A STUDY OF FUZZY TOPOLOGICAL SPACES WITH REFERENCES TO FUZZY CLOSURE AND FUZZY BOUNDARY

A THESIS

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Certificate

This is to certify that the thesis entitled **"A Study of Fuzzy Topological Spaces with Reference to Fuzzy Closure and Fuzzy Boundary"** being submitted by **Mr. Bhimraj Basumatary** for the award of the Degree of Doctor of Philosophy in Mathematics to Bodoland University, Kokrajhar, Assam, India, is a record of bonafide research work carried out by him under my guidance in the Department of Mathematical Sciences, Bodoland University, Kokrajhar with the supervision of Prof. Hemanta Kumar Baruah, Vice-Chancellor, Bodoland University and former Professor, Department of Statistics, Gauhati University.

The thesis satisfies the requirements of the regulation relating to the degree. Also, considerable parts of the thesis are published in International and National Journals. The work reported in the thesis is original and has not been submitted in any other university or institute for the award of any degree or diploma.

Dated, Kokrajhar
The 25th May, 2017

(Dr. Kangujam Priyokumar Singh)
Research Guide.

DECLARATION

I hereby declare that I have carried out the present research work entitled "**A Study of Fuzzy Topological Spaces with References to Fuzzy Closure and Fuzzy Boundary**" under the guidance and supervision of Dr. K. P. Singh, Department of Mathematical Sciences, Bodoland University, Kokrajhar, Assam, India. The thesis has been submitted to Bodoland University for the award of the degree of Doctor of Philosophy in the Faculty of Science & Technology.

I further declare that the analyses and results made in this thesis represent my original work that has not been previously submitted for a degree or diploma in any University or Institution of higher education.

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REFERENCES

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LIST OF PUBLICATIONS

Articles published

1. B. Basumatary, "Fuzzy Interior and Fuzzy Closure with Extended Definition of Fuzzy Set", *Int. J. Computational System Engineering*, Vol-2, No-2, 2015.
2. B. Basumatary, A Note on Fuzzy Closure of Fuzzy Set, *JPMNT,* Vol-3, issue-4, pp.35-39,(2015).
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7. Basumatary B., "A note on Fuzzy Boundary of Fuzzy Bitopological Spaces on the Basis of Reference Function", *Advances in Fuzzy Mathematics*,Vol-12, No-3, pp639-644.

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1. K. P. Singh & B. Basumatary,"A New View on Fuzzy Set and Fuzzy Topological Spaces with Reference to Extended definition of Fuzzy Set" in *German Journal of Advanced Mathematical Sciences (GJAMS).*

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1. Basumatary B, “A Study of Fuzzy Boundary on the Basis of Fuzzy Complement”, national seminar presented held at Cotton College State University on 21-22 Oct. 2016.
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