DARK ENERGY IN HIGHER DIMENSIONAL SPHERICALLY SYMMETRIC SPACE-TIME

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Chapter 9

Conclusions

This chapter summarizes the works presented in the thesis and conveys the relevance of the research. The chapter also explains what new knowledge has been brought to light and suggests future research ideas on the subject.

9.1 Summary

We have examined the dynamics of DE universe models in a 5D context while taking into account a SS space-time. We have anticipated some unforeseen outcomes as well as some that match the observational data. Our work, however, does have certain limitations, which is a general characteristic of a research work. We need further in-depth study and research to overcome the limitations. The works provided in the thesis are briefly summarised below, highlighting the main findings.

In **Chapter 1**, we have explained the motivation for our study and the fundamental concepts of the foundation and formulation of GR cosmological problems in a concise manner. We've also included an overview of the various ideas, space-time, and gravitational theories covered in the thesis. Above all, we have reviewed 108 articles to get a better understanding of similar works from the past and present. The review assisted us in identifying the knowledge gap as well as potential research ideas. We have summarised the review's main findings, as well as the implications and ideas for further research.

In **Chapter 2**, we have presented the research method and process employed to acquire knowledge and data to answer the research problem. The logic behind the approach used to acquire and analyse the results of the study is discussed in this chapter.

In **Chapter 3**, within the framework of SBT, we have studied an interacting model of HDE and matter in an SS space-time in a 5D context. We have established an accelerating universe dominated by phantom energy. The extra dimension maintains the theoretical foundation of the HDE scenario. The DE transits from phantom-like nature to CC, avoiding future singularity and heading to the de-Sitter phase. The Hubble parameter and DE

EoS parameter are measured to be $H = 67$ and $\omega = -1.00011$ at $t = 13.8$ Gyr. The SB scalar field φ tends to reach a positive constant value in the course of evolution when $n = -1$.

In **Chapter 4**, with minimally interacting matter and HDE in SBT, we have investigated a cosmological model in SS space-time in a 5D scenario. We have predicted that the universe will become increasingly DE dominated. We have estimated a few values of the deceleration parameter and found that they are quite close to the recently estimated values. The Hubble parameter H decreases, which is consistent with the current cosmic scenario. The DE EoS parameter has a value of $\omega = -1$, suggesting that the DE we are working with is the VE or CC. The total density parameter is predicted to be $\Omega = 0.97(\approx 1)$. The model universe begins with the Big Bang and ends with the Big Freeze singularity. Our interacting HDE model may be regarded as an alternative cosmological model to the standard ΛCDM model. Finally, we present two criteria for solving the extra dimension stability problem in GR: the first is to identify Casimir energy with the CC, or VE, and the second is to assume the extra dimension is of infinite volume. Probably, our work is the first to predict conditions to stabilize extra dimension in GR.

In **Chapter 5**, we have examined an SS metric in a 5D setting within the framework of LM. The model universe is predicted to be a DE model, with VE or CC dominating. The displacement vector serves as a time-dependent DE as well. The model is an oscillating model, with each cycle beginning with a big bang and ending with a big crunch, undergoing a series of bounces. The Hubble parameter is calculated to be $H = 67.0691$. We have created a model in LM that appeared to be a DE model; nevertheless, the work we have presented is only a toy model. The model requires further investigation, taking into account all of the observational data, which will be the focus of our forthcoming work.

In **Chapter 6**, we have investigated a SS space-time in a 5D scenario employing the $f(R,T)$ gravity framework. When $\lambda = -5.06911$ and -12.5856, the variation of pressure p and energy density ρ with cosmic time t have been investigated. It's intriguing to observe how our $f(R,T)$ model acts as a DE (vacuum energy) model in both situations. The scalar curvature R decreases over time, which is in line with previous research. In the finite time future, the model is projected to reach the de-Sitter phase. We have developed a model in which the $f(R,T)$ gravity theory itself acts like a DE (VE) model; nevertheless, the work we have presented is only a toy model. The model requires additional investigation, taking into account all of the observational data, which will be the matter of our forthcoming work.

In **Chapter 7**, we have investigated SCT in the context of a 5D SS space-time. The isotropic model acts like a phantom energy dominated model, that does not evolve from a singularity and instead tends to the de-Sitter phase, avoiding finite time future singularity (big rip). We have estimated that the value of G decreases with a variation of $\frac{\dot{G}}{G} = -7.2 \times 10^{-11}$ yr^{-1}. We have predicted $H = 68$ at t=13.8 Gyr. We have developed an SCT model that behaves as a phantom energy model, in other words that acts as a DE source. However, this developed model is a toy that requires a more in-depth study that takes into account all of the most recent cosmological facts, which we intend to do in the future.

In **Chapter 8**, in BDT framework, we have investigated a 5D SS space-time with minimally interacting DM and DE. DE is expected to become increasingly dominant. Our isotropic model universe does not start with an initial singularity, but it may encounter a big crunch singularity in the future. The values of the cosmological parameters obtained in this chapters are $\omega = -1.047$, $\Omega = 0.905988 \approx 1$ and $H = 68$, which agree with the current observational data.

9.2 Summary of key findings

Following are some of the key findings of our research.

- As DE sources, many authors have proposed various theories. Here, in our work we have shown that the modified theories of gravity, i.e. Lyra manifold, scale covariant theory, and $f(R,T)$ gravity can act as DE sources.
- In each of our constructed model, except Chapter 6, we have estimated some of the values of the cosmological parameters like Hubble parameter, DE EoS parameter and overall density parameter. Above all, we have also estimated the variation of the gravitational constant in Chapter 7.
- Generally, we can't find criteria stabilizing extra dimensions in GR. However, in Chapter 4, we have predicted two conditions for stabilizing extra dimensions in GR. Probably, our work is the first to predict conditions to stabilize extra dimension in GR.
- Because the dominant DE component is of phantom type, the big rip singularity is the most likely scenario. However, we have presented phantom energy models avoiding the cosmic doomsday, and ending at the de Sitter phase in Chapters 3 and 7. Above all, during research (Chapter 3), we come to know that in a HDE model,

big rip singularity is not permitted, as big rip violates the theoretical foundation of HDE scenario.

- Energy density should be positive to obtain a reliable cosmological model. However, we have presented models in Chapters 5 and 6 involving negative energy density, with arguments in support.

9.3 Ideas for further research

Following are some of the ideas for further research.

- We have shown that that the modified theories of gravity, i.e. Lyra manifold, scale covariant theory, and $f(R,T)$ gravity can act as DE source. Nevertheless, the models we have presented are just toy models. They require additional investigation, taking into account all of the observational data, which will be the matter of our forthcoming work. We may also see if there are any other modified theories that can be employed as DE sources.

- We have proposed two criteria for stabilising extra dimensions in GR in Chapter 4, perhaps for the first in GR. More research is needed to determine the reliability of all potential outcomes of the two criteria in terms of different cosmological factors in GR.

- In Chapter 3, we have mentioned that in a HDE model, big rip singularity is not permitted, as big rip violates the theoretical foundation of HDE scenario. This issue can be solved with the employment of an extra dimension. We can do further study to see if there is another way to solve this issue.