2016

COMPUTER SCIENCE AND TECHNOLOGY

PAPER : CSIT 2.4

MATHEMATICAL FOUNDATION OF COMPUTER SCIENCE

Full Mark : 80

Time : 3 Hrs

Figures in the right hand margin indicate full marks for the question

1 Answer the following question: $1 \times 5 = 5$

(a) Let R be a relation in the real numbers defined by$a \leq b$. Is R is symmetric.

(b) A vertex having no incident edge is …………

(i) End vertex

(ii) Isolated vertex

(iii) Pendent vertex

(iv) None of above

(c) Write the negation of the statement $(\exists x)(\forall y)P(x, y)$.

(d) A group with commutative property is called abelian group. (True or False.)

(e) Give the definition of semi-groups.

2. Answer the following question: $2 \times 5 = 10$

(a) Prove that the relation R on the set A is defined by $(a, b) \in R \Leftrightarrow a = b$ is an equivalence relation on A.

(b) Let a be any element of a Boolean algebra B. if $a + x = 1 \; and \; a * x = 0$ then show that $x = \acute{a}$

(c) Write down the principle of duality. Let B be a lattice with $a, b \in B$, then show that $ab = ba$.

(d) Show that there is only one path between every pair of vertices in a tree.

(e) If for every element a of a group G, $a^2 = e$, then prove that G is an abelian group.

3. Answer any five of the following question. $4 \times 5 = 20$

(a) Let Z be the set of integers. Let $m > 1$ be any fixed integer. Then we say a is congruent to another integer b modulo if there exists an integer k s.t. $(a - b) = km$ and it is symbolically written as $a \equiv b(mod\ m)$. Show that this is an equivalence relation.

(b) Show that the set of all positive rational numbers forms an abelian group under the composition * defined as $a * b = \frac{ab}{4}, \forall\, a, b \in Q^+$.

(c) Show that a poset $(L, \leq)$ is a lattice if and only if every non-empty finite subset of L has *glb* and *lub*.

(d) Show that the mapping $f: G \to \acute{G}$ given by $f(x) = 2x,\ \forall x \in G$ is an isomorphism of G *onto* $\acute{G}$ where G ia an additive group of integer numbers and $\acute{G}$ is the additive group of even integer numbers including zero.

(e) Consider a mapping $f: X \to Y$. For any subsets A_1 & A_2 of X and B_1 & B_2 of Y, then show that,

(1) $A_1 \subseteq A_2 \Rightarrow f(A_1) \subseteq f(A_2)$

(2) $f(A_1 \cup A_2) = f(A_1) \cup f(A_2)$

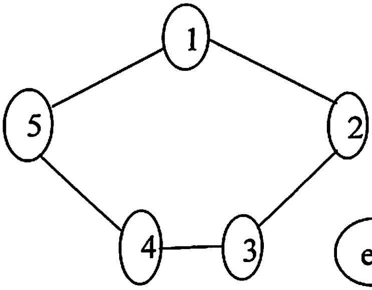
(f) Show that the relation of isomorphism in the set of all groups is an equivalence relation.

4. Answer the following question. $5 \times 5 = 25$

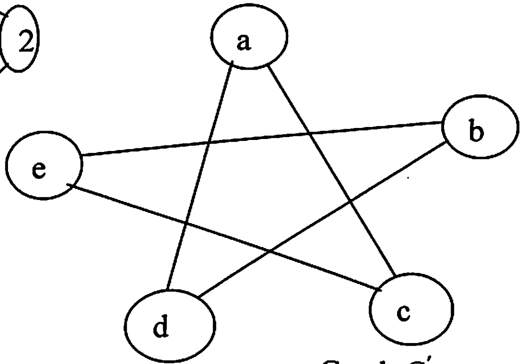
(a) Give the definition of tree. Show that a tree with n vertices has n-1 edges..

P.T.O.

(b) What do you mean by Graph Isomorphism. Show that the following graph G and G' are isomorphic:



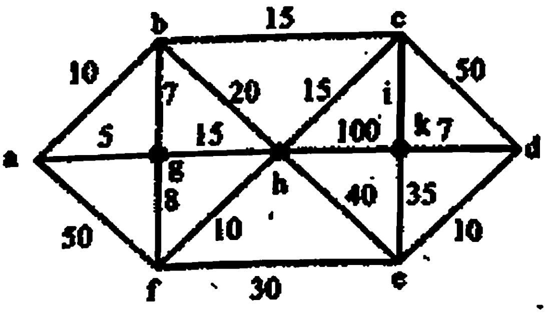
Graph-G



Graph-G'

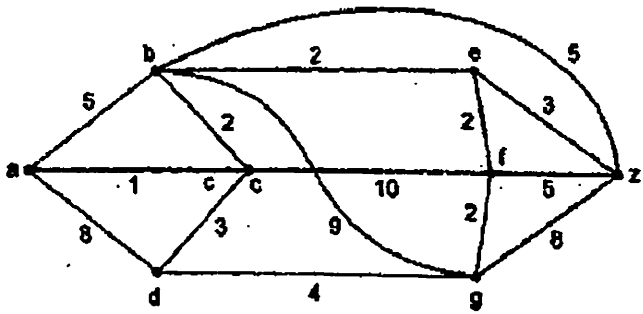
(c) Show that a connected graph G is an Euler graph if and only if all vertices of G are of even degree

(d) Apply Kruskal's algorithm to find a minimal spanning tree of the following graph.



(e) Apply Dijkstra algorithm to find out the shortest path from the vertices a to z in the following graph

P.T.O.



5. Answer any five question the following . $4 \times 5 = 20$

(a) Construct the truth tables of the following formulas:
(1) $(P \to Q) \wedge (Q \to P)$
(2) $(P \wedge \neg Q) \vee (\neg P \wedge Q)$.

(b) Show that , $\neg(P \wedge Q) \to (\neg P \vee (\neg P \vee Q)) \equiv (\neg P \vee Q)$

(c) Show that S is logically follows from the premises, $P \to Q, P \to R, \neg(Q \wedge R)$ *and* $S \vee P$

(d) Using identities, prove that , $Q \vee (P \wedge \neg Q) \vee (\neg P \wedge \neg Q)$ *is a tautology.*

(e) Find conjunction normal form and disjunction normal form for $P \rightleftharpoons \neg P \vee \neg Q$.

(f) Discuss the validity of the following argument:-
All educated persons are well behaved.
Ram is educated.
No well behaved person is quarrelsome.
Therefore, Ram is not quarrelsome

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