Advances in Fuzzy Mathematics.
ISSN 0973-533X Volume 12, Number 3 (2017), pp. 639-644

http://www.ripublication.com

A Note on Fuzzy Boundary of Fuzzy Bitopological Spaces on the Basis of Reference Function

Bhimraj Basumatary
Department of Mathematical Sciences,
Bodoland University, Kokrajhar, BTAD, India

Abstract

In this paper we define Fuzzy (τ_i, τ_j)-r-boundary of fuzzy bitopological space on the basis of reference function. We prove some propositions of Fuzzy (τ_i, τ_j)-r-boundary on the basis of reference function.

Keywords: Fuzzy Membership function, Fuzzy Reference Function, Fuzzy Membership Value, (τ_i, τ_j) –r-Fuzzy Boundary.

1. Introduction

Fuzzy set theory was discovered by Zadeh [1] in 1965. Chang [2] introduced fuzzy topology. In 1963, J.C. Kely [3] defined the study of Bitopological spaces. The concept of fuzzy boundary was introduced by Pu and Liu[4]. It is seen that when we used the existing definition of fuzzy set in fuzzy topology then many properties of fuzzy closure and fuzzy boundary do not satisfy as the properties of closure and boundary of general topology do satisfy. Present author [5] has expressed nicely in this regard with extended definition of fuzzy set and showed that closure of a fuzzy set is equal to union of a fuzzy set and its boundary.

2. New Definition Of Fuzzy Sets

Baruah [6, 7] gave an extended definition of fuzzy set. According to Baruah [6, 7] to define a fuzzy set, two functions namely fuzzy membership function and fuzzy reference function are necessary. Fuzzy membership value is the difference between fuzzy membership function and fuzzy reference function.

Let $\mu_1(x)$ and $\mu_2(x)$ be two functions such that $0 \leq \mu_2(x) \leq \mu_1(x) \leq 1$. For fuzzy number denoted by {x, $\mu_1(x)$, $\mu_2(x)$; xϵX}, we call $\mu_1(x)$ as fuzzy membership function and $\mu_2(x)$ a reference function such that $(\mu_1(x) - \mu_2(x))$ is the fuzzy membership value for any x in X.

3. PRELIMINARIES

Let A={x, $\mu_1(x)$, $\mu_2(x)$; xϵU} and B={x, $\mu_3(x)$, $\mu_4(x)$; xϵU} be two fuzzy sets defined over the same universe U.

a. $A \subseteq B$ iff $\mu_1(x) \leq \mu_3(x)$ and $\mu_4(x) \leq \mu_2(x)$ for all xϵU.
b. $A \bigcup B$ ={x, max($\mu_1(x)$, $\mu_3(x)$), min($\mu_2(x)$, $\mu_4(x)$)} for all xϵU.
c. $A \bigcap B$ ={x, min($\mu_1(x)$, $\mu_3(x)$), max($\mu_2(x)$, $\mu_4(x)$)} for all xϵU.

If for some xϵU, min($\mu_1(x)$, $\mu_3(x)$) ≤ max($\mu_2(x)$, $\mu_4(x)$)}, then our conclusion will be

$$. A \cap B = \phi$$

d. A^C={x, $\mu_1(x)$, $\mu_2(x)$; xϵU}^{C}
={x, $\mu_2(x)$, 0; xϵU} $\bigcup$ {x, 1, $\mu_1(x)$; xϵU}

e. If D = {x, $\mu(x)$, 0; xϵU} then D^C={x, 1, $\mu(x)$; xϵU} for all xϵU.

4. Fuzzy Topology

A fuzzy topology on a nonempty set X is a family τ of fuzzy set in X satisfying the following axioms

(T1) 0_X, $1_X \in \tau$

(T2) $G_1 \bigcap G_2 \in \tau$, for any $G_1, G_2 \in \tau$

(T3) $\bigcup G_i \in \tau$, for any arbitrary family $\{G_i : G_i \in \tau, i \in I\}$.

In this case the pair (X,τ) is called a fuzzy topological space and any fuzzy set in τ is known as fuzzy open set in X and clearly every element of τ^C is said to be fuzzy closed set.

5. Fuzzy τ_i-r-interior

Let (X, τ_i, τ_j) be a fuzzy bitopological space. Then for a fuzzy subset A of X, the fuzzy τ_i- r-interior of A is the union of all τ_i-r-open sets of X contained in A and is defined as follows

τ_i-r-Int(A)=$\bigcup$ {P : P is τ_i-r-open set in X and P$\subseteq$A }.

6. Fuzzy τ_i-r-closure

Let (X, τ_i, τ_j) be a fuzzy bitopological space. Then for a fuzzy subset A of X, the fuzzy τ_i-r-closure of A is the intersection of all τ_i-r-closed sets of X containing A and is defined as follows

τ_i-r-Cl(A)=$\bigcap$ {G: G is fuzzy τ_i-r-closed set in X and A$\subseteq$G}.

7. Fuzzy (τ_i, τ_j)-r-boundary

Let A be a fuzzy subset of a fuzzy bitopological space (X, τ_i, τ_j). Then the fuzzy (τ_i, τ_j)-r-boundary of A is defined by (τ_i, τ_j)–r-Bd(A)= τ_i-r-cl(τ_j-r-cl(A))$\cap\tau_i$-r-cl(τ_j-r-cl(A^C)).

8. Proposition: Let A be any fuzzy set in fuzzy bitopological space (X, τ_i, τ_j). Then (τ_i, τ_j)-r-Bd(A) = (τ_i, τ_j)-r-Bd(A^C).

Proof:

Let A be a fuzzy set of a fuzzy bitopological space (X, τ_i, τ_j).

Let A={x, μ_i(x), 0; xϵX }

(τ_i, τ_j) –r- Bd(A)= [τ_i–r-Cl(τ_j –r- Cl(A))]$\cap$[τ_i–r- Cl(τ_j –r- Cl(A^C))]

= τ_i–r- Cl [{x, $\min_{\tau_j}$ {μ_i(x)}, 0; xϵX }]$\cap$ τ_i– r-Cl [{x, 1, $\min_{\tau_j}${μ_i(x)}, 0; xϵX }]

=[{x, $\min_{\tau_i}$ $\min_{\tau_j}$ {μ_i(x)}, 0; xϵX }]$\cap$ [{x, 1, $\max_{\tau_i}$ { $\max_{\tau_j}${μ_i(x)}, 0; xϵX }]

=[{x, 1, $\max_{\tau_i}$ $\max_{\tau_j}${μ_i(x)}, 0; xϵX }]$\cap$[{x, $\min_{\tau_i}$ $\min_{\tau_j}$ {μ_i(x)}, 0; xϵX }]

$=[\{x, 1, \max_{\tau_i} \max_{\tau_j}\{\mu_i(x)\}, 0; x\epsilon X\}]\cap[\{x, \min_{\tau_i} \min_{\tau_j} \{\mu_i(x)\}, 0; x\epsilon X\}^C]^C$

$=[\tau_i\text{–r-Cl}(\tau_j \text{–r-Cl}(A^C))]\cap[\tau_i\text{– r-Cl}(\tau_j \text{–r- Cl}(A^C))]^C$

$=(\tau_i, \tau_j)$ –r- Bd(A^C).

9. Proposition: Let A be any fuzzy set in fuzzy bitopological space (X, τ_i, τ_j). A is fuzzy closed set if and only if (τ_i, τ_j) –r- Bd (A) $\subseteq$A.
The prove is straightforward.

Generally the converse part of the proposition is not true when we use the existing definition of fuzzy set. For this we cite an example.

Let X={a, b, c}and A={x, (a, 0.4), (b, 0.7), (c, 0.2)}, B={x, (a, 0.6), (b, 0.9), (c, 0.1)}, C={x, (a, 0.4), (b, 0.7), (c, 0.3)} and D={x, (a, 0.6), (b, 0.9), (c, 0.2)} be fuzzy sets on X. Then $\tau_1=\{0_X,1_X$ A, B, A⋃B, A∩B} and $\tau_2=\{0_X,1_X$,C, D, C⋃D, C∩D } be fuzzy topologies in X. Let P={ x, (a, 0.6), (b, 0.7), (c, 0.9)} be any fuzzy set in X and P^C={ x, (a, 0.4), (b, 0.3), (c, 0.1)}.

Now τ_j –r-Cl(P))=1_X and τ_i –r- Cl(1_X))= 1_X.

τ_j –r-Cl(P^C))= { x, (a, 0.4), (b, 0.3), (c, 0.8)} and

τ_i –r-Cl({ x, (a, 0.4), (b, 0.3), (c, 0.8)})= { x, (a, 0.6), (b, 0.3), (c, 0.9)}

Now (τ_i, τ_j) –r- bd (A)= 1_X∩{ x, (a, 0.6), (b, 0.3), (c, 0.9)}= { x, (a, 0.6), (b, 0.3), (c, 0.9)}.

Therefore (τ_i, τ_j)–r-Bd (P) $\subseteq$P. But P is not fuzzy closed set.

Now if we express the same problem in our definition with respect to reference function

Let X={a, b, c}and A={x, (a, 0.4, 0), (b, 0.7, 0), (c, 0.2, 0)}, B={x, (a, 0.6, 0), (b, 0.9, 0), (c, 0.1, 0)}, C={x, (a, 0.4, 0), (b, 0.7, 0), (c, 0.3, 0)} and D={x, (a, 0.6, 0), (b, 0.9, 0), (c, 0.2, 0)} be fuzzy sets on X. Then $\tau_1=\{0_X,1_X$ A, B, A⋃B, A∩B} and $\tau_2=\{0_X,1_X$ C, D, C⋃D, C∩D } be fuzzy topologies in X. Let P={ x, (a, 0.6, 0), (b, 0.7, 0), (c, 0.9, 0)} be any fuzzy sets in X and P^C={ x, (a, 1, 0.4), (b, 1, 0.3), (c, 1, 0.1)}.

Then $\tau=\{0_X,1_X$ A, B, A⋃B, A∩B} be fuzzy topology in X.

Let P={ x, (a, 0.6, 0), (b, 0.7, 0), (c, 0.9, 0)} be any fuzzy set in X and P^C ={ x, (a, 1, 0.6), (b, 1, 0.7), (c, 1, 0.9)}.

Now τ_j –r- Cl(P))=1_X and τ_i –r- Cl(1_X))= 1_X.

τ_j –r- Cl(P^C))={ x, (a, 1, 0.4), (b, 1, 0.7), (c, 1, 0.2)}.

τ_i –r- Cl{x, (a, 1, 0.4), (b, 1, 0.6), (c, 1, 0.2)}= { x, (a, 1, 0.4), (b, 1, 0.7), (c, 1, 0.1)}.

Now (τ_i, τ_j) –r-Bd (A)= 1_X∩{ x, (a, 1, 0.4), (b, 1, 0.7), (c, 1, 0.1)}

= { x, (a, 1, 0.4), (b, 1, 0.7), (c, 1, 0.1)}.

Now we have (τ_i, τ_j) –r- Bd (A)={ x, (a, 1, 0.4), (b, 1, 0.7), (c, 1, 0.1)} and P={ x, (a, 0.6, 0), (b, 0.7, 0), (c, 0.9, 0)} which is not comparable. This is happened because P is not closed set.

Thus it is clearly seen that the converse part of the proposition is also true when we use our definition of fuzzy set with respect to reference function.

10. Proposition: Let A be any fuzzy set in fuzzy bitopological space (X, τ_i, τ_j). A is fuzzy open set if and only if (τ_i, τ_j)-r-Bd(A) $\subseteq A^C$.
Prove is straightforward.

11. Proposition: Let A be any fuzzy set in fuzzy bitopological space (X, τ_i, τ_j).
Then (τ_i, τ_j)-r-Bd(Cl(A)) ⊆ (τ_i, τ_j)-r-Bd(A).
The proposition is true when we use our definition of fuzzy set.

12. Proposition: Let A and B be fuzzy sets in fuzzy bitopological space (X, τ_i, τ_j).
Then (τ_i, τ_j)-r-Bd(A∪B) ⊆ (τ_i, τ_j)-r-Bd(A) ∪ (τ_i, τ_j)-r-Bd(B).
The proposition is true when we use our definition of fuzzy set.

13. Proposition: Let A and B be fuzzy sets in fuzzy bitopological space (X, τ_i, τ_j).
Then (τ_i, τ_j)-r-Bd(A∩B) ⊆ (τ_i, τ_j)-r-Bd(A) ∪ (τ_i, τ_j)-r-Bd(B).
The proposition is true when we use our definition of fuzzy set.

Conclusion:

In this paper we have used new definition of fuzzy set on the basis of reference function so that we can over come from the drawbacks of fuzzy complement. It is seen that when we used our definition of fuzzy set then some propositions of (τ_i, τ_j)–r-Boundary do satisfied as the proposition of general (τ_i, τ_j) – Boundary of bitopology satisfied whereas these proposition do not satisfy in fuzzy bitopology when we use existing definition of fuzzy set.

References

[1] Zadeh L. A. , Fuzzy sets, *Information and Control*, 8, 338-353 (1965).

[2] Chang C. L. , Fuzzy Topological Space, Journal of Mathematical Analysis and Application 24, 182- 190 (1968).

[3] Kelly J.C, Bitopoligicaal spaces, Pro. London math soc., 313 (1963), 71-89

[4] Pu, P. M. and Liu, Y. M. Fuzzy topology I: neighbourhood structure of a fuzzy point and Moore-Smith convergence, J. Math. Anal. Appl. 76, 571–599, 1980.

[5] Basuatary B., Towards Forming the Field of Fuzzy Closure with Reference to Fuzzy Boundary, *JPMNT*, Vol. 4, No.1, pp30-34.

[6] Baruah H. K., Towards Forming a Field of Fuzzy Sets, Int. Jr. of Energy, Information and Communications, Vol. 2, issue 1, Feb. (2011).

[7] Baruah H. K., The Theory of Fuzzy Sets: Belief and Realities, Int. Jr. of Energy, Information and Communications, 1-22Vol. 2, issue 2, May (2011).

[8] Warren, R. H. Boundary of a fuzzy set, Indiana Univ. Math. J. 26, 191–197, (1977)