2018

CSIT

CSIT: 2.4

MATHEMATICAL FOUNDATION OF COMPUTER SCIENCE

Full Marks: 80

Time: 3 Hours

The figures in the margin indicates full marks for the questions

1. Answer the following question: $1 \times 10 = 10$

(a) A ___________ is an ordered collection of objects

(i) Relation
(ii) Function
(iii) Set
(iv) Proposition

(b) Which of the following two sets are equal?

(i) $A = \{1,2\}$ & $B = \{1\}$
(ii) $A = \{1,2\}$ & $B = \{1,2,,3\}$
(iii) $A = \{1,2,3\}$ & $B = \{2,1,3\}$
(iv) $A = \{1,2,4\}$ & $B = \{1,2,3\}$

(c) A graph G is called a ———— if it is a connected acyclic graph.

(i) Cyclic graph (ii) Regular graph
(iii) Tree (iv) Not a graph

(d) The relation $\{(1,2),(1,3),(3,1),(1,1),(3,3),(3,2),(1,4),(4,2),(3,4)\}$ is

(i) Reflexive (ii) Transitive
(iii) Symmetric (iv) Asymmetric

(e) Which of the following statement is the negation of the statement "2 is even and 3 is negative"?

(i) 2 is even and 3 is not negative

(ii) 2 is odd and 3 is not negative
(iii) 2 is even or 3 is not negative
(iv) 2 is odd or 3 is not negative

(f) If $f(x) = \cos x$ & $g(x) = x^3$, then $(f_0 g)(x)$ is

(i) $(\cos x)^3$ *(ii)* $(\cos^3 x)$

(iii) $x^{(\cos x)^3}$ *(iv)* $\cos x^3$

(g) $P \longrightarrow Q$ is logically equivalent to

(i) $\neg Q \longrightarrow P$
(ii) $\neg P \longrightarrow Q$
(iii) $\neg P \wedge Q$
(iv) $\neg P \vee Q$

(h) Which of the following propositions is tautology?

(i) $(P \vee Q) \longrightarrow Q$
(ii) $P \vee (Q \longrightarrow P)$
(iii) $P \vee (P \longrightarrow Q)$
(iv) Both (ii) and (iii)

(i) The complete graph with four vertices has k edges when k is

(i) 3 (ii) 4 (iii) 5 (iv) 6

(j) The set of integers Z with the binary operation *defined as $a * b = a + b + 1, \forall a, b \in Z$ is a group. The identity element of this group is,

(i) 0
(ii) 1
(iii) -1
(iv) 12

2. Answer any six questions of the following. $5 \times 6 = 30$

(a) Let Z be the set of integers. Let $m > 1$ be any fixed integer. Then we say a is congruent to another integer b modulo if $(a - b)$ is divisible by m, i.e. there exists an integer k such that $(a - b) = km$ and it is symbolically written as, $a \cong b(mod\ m)$. Show that this defines an equivalence relation.

(b) If $f: X \rightarrow Y$ is a one-one and onto mapping and $g: Y \rightarrow X$ is the inverse of f, then show that $f_0 g = I_y$ and $g_0 f = I_x$ where I_x & I_y are the identity functions on the set X &Y respectively. Also show that $I_{y_0} f = f$ and $f_0 I_x = f$.

(c) Show that the set of all positive rational numbers forms an abelian group under the composition * defined as $a * b = \frac{ab}{4}, \forall a,b \in Q^+$.

(d) Show that,

(i) A group G is abelian if and only if $(ab)^{-1} = a^{-1}b^{-1}, \forall a,b \in G$

(ii) If for every element of a group G, $a^2 = e$, then prove that G is an abelian group.

(e) Let a, b, c be any elements in a Boolean algebra B. Then show that, $(a * b) * c = a * (b * c)$.

(f) Show that a poset $(L, \leq)$ is a lattice if and only if every non-empty finite subset of L has glb and lub. Also show that in poset glb and lub both are unique.

(g) If G be a group, then show that,

(i) Every $a \in G$ has an unique inverse in G.

(ii) For all $a, b \in G, (ab)^{-1} = b^{-1}a^{-1}$

(h) Solve the following,

(i) Find the solution of the equation $abxax = cbx$ in a group G, where a, b are given element of G.

(ii) If G is a group such that $(ab)^n = a^n b^n$ for three consecutive integers n, for all $a, b \in G$. Show that G is abelian.

3. Answer the following questions. $4 \times 5 = 20$

(a) Construct the truth table for the followings:

(i) $(P \vee (Q \wedge R)) \rightleftharpoons [(P \vee Q) \wedge (P \vee R)]$

(ii) $(\neg P \rightleftharpoons \neg Q) \rightleftharpoons (P \rightleftharpoons Q)$

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(b) Show that, $\neg(P \vee (\neg P \wedge Q)) \equiv \neg P \wedge \neg Q$ and $(P \wedge Q) \rightarrow (P \vee Q)$ is a tautology by using logical equivalent formulas.

(c) Test the validity of the argument:
If milk is black then every cow is white. If every cow is white then it has four legs. If every cow has four legs then every buffalo is white and brish. The milk is black. Therefore, the buffalo is white.

(d) Find the conjunction normal form and disjunction normal form for, $\neg(P \vee Q) \rightleftharpoons (P \wedge Q)$.

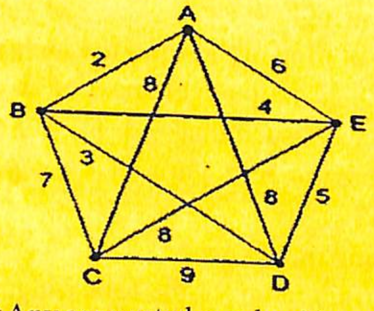
(e) Discuss the validity of the following argument:
Babies are illogical. Nobody is despised who can manage a crocodile. Illogical persons are despised. Therefore babies can not manage crocodile.

4. Answer the following questions: $4 \times 5 = 20$

(a) Let G be a disconnected graph with n vertices where n is even. If G has two components each of which is complete, prove that G has a minimum of $\frac{n(n-2)}{4}$ edges.

(b) Show that a connected graph G is an Euler graph if and only if it can be decomposed into circuits.

(c) Find the minimum spanning tree of the following graph with the help of Prime algorithm,



(d) Show that Any connected graph with n vertices and (n-1) edges is a tree.

(e) What do you mean by graph operations defined with proper example.

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