Chapter 8

Dark energy on higher dimensional spherically symmetric Brans-Dicke universe

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8.1 Introduction

Topics on the accelerated expansion of the universe have attracted wide attention from many theoretical physicists and cosmologists around the world energizing them for further investigations and many clear and convincing evidence have been produced in support. This accelerated expansion is explained by the so-called dark energy (DE) (Riess et al. 1998; Perlmutter et al. 1999), a completely mysterious form of energy with an exotic property of negative pressure which generates a negative gravity that causes the acceleration by emitting a strong repulsive force resulting in an anti-gravity effect. This uniformly distributed mystical component dominating the universe is slowly varying with time and space (Carroll 2001a, 2001b; Peebles & Ratra 2003). Since its discovery, it has become one of the most discussed topics among the cosmological society and great scientific efforts have been invested in order to explore its bizarre nature, properties, future characteristics and applications to modern cosmology. Frampton & Takahashi (2003) obtain that the universe might be DE dominant or free from DE in future time. Steinhardt et al. (2003) studies the quintessential introduction to DE. Sahoo & Mishra (2014b) investigate wet dark fluid, a DE candidate. Sahoo & Mishra (2014a) further study an axially symmetric cosmological model in the presence of anisotropic DE in which the solutions obtained could give us an appropriate description of the evolution of the universe. DE Survey Collaboration (Collaboration et al. 2016) describes the future prospect and discovery potential of the Dark Energy Survey (DES) beyond cosmological studies. Singh et al. (2017a) examine if DE could neutralize the global warming. Singh et al. (2017b) put forward interesting explanations to show that Lyra's manifold could be the hidden source of DE. Nair & Jhingan

(2013) examine whether DE is evolving or not. Abbott et al. (2018) provide us the first public data release of the DES. Risaliti & Lusso (2018) observe that the DE density is increasing with time. According to Cooper (2018), there are cosmologists who doubt if DE is behind the increasing expansion of the universe and the author analysed the arguments. Lastly, Calder & Lahav (2008) hunt down the origin of DE as far back as Newton and Hooke and presented a comprehensive summary of 90 years old history of the cosmological constant.

To understand this dark component as precisely as possible to obtain hints as to exactly predict its nature and properties, cosmologists have opted for analysing the equation of state (EoS) parameter ωwhich is the ratio of the pressure to the density of the DE. In recent years, different authors have calculated different viable limits on the value of ω with strong evidence in support. Knop et al. (2003) calculate two different ranges -1.61< ω <-0.78 and -1.67 < ω < -0.62 on two different situations. Melchiorri (2003) measures a bound of -1.38< <-0.82, whereas according to the most recent Planck 2018 results (Collaboration et al. 2020), the value of ω is measured to be ω = -1.03±0.03.

We can describe the accelerated expansion of the universe by two approaches: (i) DE approach in which different viable candidates of DE are developed (ii) Modified theories of gravitation approach in which ETG is modified to many optimized forms. Besides these approaches, many authors have put forward other possible ways to explain the late time acceleration of the universe. It is shown that the acceleration of the universe is the result of the back reaction of cosmological perturbations, rather than the effect of a negative pressure DE fluid or a modification of general relativity (Kolb et al. 2006). Gorji (2016) addresses the late-time cosmic acceleration the infrared corrections. An interesting explanation of cosmic acceleration using only dark matter (DM) and ordinary matter can be seen in the study of Berezhiani et al. (2017). An approach is also suggested by Narain & Li (2018) where the late time cosmic acceleration is obtained from an Ultraviolet Complete Theory.

The natural candidate for DE is the cosmological constant or the vacuum energy (VE) with $\omega = -1$. But, VE fails to illustrate many riddles of physics, one of which worth mentioning is the coincidence problem (Zlatev et al. 1999) in which the similar densities, at the present epoch, of the differently evolved DE and DM remains a mystery. Therefore, many other viable candidates of DE have been introduced (Copeland et al. 2006). Cosmologist started to construct models which involve the interaction of these two dark components to explain the small value of Λ (Wetterich 1995, 1988). Afterwards, these constructed models were found applicable to mollify the coincidence problem (Amendola & Valentini

2001; Zimdahl et al. 2001; Zimdahl & Pavon 2004; Cai & Wang 2005). During the last decade, evidence have been put forward which confirm that modified gravity can be presented in terms of interaction of these two dark components in the Einstein frame (Felice & Tsujikawa 2010; He et al. 2011; Zumalacarregui et al. 2013; Kofinas et al. 2016; Cai et al. 2016). This can enable us to broaden the gravitational theory beyond the breadth of general relativity if we can figure out the specific interaction term. Recently, great scientific efforts have been utilized to study the DE-DM interaction, for both theoretical and observational point of view, in the holographic dark energy (HDE) setting (Sadjadi 2007; Sadjadi & Vadood 2008; Setare & Vagenas 2009; Chimento et al. 2013; Kiran et al. 2014; Adhav et al. 2014a, 2014b; Umadevi & Ramesh 2015; Reddy et al. 2016a; Raju et al. 2016). HDE, a consequence of the application of the holographic principle (Wang et al. 2017) to the repulsive dark entity, was introduced by Gerard 't Hooft (Hooft 2009). Interacting models involving this dark holographic entity and matter in spherically symmetric (SS) space-time were studied in (Raju et al. 2016; Reddy et al. 2016). The mysterious nature of these two dark components have arisen many fundamental questions indicating that there are many new physics yet to be uncovered.

In the past few decades, many modified theories of gravitation challenging Einstein's theory have been put forward and these theories succeeded to fit the present cosmological trends in a quite satisfactory way, a handful of which that have not escaped our notice are Weyl's theory (Weyl 1918), Lyra geometry (Scheibe 1952), Brans-Dicke theory (Brans & Dicke 1961), $f(R)$ gravity (Chiba al. 2007), $f(R,T)$ gravity (Harko et al. 2011), Mimetic $F(R)$ gravity (Nojiri & Odintsov 2014) etc. Brans-Dicke theory (BDT) of gravitation has become one of the favourite choices among many cosmological audiences and enormous efforts have been employed to study its modern cosmological aspects (Miyazaki 2000; Kim 2005; El-Nabulsi 2007, 2010; Hrycyna & Szydlowski 2013b; Rani et al. 2018; Cruz & Peracaula 2018; Sadri & Vakili 2018; Brando et al. 2018). In this theory, a metric tensor g_{ij} is introduced along with a scalar filed φ which represents the space-time varying gravitational constant. In Einstein theory, gravity is explained by the lone entity - the space-time metric tensor or, in simple word, geometry. Whereas, in this modified theory, all matters are the reason for the gravitational behaviour of φ, so that, in this logic, it can be treated as a modification from purely geometric to geometric-scalar nature and thus, becoming a part of the family of scalar-tensor theory.

BDT can be of good choice to study DE and the expansion of the universe. It can be considered as the most natural choice of the scalar-tensor generalization of general relativity due to its easiness and is less stringent than general relativity. Above all, the scalar field and the theory itself are of classical origin and can be considered as viable candidates to contribute in the late time evolution of the universe (Kim 2005). BDT or its modified versions are also the possible agents generating the present cosmic acceleration (Banerjee & Pavon 2001a; Brunier et al. 2004). It has also been shown that the theory can potentially generate sufficient acceleration in the matter dominated era (Banerjee & Pavon 2001b). In most of the studies in the BDT setting, it can be seen that the accelerated expansion of the universe needs a very small value of ω, in the order of unity (Das & Mamon 2014) and to be negative. It is shown that if the Brans-Dicke scalar field interacts with the DM, a generalized BDT may cause the acceleration of the universe even with a high value of ω (Das & Banerjee 2006). Interestingly, Joyce et al. (2016) show that the theory is essentially equivalent to a DE model. At present, both BDT and general relativity are generally held to be in agreement with observation.

Sadjadi (2007) studies a spatially homogeneous and anisotropic Bianchi type-V universe filled with minimally interacting fields of HDE and matter obtaining a universe which decelerate initially and accelerate in infinite time. Sadjadi & Vadood (2008) and Setare & Vagenas (2009) examine interacting models in Bianchi type-I and Bianchi type-V universe respectively showing that for suitable choice of interaction between matter and DE, there is no coincidence problem. Chimento et al. (2013) find an interacting models between the two dark components in BDT setting and the authors obtained a model that exhibits early inflation and late time acceleration. Kiran et al. (2014) present an five dimensional interaction model in BDT obtaining an anisotropic universe. In the paper, the authors further mentioned that their universe will become isotropic in finite time due to cosmic re-collapse. Adhav et al. (2014) obtain an interacting model in a 5D spherically universe where the model experiences a transition from decelerated to accelerated phase due to cosmic re-collapse. Reddy et al. (2016a) study DE and matter using a relation between metric potentials and an equation of state representing disordered orientation obtaining the flat ΛCDM model as a particular case.

Inspired by the above studies, in this chapter, the minimal interaction model of the two dark entities has been presented with a 5D SS space-time in BDT of gravitation. Here, we consider some reasonable assumptions in agreement with the present cosmological observations. With particular choices of the constants involved, the values of the overall density parameter and the Hubble's parameter are obtained to be very close to the latest observational values. We obtain a model universe which will be increasing DE dominated. We also obtain that the model universe will face the big crunch singularity in the far future. The chapter has been structured into sections. In Sect. 8.2, the formulation of the problem is presented along with the solutions of the field equations. Related cosmological parameters are also solved in this section. In Sect. 8.3, the graphs of the parameters are plotted and the physical and kinematical aspects of our model in comparison with the present observational findings are discussed. Considering everything, a concluding note is provided in Sect. 8.4.

8.2 Formulation of problem with solutions

For our universe, we consider the 5D spherically symmetric metric (Samanta & Dhal 2013))

$$ds^2 = dt^2 - e^{\alpha}\left(dr^2 + r^2 d\Theta^2 + r^2 \sin^2\Theta d\phi^2\right) - e^{\beta} dy^2 \tag{8.2.1}$$

where μ and δ are cosmic scale factor which are functions of time only.

Here, BD field equations take the form

$$R_{i\,j} - \frac{1}{2} g_{i\,j} R + \omega_{_{BD}} \varphi^{-2}\left(\varphi_{,i}\varphi_{,j} - \frac{1}{2} g_{i\,j}\varphi_{,k}\varphi^{,k}\right) + \varphi^{-1}\left(\varphi_{i\,;j} - g_{i\,j}\varphi^{k}_{;k}\right) = -8\pi\varphi^{-1}\left(T_{i\,j} + S_{i\,j}\right) \tag{8.2.2}$$

where φ is the BD scalar field and $T_{i\,j}$ and $S_{i\,j}$ are respectively the energy momentum tensors for matter and HDE, whereas R is the Ricci scalar and $R_{i\,j}$ is the Ricci tensor.

In our study we define $T_{i\,j}$ and $S_{i\,j}$ as follows

$$T_{i\,j} = \rho_m u_i u_j \tag{8.2.3}$$

$$S_{i\,j} = (\rho_d + p_d)\, u_i u_j - g_{i\,j} p_d \tag{8.2.4}$$

where ρ_m is the energy density of matter whereas ρ_d and p_d are respectively the energy density and the pressure of the HDE.

The wave equation satisfied by the scalar field is written as

$$\varphi^{k}_{;k} = 8\pi\left(3 + 2\omega_{_{BD}}\right)^{-1}(T + S) \tag{8.2.5}$$

The energy conservation equation in its obvious form is given by

$$T^{i\,j}_{;j} + S^{i\,j}_{;j} = 0 \tag{8.2.6}$$

We consider the co-moving co-ordinate system so that the flow vector satisfies the relation

$$g_{\mu\upsilon}u^{\mu}u^{\upsilon}=1 \tag{8.2.7}$$

We obtain the field equations as follows

$$\frac{3}{4}\left(\dot{\mu}^2+\dot{\mu}\dot{\delta}\right)-\frac{\omega_{BD}}{2}\frac{\dot{\varphi}^2}{\varphi^2}+\frac{\dot{\varphi}}{\varphi}\left(\frac{3\dot{\mu}+\dot{\delta}}{2}\right)=8\pi\varphi^{-1}\left(\rho_m+\rho_d\right) \tag{8.2.8}$$

$$\ddot{\mu}+\frac{3}{4}\dot{\mu}^2+\frac{\ddot{\delta}}{2}+\frac{\dot{\delta}^2}{4}+\frac{\dot{\mu}\dot{\delta}}{2}+\frac{\omega_{BD}}{2}\frac{\dot{\varphi}^2}{\varphi^2}+\frac{\ddot{\varphi}}{\varphi}+\frac{\dot{\varphi}}{\varphi}\left(\dot{\mu}+\frac{\dot{\delta}}{2}\right)=-8\pi\varphi^{-1}p_d \tag{8.2.9}$$

$$\frac{3}{2}\left(\ddot{\mu}+\dot{\mu}^2\right)+\frac{\omega_{BD}}{2}\frac{\dot{\varphi}^2}{\varphi^2}+\frac{\ddot{\varphi}}{\varphi}+\frac{3}{2}\frac{\dot{\varphi}}{\varphi}\dot{\mu}=-8\pi\varphi^{-1}p_d \tag{8.2.10}$$

And Eq. (8.2.6) gives

$$\ddot{\varphi}+\dot{\varphi}\left(\frac{3\dot{\mu}+\dot{\delta}}{2}\right)=8\pi\left(3+2\omega_{BD}\right)^{-1}\left(\rho_m+\rho_d-4p_d\right) \tag{8.2.11}$$

where an overhead dot represents differentiation with respect to time t.

Taking ω as the equation of state (EoS) parameter of HDE, we have

$$p_d=\omega\rho_d \tag{8.2.12}$$

Then, the conservation equation takes the form

$$\dot{\rho}_d+(1+\omega)\left(\frac{3\dot{\mu}+\dot{\delta}}{2}\right)\rho_d+\dot{\rho}_m+\rho_m\left(\frac{3\dot{\mu}+\dot{\delta}}{2}\right)=0 \tag{8.2.13}$$

Since the HDE and matter are interacting minimally, both the components will conserve separately. Thus, we can write (Sarkar 2014a, 2014b)

$$\dot{\rho}_m+\rho_m\left(\frac{3\dot{\mu}+\dot{\delta}}{2}\right)=0 \tag{8.2.14}$$

$$\dot{\rho}_d+(1+\omega)\,\rho_d\left(\frac{3\dot{\mu}+\dot{\delta}}{2}\right)=0 \tag{8.2.15}$$

Also, we have

$$\dot{\rho}+(\rho+p)\left(\frac{3\dot{\mu}+\dot{\delta}}{2}\right)=0 \tag{8.2.16}$$

Now, from Eqs. (8.2.9) and (8.2.10), we have

$$\frac{1}{2}\ddot{\mu} + \frac{3}{4}\dot{\mu}^2 - \frac{\ddot{\delta}}{2} - \frac{\dot{\delta}^2}{4} - \frac{\dot{\mu}\dot{\delta}}{2} + \frac{1}{2}\frac{\dot{\varphi}}{\varphi}\left(\dot{\mu} - \dot{\delta}\right) = 0 \tag{8.2.17}$$

From Eq. (8.2.14), we have

$$\rho_m = a_0 e^{-\left(\frac{3\mu+\delta}{2}\right)} \tag{8.2.18}$$

Similarly,

$$\rho_d = b_0 e^{-(1+\omega)\left(\frac{3\mu+\delta}{2}\right)} \tag{8.2.19}$$

where a_0 and b_0 are arbitrary constants.

From Eqs. (8.2.9) and (8.2.10), we get

$$\mu = a_1 - \log\left(c_1 - t\right)^{\frac{2}{3}} \tag{8.2.20}$$

$$\delta = b_1 - \log\left(c_1 - t\right)^{\frac{2}{3}} \tag{8.2.21}$$

where a_1 and b_1 are arbitrary constants.

Thus, from Eqs. (8.2.18)-(8.2.21), we obtain

$$\rho_m = a_0 e^{-\frac{1}{2}(3a_1+b_1)} \left(c_1 - t\right)^{\frac{4}{3}} \tag{8.2.22}$$

$$\rho_d = b_0 e^{-\frac{1}{2}(3a_1+b_1)(1+\omega)} \left(c_1 - t\right)^{\frac{4}{3}(1+\omega)} \tag{8.2.23}$$

Now, using Eqs. (8.2.12), (8.2.20)-(8.2.23) in Eq. (8.2.11), we obtain the expression of the scalar field φ as

$$\varphi = M_0 \left(c_1 - t\right)^{\frac{10}{3}} + N_0 \left(c_1 - t\right)^{\frac{10}{3}+\frac{4}{3}\omega} \tag{8.2.24}$$

where

$$M_0 = \frac{36}{65}\pi \left(3 + 2\omega\right)^{-1} a_0 e^{-\frac{1}{2}(3a_1+b_1)} \tag{8.2.25}$$

$$N_0 = 8\pi b_0 (1-4\omega)(3+2\omega)^{-1}\left(\frac{10}{3}+\frac{4}{3}\omega\right)^{-1}\left(\frac{11}{3}+\frac{4}{3}\omega\right)^{-1} e^{-\frac{1}{2}(1+\omega)(3a_1+b_1)} \quad (8.2.26)$$

From Eqs. (8.2.22) and (8.2.23), we obtain the expression for the energy density ρ as

$$\rho = \rho_m + \rho_d = a_0 e^{-\frac{1}{2}(3a_1+b_1)}(c_1-t)^{\frac{4}{3}} + b_0 e^{-\frac{1}{2}(1+\omega)(3a_1+b_1)}(c_1-t)^{\frac{4}{3}(1+\omega)} \quad (8.2.27)$$

Using Eqs. (8.2.20), (8.2.21) and (8.2.27) in Eq. (8.2.16), we obtain the expression for the pressure as

$$p = \frac{1}{3}a_0 e^{-\frac{1}{2}(3a_1+b_1)}(c_1-t)^{\frac{4}{3}} + \left(\frac{1}{3}+\frac{4}{3}\omega\right) b_0 e^{-\frac{1}{2}(1+\omega)(3a_1+b_1)}(c_1-t)^{\frac{4}{3}\omega+\frac{4}{3}} \quad (8.2.28)$$

From Eqs. (8.2.12) and (8.2.23), we the pressure of the DE is given by

$$p_d = \omega b_0 e^{-\frac{1}{2}(3a_1+b_1)(1+\omega)}(c_1-t)^{\frac{4}{3}(1+\omega)} \quad (8.2.29)$$

Now, at any time $t = t_0$, we can take

$$p = p_d \quad (8.2.30)$$

Therefore, from Eqs. (8.2.28), (8.2.29) and (8.2.30), we get

$$\left(a_0 e^k + b_0 (1+\omega) e^{k(1+\omega)} (c_1 - t_0)^{\frac{4\omega}{3}}\right)(c_1-t_0)^{\frac{1}{3}} = 0 \quad (8.2.31)$$

where $k = -\frac{1}{2}(3a_1+b_1)$

Eq. (8.2.31) will give us the expression for the EoS parameter ω.

Now, we obtain the values of the different cosmological parameters as follows.

Spatial volume:

$$V = e^{\frac{3a_1+b_1}{2}}(c_1-t)^{-\frac{4}{3}} \quad (8.2.32)$$

Scalar expansion:

$$\theta = \frac{4}{3}(c_1-t)^{-1} \quad (8.2.33)$$

Hubble parameter:

$$H = \frac{1}{3}\left(c_1 - t\right)^{-1} \tag{8.2.34}$$

Shear scalar:

$$\sigma^2 = \frac{2}{9}\left(1 - \frac{1}{c_1 - t}\right)^2 \tag{8.2.35}$$

Anisotropic parameter:

$$A_h = 0 \tag{8.2.36}$$

DE density parameter:

$$\Omega_d = \frac{\rho_d}{3H^2} = 3b_0 e^{-\frac{1}{2}(3a_1+b_1)(1+\omega)}\left(c_1 - t\right)^{\frac{2}{3}(5+2\omega)} \tag{8.2.37}$$

Matter density parameter:

$$\Omega_m = \frac{\rho_m}{3H^2} = 3a_0 e^{-\frac{1}{2}(3a_1+b_1)}\left(c_1 - t\right)^{\frac{10}{3}} \tag{8.2.38}$$

Overall density parameter:

$$\Omega = \Omega_d + \Omega_m = 3\left(a_0 e^{-\frac{1}{2}(3a_1+b_1)} + b_0\left(c_1 - t\right)^{\frac{4\omega}{3}} e^{-\frac{1}{2}(3a_1+b_1)(1+\omega)}\right)\left(c_1 - t\right)^{\frac{10}{3}} \tag{8.2.39}$$

8.3 Discussion

For different values of the constants involved, we will obtain different graphs. So, we opt to take particular values of the constants i.e., a_0=b_0=a_1=b_1=1, c_1=14.301443981790266 and plot the graphs of some of the parameters showing their variations with time as shown in the figures of this section.

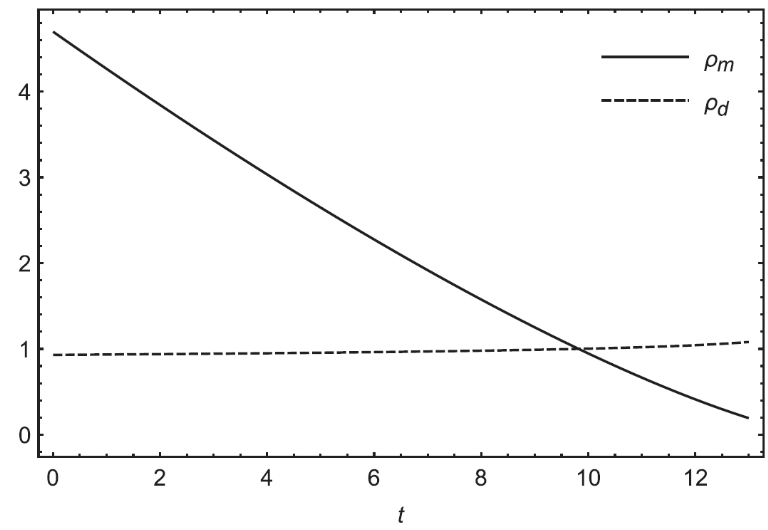

Figure 8.1: Variation of energy density of DE ρ_d and matter ρ_m with time t when a_0=b_0=a_1=b_1=1, c_1=14.301443981790266 showing that ρ_m decreases throughout evolution whereas ρ_d tends to increase very slowly or is nearly unchanged.

From Eqs. (8.2.22) and (8.2.23), it is obvious that energy densities of matter ρ_m and DE ρ_d are functions of cosmic time. To examine their nature, we plot their graphs showing their variations with cosmic time t as shown in Fig. 8.1. Here, it can be seen that ρ_m decreases throughout the evolution, as with the expansion of the universe, the galaxies get farther away from each other so that the matter density continues to diminish (Carroll 2001b). But, ρ_d tends to increase very slowly or is nearly unchanged. This may be a result of this anti-gravity dark component varying slowly with time and space (Carroll 2001a, 2001b; Peebles & Ratra 2003). So, our model universe will be increasingly dominated by dark energy in the far future.

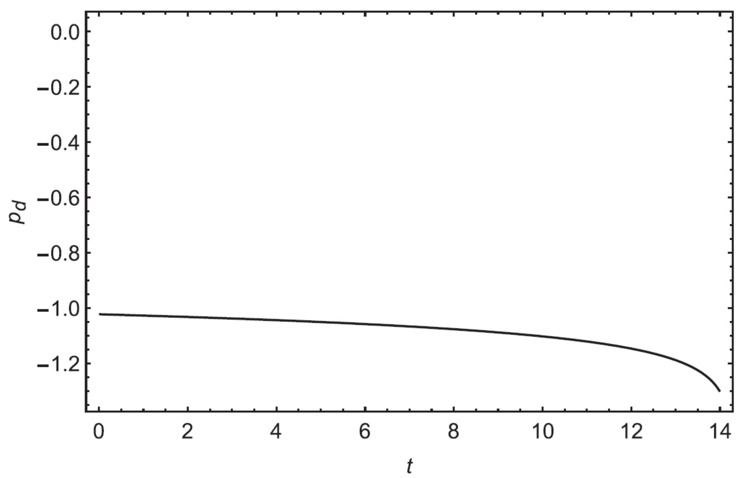

Figure 8.2: Variation of pressure of DE p_d with time t when b_0=a_1=b_1=1, c_1=14.301443981790266 showing it varies in the negative plane throughout evolution.

From Fig. 8.2, it can be clearly seen that the pressure of DE varies in the negative region throughout the evolution which is in agreement with the exotic property of dark energy that causes the universe to expand.

Figs. 8.3 and 8.4 respectively show that spatial volume V and the scalar expansion θ increases with t showing the accelerating spatial expansion of the universe. From Eq. (8.2.32), the spatial volume V of the universe is constant ($V \neq 0$) at time t=0. Also, other related parameters are also constant at time t=0. These show that our universe is free from initial singularity. But, when $t \rightarrow \infty$, both V and $\theta \rightarrow 0$ which indicates that after an infinite period of time, there will be a phase transition in which the expansion of the universe will cease. This may be supported by the fact that dark energy which causes the expansion of the universe varies slowly with time and space (Carroll 2001a, 2001b; Peebles & Ratra 2003). Also, the energy density of dark energy may decreases faster than that of matter leading to the disappearance of dark energy at $t \rightarrow \infty$ (Peebles & Ratra 2003). Then, our model universe will expand up to a finite degree; the expansion will tend to decrease. So, in the far future, this would lead our universe to be dominated by gravity causing it to shrink; finally collapsing resulting to the big crunch singularity.

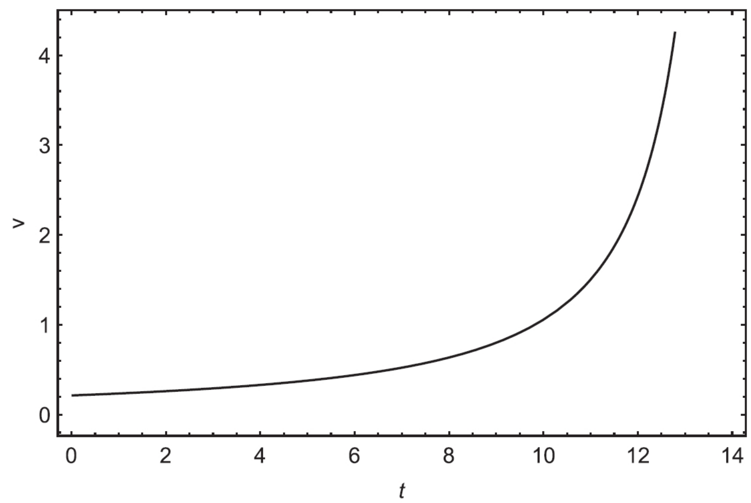

Figure 8.3: Variation of spatial volume V with time t when a_1=b_1=1, c_1=14.301443981790266 showing that it increases with time.

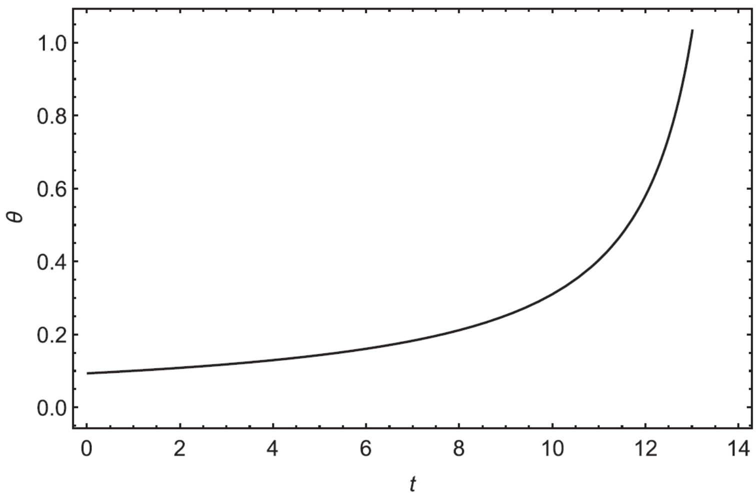

Figure 8.4: Variation of scalar expansion θ with time t when c_1=14.301443981790266 showing that it increases with time.

Fig. 8.5 shows that shear scalar (σ^2) tends to remain constant in the initial stage. Then, it start to converge and finally, diverges with the increase of cosmic time. Shear scalar provides us the rate of distortion of the matter flow of the large scale structure of cosmology (Ellis & Elst 1999). Hence, the model universe expands with a slow and uniform change of shape in the initial stage. Then, the change become more slower and finally, the change becomes faster. From Eq. (8.2.36), it is clear that anisotropic parameter A_h=0 all the time which indicates that our model universe is isotropic throughout the evolution.

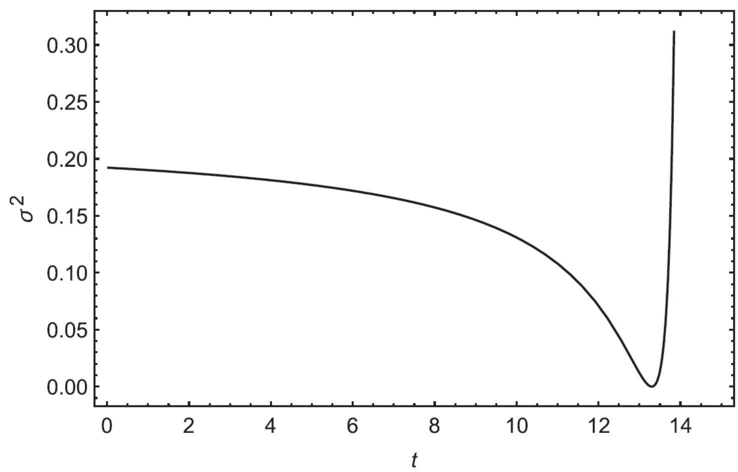

Figure 8.5: Variation of shear scalar σ^2 with time t when c_1=14.301443981790266.

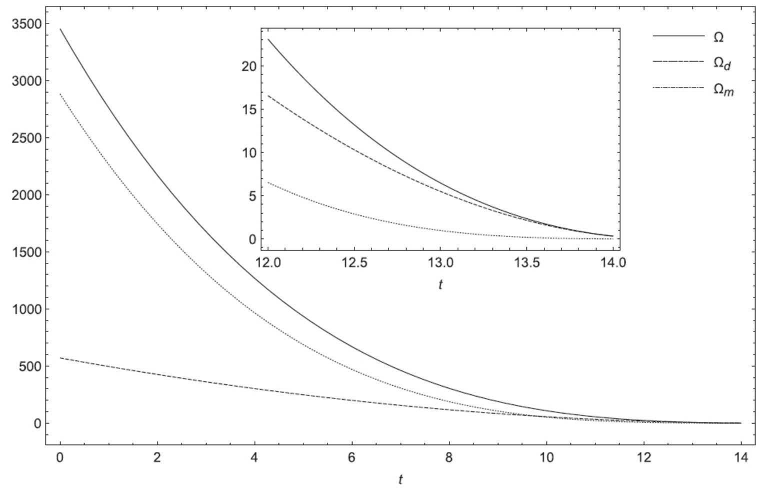

Figure 8.6: Variation of overall density parameter Ω, DE density parameter Ω_d and matter density parameter Ω_m with time t when a_0=b_0=a_1=b_1=1, c_1=14.301443981790266 showing that Ω and Ω_d decrease and tend to become constant whereas Ω_m decreases with a greater extent.

The variations of Ω, Ω_d and Ω_m with cosmic time t are shown in Fig. 8.6. Here, Ω and Ω_d are decreasing with the increase of cosmic time t and tend to become constant whereas Ω_m decreases but with a greater extent which might be supported by the fact that the matter density is diminishing with the accelerated expansion of the universe (Carroll 2001b). Here, it may be predicted that our model universe will become increasingly DE dominated in the far future. Moreover, on assuming that a_0=b_0=a_1=b_1=1, c_1=14.301443981790266 and taking EoS parameter ω=-1.047 which is in agreement with the latest observational value of ω (Knop et al. 2003; Melchiorri 2003; Collaboration et al. 2020), we find that the expression for EoS given by Eq. (8.2.31) is satisfied by time t_0=13.8 which is age of the universe at the present epoch. Also, under these assumptions, Eq. (8.2.39) gives us the value of the overall density parameter Ω=0.905988($\approx$1) at t=13.8 which is consistent with the present cosmological belief. Above all, at t=13.8, Eq. (8.2.34) gives us the value of Hubble parameter H=68 which is very close to $H_0 = 67.36 \pm 0.54 \text{kms}^{-1}\text{Mpc}^{-1}$, the value of Hubble parameter by the most recent Planck 2018 results (Collaboration et al. 2020).

8.4 Conclusions

In this chapter, we have studied a 5D SS space-time accompanied by minimally interacting fields - DM and DE components in BDT. It is predicted that our model universe will be increasingly dominated by DE. It is observed that the model universe is isotropic throughout the evolution. Our model universe is free from initial singularity but may face the big crunch singularity in the far future. With reasonable assumptions of the values of the constants and ω=-1.047 which is consistent with the value of ω of the most recent Planck

2018 results (Collaboration et al. 2020), we obtain the value of overall density parameter Ω=0.905988($\approx$1) which agrees with the present cosmological observation. Above all, at t=13.8, we obtain the value of Hubble parameter H=68 which is very close to $H_0 = 67.36 \pm 0.54 \mathrm{kms}^{-1}\mathrm{Mpc}^{-1}$, the value predicted by the most recent Planck 2018 results (Collaboration et al. 2020).