DE to specific categories, the equation of state (EoS) parameter ω is considered as a good choice. $\omega = -1$ represents the cosmological constant or vacuum energy. Phantom energy has $\omega < -1$ whereas the range $-1 < \omega < \frac{-1}{3}$ signifies quintessence. The latest Planck 2018 result (Collaboration et al. 2020) predicts its possible bound to be $\omega = -1.03 \pm 0.03$, which is an indication that the form of DE in the present universe is highly likely to be phantom energy.

Over the years, many cosmologists have successfully introduced many well appreciated optimized modifications of ETG which align with the current cosmological trends quite convincingly (Weyl 1918; Nojiri and Odintsov 2014; Harko et al. 2011; Chiba et al. 2007; Brans & Dicke 1961; Scheibe 1952). Cosmologists and theoretical physicists prefer to opt such modified theories to study the late time accelerating phenomenon as Einstein's General Relativity doesn't provide the accurate explanation of gravity (Sbisa 2014). One of such modifications which has not escaped our attention is the scale covariant theory (SCT) introduced by Canuto et al. (1977a) and Canuto et al. (1977b). They developed the theory by applying scale transformation in order to calculate space-time distances Canuto et al. (1977a). According to them, the generalized Einstein's field equations (EFE) are invariant under scale transformation and they successfully investigated many astrophysical tests with the theory Canuto et al. (1977b). In SCT, the EFE are valid in gravitational units, on the other hand, atomic units are used for physical quantities. The metric tensors associated with these two unit systems are connected by the scale transformation $\overline{g_{ij}} = \varphi^2(x^k)g_{ij}$, where the bar and unbar respectively represent the gravitational units and atomic quantities. φ is a gauge function satisfying $0 < \varphi < \infty$, without possessing any wave equation. Using this transformation, Canuto et al. (1977a, 1977b) transform the usual EFE into

$$R_{ij} - \frac{1}{2}g_{ij}R + f_{ij}(\varphi) = -8\pi G(\varphi)T_{ij} + \Lambda(\varphi)\, g_{ij} \tag{7.1.1}$$

such that

$$\varphi^2 f_{ij} = 2\varphi\varphi_{i;j} - 4\varphi_{,i}\varphi_{,j} - g_{ij}\left(\varphi\varphi^{,k}_{;k} - \varphi^{,k}\varphi_{,k}\right) \tag{7.1.2}$$

where all the symbols have their usual meanings.

According to Katore et al. (2014), SCT is one of the best alternatives to ETG. This theory permits the variation of the gravitational constant G (Wesson 1980; Will 1984). The ambiguous DE and the mysterious expanding phenomenon have been successfully studied by many authors within the framework of SCT. In the recent study by Singh et al. (2020), it is asserted that SCT might be one of the probable contributors to the late time accelerated expanding phenomenon. Zeyauddin et al. (2020) present a cosmological model that decelerates during the initial phase and accelerates during the present evolution. Ram et al. (2015) present a forever expanding DE dominated universe which tends to de-sitter universe in the future. Naidu et al. (2015) present an DE model with early inflation and late time acceleration. Katore et al. (2014) investigates three Bianchi type space-times involving magnetized anisotropic DE. Singh & Sharma (2014b) investigate a Bianchi type-II space-time with variable ω. Zeyauddin & Saha (2013) study an endlessly expanding and shearing model with an initial singularity. Reddy et al. (2012) construct an expanding DE model, which doesn't evolve from a singularity in the initial epoch. In the present scenario, SCT paired with DE is considered to align with cosmological observations.

Cosmological models based on higher dimension have become a preferred choice among many authors. The concept of higher dimension in cosmology was put forward by Kaluza (1921) and (Klein 1926). Banik & Bhuyan (2017) assert that models based on higher dimension can be considered as means to explain the expanding phenomenon of the universe. An explanation in support of the extra dimension can be seen in the work of Marciano (1984). Most probably, the unknown fifth dimension might correspond to the two ambiguous and unseen dark entities - DE and DM (Chakraborty & Debnath 2010). Many well known authors have put forward noteworthy discussions on higher dimension during the past few decades (Astefanesei et al. 2020; Ghaffarnejad et al. 2020; Montefalcone et al. 2020; Saha & Ghose 2020; Demirel 2019; Bahrehbakhsh 2018; Shinkai & Torii 2015; Samanta et al. 2014; Oli 2014; Singh & Desikan 1997).

Taking into consideration the above noteworthy related studies, we try to find out if SCT itself can behave as a DE model, within the framework of a 5D spherically symmetric (SS) space-time. In this chapter, we present an in-depth discussion on every cosmological parameter obtained. We estimate the variation of gravitational constant G. After the introduction, in Sect. 7.2, we present the formulation of the problem with solutions to the parameters. In Sect. 7.3, the solutions are discussed with graphical representations. Lastly, to sum up the observations, a concluding note is provided in Sect. 7.4.

7.2 Problem formulation with solutions

We consider a SS metric in 5D of following the form (Samanta & Dhal 2013)

$$ds^2 = dt^2 - e^{\mu}\left(dr^2 + r^2 d\Theta^2 + r^2 \sin^2\Theta d\phi^2\right) - e^{\delta} dy^2 \tag{7.2.1}$$

where $\mu = \mu(t)$ and $\delta = \delta(t)$ are cosmic scale factors.

The energy-momentum tensor is given by

$$T_{ij} = (\rho + p) u_i u_j - p g_{ij} \tag{7.2.2}$$

where p and ρ represent pressure and energy density, whereas u^i satisfies $u^i u_i = 1$, in co-moving co-ordinate system.

Now, the surviving field equations from Eqs. (7.1.1) and (7.1.2) for the spherically symmetric metric are obtained as

$$\frac{3}{4}\left(\dot{\mu}^2 + \dot{\mu}\dot{\delta}\right) - \frac{\dot{\varphi}}{\varphi} + 3\left(\frac{\dot{\varphi}}{\varphi}\right)^2 + \frac{\dot{\varphi}}{\varphi}\left(\frac{3\dot{\mu} + \dot{\delta}}{2}\right) = 8\pi G(\varphi)\rho \tag{7.2.3}$$

$$\ddot{\mu} + \frac{3}{4}\dot{\mu}^2 + \frac{\ddot{\delta}}{2} + \frac{\dot{\delta}^2}{4} + \frac{\dot{\mu}\dot{\delta}}{2} + \frac{\ddot{\varphi}}{\varphi} + \frac{\dot{\varphi}}{\varphi}\left(\frac{\dot{\mu} + \dot{\delta}}{2}\right) - \left(\frac{\dot{\varphi}}{\varphi}\right)^2 = -8\pi G(\varphi) p \tag{7.2.4}$$

$$\frac{3}{2}\left(\ddot{\mu} + \dot{\mu}^2\right) + \frac{\ddot{\varphi}}{\varphi} - \left(\frac{\dot{\varphi}}{\varphi}\right)^2 + \frac{\dot{\varphi}}{\varphi}\left(\frac{3\dot{\mu} - \dot{\delta}}{2}\right) = -8\pi G(\varphi) p \tag{7.2.5}$$

where a superscribed dot represents derivative w.r.t. t.

From Eqs. (7.2.6) and (7.2.7), we have

$$\mu = x - \frac{2}{3}\log(z - t) \tag{7.2.6}$$

$$\delta = y - \frac{2}{3}\log(z - t) \tag{7.2.7}$$

where x, y and z are arbitrary constants.

The expressions for the parameters are obtained as follows.

Spatial volume:

$$V = e^{\frac{3\mu+\delta}{2}} = e^{\frac{3x+y}{2}} (z-t)^{-\frac{4}{3}} \tag{7.2.8}$$

Scale factor:

$$a(t) = V^{\frac{1}{4}} = e^{\frac{3x+y}{8}} (z-t)^{-\frac{1}{3}} \tag{7.2.9}$$

Scalar expansion:

$$\theta = u^i_{;j} = \frac{3\dot{\mu}}{2} + \frac{\dot{\delta}}{2} = \frac{4}{3}(z-t)^{-1} \tag{7.2.10}$$

Hubble parameter:

$$H = \frac{\theta}{4} = \frac{1}{3}(z-t)^{-1} \tag{7.2.11}$$

The most recent Planck 2018 results (Collaboration et al. 2020) estimates the value of the Hubble parameter to be $H = 67.4 \pm 0.5\text{kms}^{-1}\ \text{Mpc}^{-1}$.

With $\Delta H_i = H_i - H, (i = 1, 2, 3, 4)$ representing the directional Hubble parameters, the anisotropic parameter A_h is given by

$$A_h = \frac{1}{4}\sum_{i=1}^{4}\left(\frac{\Delta H_i}{H}\right)^2 = 0 \tag{7.2.12}$$

Shear Scalar:

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{1}{2}\sum_{i=1}^{4}\left(H_i^2 - 4H\right) = \frac{2}{9}\left(1 - \frac{1}{z-t}\right)^2 \tag{7.2.13}$$

In our study, φ is time dependent and we consider the well appreciated relation $\varphi = m\,(a(t))^n$ (Zeyauddin et al. 2020; Zeyauddin & Saha 2013; Singh & Sharma 2014b), where m and n are arbitrary constants so that from Eq. (7.2.11), we have

$$\varphi = me^{\frac{n(3x+y)}{8}} (z-t)^{-\frac{n}{3}} \tag{7.2.14}$$

Using Eqs. (7.2.8), (7.2.9) and (7.2.16) in Eqs. (7.2.5) and (7.2.7), we have

$$\rho = \frac{6 + 3n^2 + n\,(4 - 3(z-t))}{72\pi G(\varphi)(z-t)^2} \tag{7.2.15}$$

$$p = -\frac{5(3+n)}{72\pi G(\varphi)(z-t)^2} \tag{7.2.16}$$

Dirac (1937) asserts that G decreases with cosmic time t. The study on the accelerating universe with decreasing G can be seen in the studies of Hova (2020) and Tiwari et al. (2010) whereas increasing G is presented in the studies of Oli (2014), Massa (1995) and Levit (1980). Models with variable G are also investigated by many other authos (Sahni & Shtanov 2014; Kordi 2009; Srivastava 2008; Debnath & Paul 2006; Grigorian & Saharian

1990; Narlikar 1983). Beesham (1986) assumes G to be $G \propto t^{\alpha}$, where α is a constant, Sistero (1991) considers $G \propto (a(t))^{\alpha}$ whereas Ram et al. (2009) assert $G = \varepsilon t$ where ε is a proportionality constant. In our study, we consider G in the following form as suggested by Dirac (1937).

$$G = ct^{-1} \tag{7.2.17}$$

where c is a proportionality constant.

Using Eq. (7.2.19) in Eqs. (7.2.17) and (7.2.18), we obtain the expressions for ρ and p as functions of t only as follows

$$\rho = \frac{6 + 3n^2 + n\,(4 - 3(z-t))}{72\pi c t^{-1}(z-t)^2} \tag{7.2.18}$$

$$p = -\frac{5(3+n)}{72\pi c t^{-1}(z-t)^2} \tag{7.2.19}$$

7.3 Discussion

In this section, for simplicity purposes and reasonable outcomes, we opt to choose $c = m = x = y = 1, n = 0.2$ and $z = 13.8049$ and the parameters are plotted with respect to the cosmic time t.

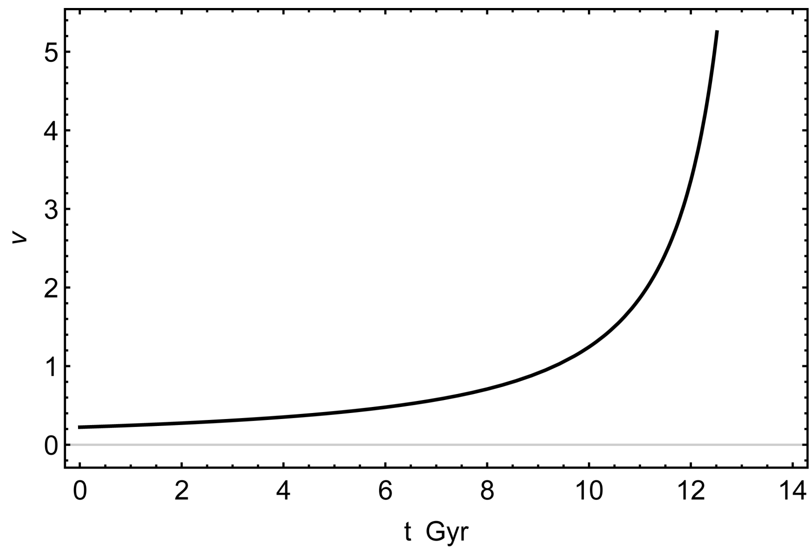

Figure 7.1: Volume V increasing with t when $x = y = 1, z = 13.8049$.

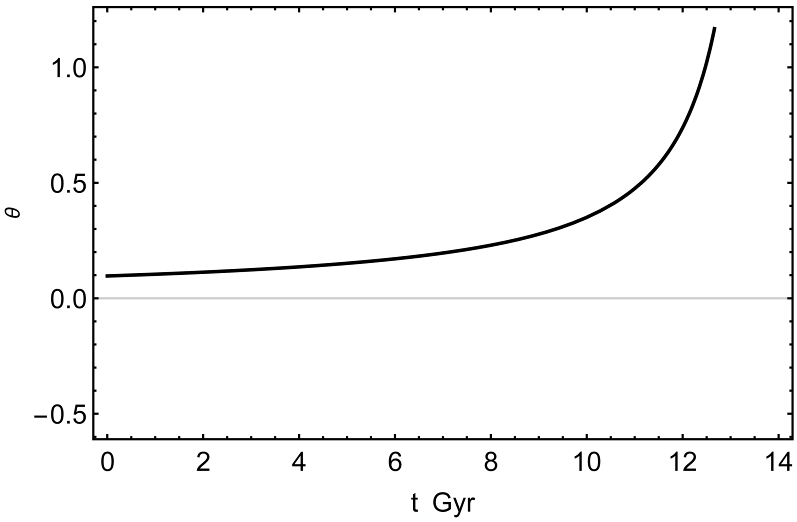

Figure 7.2: Expansion scalar θ increasing with t when $z = 13.8049$.

It can be seen from Figs. 7.1 and 7.2 respectively that V and θ diverge with t, which favour the expansion of the universe at an increasing rate. Above all, V is constant ($\neq 0$) at $t = 0$. Hence, it won't be wrong to conclude that the model doesn't evolve from a singularity.

From Fig. 7.3, it can be seen that throughout the evolution, the graph of p lies within the negative plane. So, the model undergoes accelerated spatial expansion with negative pressure. This is the indication that our SCT model is in fact a DE dominated model. From Fig. 7.4, we can witness the increasing nature of ρ during the course of evolution. To precisely understand the nature of the DE, in Fig. 7.5, we have plotted the graph of $\omega = \frac{p}{\rho}$ which is the DE EoS parameter showing that $\omega < -1$. From these, we can conclude that the model is a phantom energy dominated model which agrees with the present observation. It may be noted that phantom energy is also characterised by the increasing positive energy density with cosmic time (Ram et al. 2009; Baushev 2010; Caldwell et al. 2003). Above all, in Fig. 7.5, we can observe that during the course of evolution, ω tends very close to -1 in the future which agrees with observations of Amirhashchi (2017) and Carroll et al. (2003) asserting that the value of ω in phantom energy model should reduce to -1 in the far future so that the dominating DE will be transformed to VE or the CC. This will make sure the model universe bypass the future finite time big rip singularity thereby, ultimately, leading to the de-Sitter phase. The concept of de-Sitter phase avoiding future singularity is also presented by many other authors (Dymnikova 2019; Sakharov 1966; Gliner 1966).

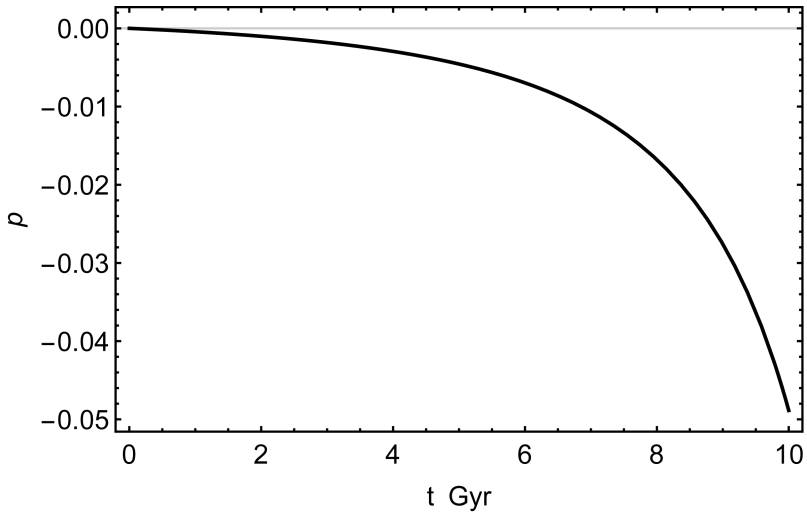

Figure 7.3: Pressure p ranging in the negative plane when $c = 1, n = 0.2, z = 13.8049$.

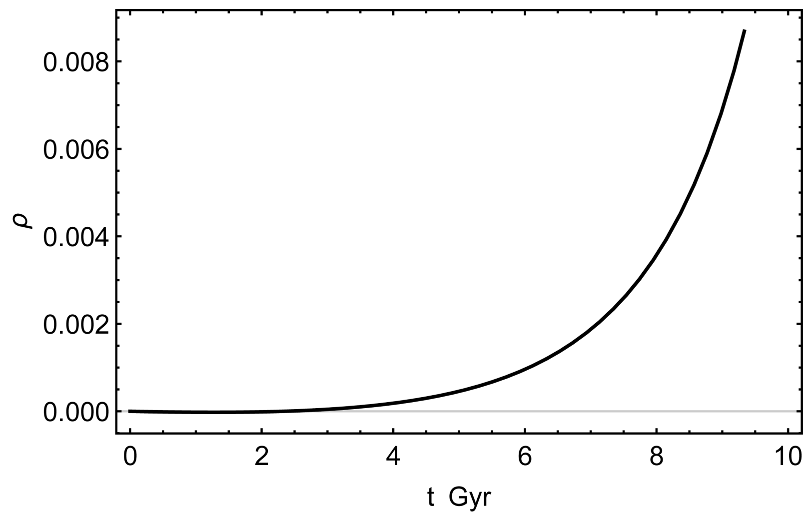

Figure 7.4: Energy density ρ increasing with t when $c = 1, n = 0.2, z = 13.8049$.

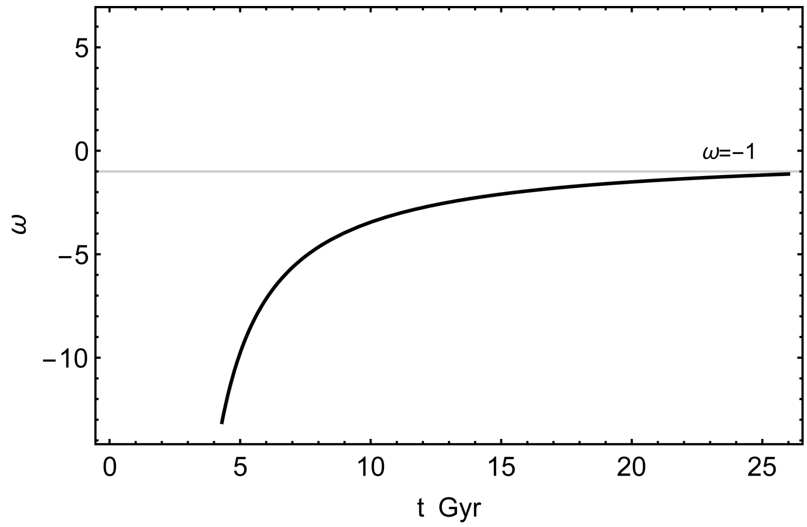

Figure 7.5: EoS parameter ω with t when $n = 0.2$ showing that it tends very close to -1 in the future.

Fig. 7.6 shows that σ^2 decreases slowly in the early state of evolution and then, it starts to decrease with a greater extent and finally, it tends to diverge. σ^2 estimates the rate of distortion of the matter flow (Ellis & Elst 1999). So, we can conclude that the model expands with a steady and consistent change of structure in the early evolution and then, the change become more steady and consistent and finally, the change tends to become faster at late times. From Fig. 7.7, we can see the decreasing nature of G which is supported by the observation of Dirac (1937). Similar plots of decreasing G with accelerating universe can also be seen in the investigation of Hossain et al. (2017) and Tiwari et al. (2010). From Eq. (7.2.14), $A_h = 0$ so that the constructed model in isotropic throughout.

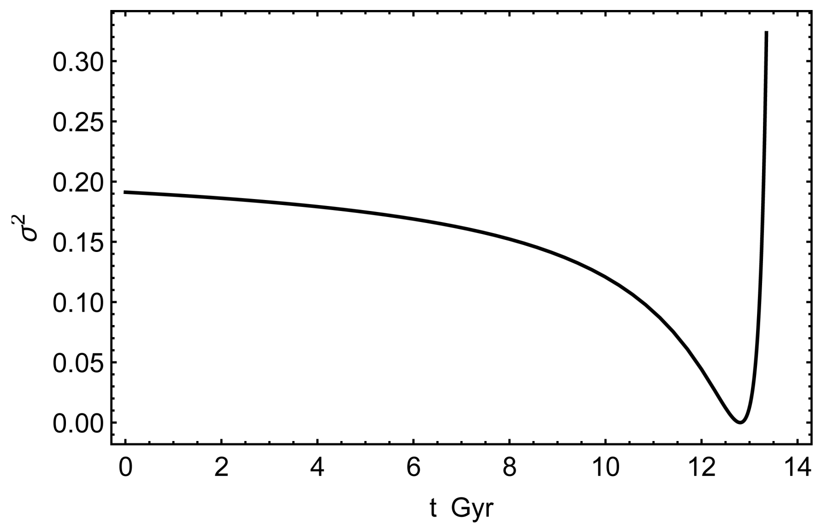

Figure 7.6: Shear scalar σ^2 with t when $z = 13.8049$.

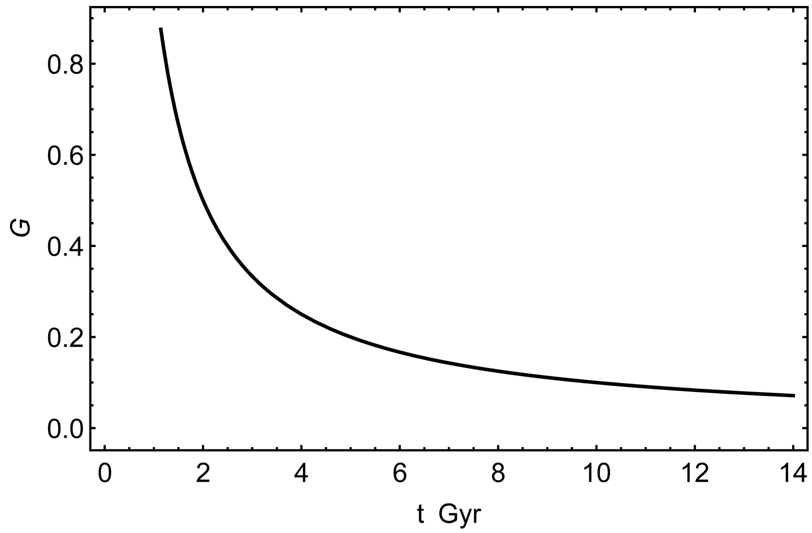

Figure 7.7: Gravitational constant G decreasing with t when $c = 1$.

$\dot{G}/G$ **in** $\mathbf{yr}^{-1}$	**Reference**
-7×10^{-11}	Kordi (2009)
-7.4×10^{-11}, 10^{-14}—10^{-11}	Steinhardt & Wesley (2010)
$-10^{-11} \leq \frac{\dot{G}}{G} \leq 0$	Gaztanaga et al. (2001)
$-2.3 \times 10^{-11} < \frac{\dot{G}}{G} < +0.3 \times 10^{-11}$	Loren-Aguilar et al. (2003)
-5×10^{-11}, -1×10^{-11}, -1×10^{-12}	Althaus (2011)
$-2.50 \times 10^{-10} \leq \lvert\frac{\dot{G}}{G}\rvert < 0$	Benvenuto et al. (2004)
$\lvert\frac{\dot{G}}{G}\rvert \leq 4.1 \times 10^{-10}$	Biesiada & Malec (2004)
$\lvert\frac{\dot{G}}{G}\rvert = 2 \times 10^{-10}$	Sahoo & Singh (2003)
$0.7^{+3.8}_{-4.3} \times 10^{-12}$	Alvey et al. (2020)
$\lvert\frac{\dot{G}}{G}\rvert \leq 3 \times 10^{-12}, 1.5 \times 10^{-12}$	Zhao et al. (2018)
$(-0.7 \pm 3.8) \times 10^{-13}$	Hofmann et al. (2010)
-2.8×10^{-13}	Li (2018)
$(7.1 \pm 7.6) \times 10^{-14}$	Hofmann & Muller (2018)

Table 7.1: Estimated values of variation of G during the past few decades.

Since the prediction of the decreasing nature of the gravitational constant G with time (Dirac 1937), a number of authors have put forward different values of the variation of G i.e., $\frac{\dot{G}}{G}$ with convincing arguments and evidences in support, some of which are presented in Table 7.1. Banerjee & Pavon (2001) predict that $\frac{\dot{G}}{G}$ is safely below 4×10^{-10} yr^{-1}. G changes with a fraction of $\frac{\dot{G}}{G}$ per year (Kordi 2009). In our study, with the current age of the universe to be 13.8 Gyr, i.e. 13.8×10^9 years (Collaboration et al. 2020), the variation of G is measured to be $\frac{\dot{G}}{G} = -7.2 \times 10^{-11}$ yr^{-1}. Additionally, at t=13.8 Gyr, from Eq. (7.2.13), we obtain $H = 68$ which align with the result of the most recent Planck 2018 results (Collaboration et al. 2020).

7.4 Conclusions

In this chapter, we have studied SCT within the framework of a SS space-time in 5D. For simplicity purposes and reasonable outcomes, we opt to choose $c = m = x = y = 1, n = 0.2$ and $z = 13.8049$. The universe is isotropic. The model behaves as a phantom energy dominated model, which doesn't evolve from a singularity and tends to the de-Sitter phase avoiding finite time future singularity (big rip). During the early evolution, the universe expands with a steady and consistent change of structure and then, the change become more steady and consistent and finally, the change tends to become faster at late times. The value of G is predicted to be decreasing with a variation of $\frac{\dot{G}}{G} = -7.2 \times 10^{-11}$ yr^{-1}. At t=13.8 Gyr, we obtain $H = 68$ which align with the result of the most recent Planck 2018 results. In the study, an SCT model, which acts as a phantom energy model is presented. In other words, the SCT model acts as a DE source. However, this constructed model is a toy model which requires more in depth analysis taking into account all the latest cosmological findings, which we are planning to work on.