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Towards Forming the Field of Fuzzy Boundary on the Basis of Reference Function

Bhimraj Basumatary
Department of Mathematical Sciences,
Bodoland University, Kokrajhar, BTAD, India.

Sahalad Borgoyary
Central Institute of Technology, Kokrajhar, BTAD, India.

Kangujam Priyokumar Singh
Department of Mathematical Sciences, Bodoland University,
Kokrajhar, BTAD, India.

Hemanta K. Baruah
Bodoland University, Kokrajhar, BTAD, India.

Abstract

It is seen that some propositions of fuzzy boundary are note satisfies same as propositions of classical boundary satisfy if we use existing definition of fuzzy set. In this article we use our definition of fuzzy set on the basis of reference function so that the propositions of fuzzy boundary which is not satisfy as propositions of classical boundary could satisfy. This article proposes to revisit on the comments of fuzzy boundary propositions and theorems. The main contribution of this article is to suggest that the propositions of fuzzy boundary can also satisfy as propositions of classical boundary could satisfy if we use our definition of fuzzy set and complementation of fuzzy set on the basis of reference function.

Keywords: Fuzzy Membership function, Fuzzy Reference Function, Fuzzy Membership Value, Fuzzy Interior, Fuzzy Closure, Fuzzy Boundary.

INTRODUCTION

Fuzzy set theory was discovered by Zadeh [1] in 1965. The theory of fuzzy sets actually has been a generalization of the classical theory of sets in the sense that the theory of sets should have been a special case of the theory of fuzzy sets. But unfortunately it has been accepted that for fuzzy set A and its complement A^C, neither $A\cap A^C$ is empty set nor $A\cup A^C$ is the universal set. Whereas the operations of union and intersection of crisp sets are indeed special cases of the corresponding operation of two fuzzy sets, they end up giving peculiar results while defining $A\cap A^C$ and $A\cup A^C$. In this regard Baruah [2, 3] has forwarded an extended definition of complement of fuzzy sets which enable us to define complement of fuzzy sets in a way that give us $A\cap A^C$ is empty and $A\cup A^C$ is universal set.

Chang [4] introduced fuzzy topology. After the introduction of fuzzy sets and fuzzy topology, several researches were conducted on the generalizations of the notions of fuzzy sets and fuzzy topology. In fuzzy topology also many results are not same as general topology. It is seen that when we used the existence definition of fuzzy set in fuzzy boundary then closure of any fuzzy set is not equal to union of a fuzzy set and its boundary. The main reason behind it is expression of complement of existing definition of fuzzy set. Present author [5] has expressed nicely in this regard with extended definition of fuzzy set and showed that closure of a fuzzy set is equal to union of a fuzzy set and its boundary.

1. Problem Statement:

In this article we try to prove some propositions of fuzzy boundary by taking our definition of fuzzy set on the basis of reference function. Actually some of the propositions of fuzzy boundary are not same as classical boundary satisfied if we use existing definition of complement of fuzzy set. In that matter we use definition of complement of fuzzy set on the basis of reference function so that we could get same result.

2. Related work:

In GIS the most important notions concerned with is the fuzzy boundary. Many Authors has discussed in different way in fuzzy boundary, Warren[6] discussed on fuzzy boundary and according to him The fuzzy boundary of A is the infimum of all the closed fuzzy sets D in X with the property $Cl(A(x)) \leq D(x)$ for all $x\epsilon X$ for which $(Cl(A)\wedge Cl(A^C))(x) > 0$ or $Int(A(x)) \neq 1$.According to Pu and Liu[7] the fuzzy boundary of A is defined as $Cl(A)\cap Cl(A^C)$. Cuchillo-Ibanez and Tarres [8] gave the definition of fuzzy boundary and according to him fuzzy boundary of Ais the infimum of all closed fuzzy sets D in X with the property $Cl(A(x)) \leq D(x)$ for all $x\epsilon X$ for which $(Cl(A) - Int(A))(x) \geq 0$.

In this article we would like to express some propositions of fuzzy boundary on the basis of reference function.

3. Material and methods:

New definition of fuzzy sets**:**

Baruah [2, 3] gave an extended definition of fuzzy set. According to Baruah [2, 3] to define a fuzzy set, two functions namely fuzzy membership function and fuzzy reference function are necessary. Fuzzy membership value is the difference between fuzzy membership function and fuzzy reference function.

Let $\mu_1(x)$ and $\mu_2(x)$ be two functions such that $0 \leq \mu_2(x) \leq \mu_1(x) \leq 1$. For fuzzy number denoted by $\{x, \mu_1(x), \mu_2(x) ; x \epsilon X\}$, we call $\mu_1(x)$ as fuzzy membership function and $\mu_2(x)$ a reference function such that $(\mu_1(x) - \mu_2(x))$ is the fuzzy membership value for any x in X.

4. Preliminaries

Let $A=\{x, \mu_1(x), \mu_2(x); x \epsilon U\}$ and $B=\{x, \mu_3(x), \mu_4(x); x \epsilon U\}$ be two fuzzy sets defined over the same universe U.

a. $A \subseteq B$ iff $\mu_1(x) \leq \mu_3(x)$ and $\mu_4(x) \leq \mu_2(x)$ for all $x \epsilon U$.
b. $A \bigcup B = \{x, \max(\mu_1(x), \mu_3(x)), \min(\mu_2(x), \mu_4(x))\}$ for all $x \epsilon U$.
c. $A \bigcap B = \{x, \min(\mu_1(x), \mu_3(x)), \max(\mu_2(x), \mu_4(x))\}$ for all $x \epsilon U$.
If for some $x \epsilon U$, $\min(\mu_1(x), \mu_3(x)) \leq \max(\mu_2(x), \mu_4(x))\}$, then our conclusion will be $A \cap B = \phi$.

d. $A^C=\{x, \mu_1(x), \mu_2(x); x \epsilon U\}^C$
$=\{x, \mu_2(x), 0; x \epsilon U\} \bigcup \{x, 1, \mu_1(x); x \epsilon U\}$

e. If $D = \{x, \mu(x), 0; x \epsilon U\}$ then $D^C=\{x, 1, \mu(x); x \epsilon U\}$ for all $x \epsilon U$.

5. Fuzzy Topology

A fuzzy topology on a nonempty set X is a family τ of fuzzy set in X satisfying the following axioms

(T1) $0_X, 1_X \in \tau$

(T2) $G_1 \bigcap G_2 \in \tau$, for any $G_1, G_2 \in \tau$

(T3) $\bigcup G_i \in \tau$, for any arbitrary family $\{G_i : G_i \in \tau, i \in I\}$.

In this case the pair (X,τ) is called a fuzzy topological space and any fuzzy set in τ is known as fuzzy open set in X and clearly every element of τ^C is said to be fuzzy closed set.

5.1 Interior of a fuzzy set

Definition Let (X, τ) be fuzzy topology also A={x, μ_A(x), γ_A(x)} be fuzzy set on X.

Then interior of a fuzzy set A is defined as union of all open subsets contained in A, denoted it as int(A) and is defined as follows

Int(A)=$\bigcup$ {P : P is open set in X and P$\subseteq$A }.

5.2 Closure of fuzzy set

Let (X, τ) be fuzzy topology and A={x, μ(x), γ(x); x∈X} be fuzzy set in X. Then fuzzy closure of A are defined by

Cl(A)=$\bigcap$ {G: G is fuzzy closed set in X and A$\subseteq$G}

6. Fuzzy Boundary: Let A be a fuzzy set in fuzzy topological space X. Then the fuzzy boundary of A is defined as

Bd(A)=cl(A)∩cl(A^C).

Note: The present Author [5] has shown that cl(A)=A$\bigcup$Bd(B), when we express fuzzy set on the basis of reference function.

Now we discuss some propositions of fuzzy boundary which do form field if we apply our definition of fuzzy set.

7. Proposition: Let A be any fuzzy set in fuzzy topological spaces X. Then Bd(A) = Bd(A^C).
Proof:

$$\begin{aligned} bd(A) &= Cl(A) \cap Cl(A^C) \\ &= Cl(A^C) \cap Cl(A) \\ &= Cl(A^C) \cap (Cl(A^C))^C \\ &= Bd(A^C). \end{aligned}$$

Now let us try to prove this proposition with our extended definition of fuzzy set.

Suppose A={x, μ_i(x), 0; x∈X } then A^C={x, 1, μ_i(x); x∈X }.

Therefore $Cl(A) = \cap\{x, \mu_i(x), 0; x \in X\} = \{x, \min\{\mu_i(x)\}, 0; x \in X\}$ and

$$Cl(A^C) = \cap\{x, 1, \mu_i(x); x \in X\} = \{x, 1, \max\{\mu_i(x)\}, 0; x \in X\}$$

So,

$$Bd(A) = Cl(A) \cap Cl(A^C)$$

$$= \{x, \min\{\mu_i(x)\}, 0; x \in X\} \cap \{x, 1, \max\{\mu_i(x)\}, 0; x \in X\}$$

$$= \{x, 1, \max\{\mu_i(x)\}, 0; x \in X\} \cap \{x, \min\{\mu_i(x)\}, 0; x \in X\}$$

$$= \{x, 1, \max\{\mu_i(x)\}, 0; x \in X\} \cap \{x, 1, \min\{\mu_i(x)\}; x \in X\}^C$$

$$= \{x, 1, \max\{\mu_i(x)\}, 0; x \in X\} \cap \{\{x, \min\{\mu_i(x)\}, 0; x \in X\}^C\}^C$$

$$= Cl(A^C) \cap (Cl(A^C))^C$$

$$= Bd(A^C).$$

8. Proposition: Let A be any fuzzy set in fuzzy topological spaces X. A is fuzzy closed set if and only if $bd(A) \subseteq A$.

Proof:

Let A be fuzzy closed set. So, $cl(A) = A$.

$Bd(A) = cl(A) \cap cl(A^C) \subseteq cl(A) = A$.

Therefore $Bd(A) \subseteq A$.

Conversely let $Bd(A) \subseteq A$.

Also $A \subseteq cl(A)$.

Now $cl(A) = A \cup bd(A)$

$$\subseteq A \cup A$$

$$= A$$

$\Rightarrow Cl(A) \subseteq A$

Therefore $cl(A) = A$.

Hence A is fuzzy closed set.

Note: If we expressed the fuzzy set and fuzzy closed set as existence definition of fuzzy set then the converse of the proposition is not true. But if we expressed with our new definition of fuzzy set then the converse part is also true. Let us cite an example.

Let X={a, b, c}and A={x, (a, 0.4), (b, 0.7), (c, 0.2)} and B={x, (a, 0.6), (b, 0.9), (c, 0.1)} be fuzzy sets on X. Then $\tau=\{0_X, 1_X$ A, B, $A\cup B$, $A\cap B\}$ be fuzzy topology in X.

Let P={ x, (a, 0.6), (b, 0.7), (c, 0.9)} be any fuzzy set in X.

Now $Cl(P)=\cap\{M \mid M$ is fuzzy closed set in X and $P\subseteq M\}$

$=1_X$

And $Cl(P^C)=\cap\{M \mid M$ is fuzzy closed set in X and $P^C\subseteq M\}$

$=A^C\cap(A\cap B)^C$

$=A^C$

$Bd(P)=Cl(P)\cap Cl(P^C)=1_X\cap A^C= A^C$

Hence $Bd(P)\subseteq P$. But P is not fuzzy closed set.

Now if we express the same problem in our definition with respect to reference function

Let X={a, b, c}and A={x, (a, 0.4, 0), (b, 0.7, 0), (c, 0.2, 0)} and B={x, (a, 0.6, 0), (b, 0.9, 0), (c, 0.1, 0)} be fuzzy sets on X. Then $\tau=\{0_X, 1_X$ A, B, $A\cup B$, $A\cap B\}$ be fuzzy topology in X.

Let P={ x, (a, 0.6, 0), (b, 0.7, 0), (c, 0.9, 0)} be any fuzzy set in X.

Now $Cl(P)=\cap\{M \mid M$ is fuzzy closed set in X and $P\subseteq M\}$

$=1_X$

And $Cl(P^C)=\cap\{M \mid M$ is fuzzy closed set in X and $P^C\subseteq M\}$

$=A^C\cap(A\cap B)^C$

$=A^C$

$Bd(P)=Cl(P)\cap Cl(P^C)=1_X\cap A^C=A^C=$ {x, (a, 1, 0.4), (b, 1, 0.7), (c, 1, 0.2)}.

So, Bd(P)= {x, (a, 1, 0.4), (b, 1, 0.7), (c, 1, 0.2)} and P = {x, (a, 0.6, 0), (b, 0.7, 0), (c, 0.9, 0)}, which is not comparable. This is happened because P is not closed set.

Thus it is clearly seen that the converse part of the proposition is also true when we use our definition of fuzzy set with respect to reference function.

9. Proposition: Let A be any fuzzy set in fuzzy topological spaces X. A is fuzzy open set if and only if bd(A) $\subseteq A^C$.
Proof:

Prove is straightforward.

10. Proposition: $(Bd(A))^C = Int(A) \cup Int(A^C)$

Proof: We have from the definition of fuzzy boundary $Bd(A) = Cl(A) \cap Cl(A^C)$.

Now

$(Bd(A))^C = (Cl(A) \cap Cl(A^C))^C$

$= (Cl(A))^C \cup (Cl(A^C))^C$

$= Int(A) \cup Int(A^C)$.

Example: Let X={a, b, c}and A={x, (a, 0.4, 0), (b, 0.7, 0), (c, 0.2, 0)} and B={x, (a, 0.6, 0), (b, 0.9, 0), (c, 0.1, 0)} be fuzzy sets on X. Then $\tau=\{0_X, 1_X\ A, B, A \cup B, A \cap B\}$ be fuzzy topology in X.

Let G={ x, (a, 0.5, 0), (b, 0.7, 0), (c, 0.9, 0)} be any fuzzy set in X.

Then $Cl(G)=1_X$, $Cl(G^C)=A^C \cap (A \cap B)^C = A^C$.

$Bd(G) = Cl(G) \cap Cl(G^C)$

$= 1_X \cap A^C$

$= A^C$

Also $(Bd(G))^C=A$.

Now

$$Int(G)=\{P|\ P \text{ is fuzzy open set in X and } P\subseteq G\}$$

$$=A\cup(A\cap B)$$

$$=A.$$

$$(Int(G^C))=0_X$$

$Int(G)\cup(Int(G^C))=A\cup 0_X=A$.

Hence $(Bd(A))^C=Int(A)\cup Int(A^C)$.

11. Proposition: Let A be any fuzzy set in fuzzy topological spaces X. $Bd(A)=Cl(A)-Int(A)$.

Proof: We have $(Cl(A^C))^C=Int(A)$.

Now

$$Bd(A)= Cl(A)\cap Cl(A^C)$$

$$=Cl(A)-(Cl(A^C))^C$$

$$=Cl(A)-Int(A).$$

Hence $Bd(A)=Cl(A)-Int(A)$.

Example: We try to prove this proposition from previous example.

Let X={a, b, c}and A={x, (a, 0.4, 0), (b, 0.7, 0), (c, 0.2, 0)} and B={x, (a, 0.6, 0), (b, 0.9, 0), (c, 0.1, 0)} be fuzzy sets on X. Then $\tau=\{0_X,1_X\ A, B, A\cup B, A\cap B\}$ be fuzzy topology in X.

Let G={ x, (a, 0.5, 0), (b, 0.7, 0), (c, 0.9, 0)} be any fuzzy set in X.

Then $Cl(G)=1_X$, $Cl(G^C)=A^C\cap(A\cap B)^C=A^C$.

$$Bd(G)= Cl(G)\cap Cl(G^C)$$

$=1_X \cap A^C$

$=A^C$

Also $(Bd(G))^C=A$.

Since, $Cl(G)=1_X$ and $Int(G)=A$.

Therefore $Cl(G) – Int(A) = Cl(G) \cap (Int(A))^C$

$=1_X \cap A^C$

$=A^C$.

Hence $Bd(G)=Cl(G) – Int(G)$.

12. Proposition: Let A be any fuzzy set in fuzzy topological spaces X. $Bd(Int(A)) \subseteq Bd(A)$.

Proof:

$Bd(Int(A)) = Cl(Int(A)) \cap Cl((Int(A))^C)$

$=Cl(Int(A)) – (Cl((Int(A))^C))^C$

$= Cl(Int(A)) – Int(Int(A))$

$= Cl(Int(A)) – Int(A)$

$\subseteq Cl(A) – Int(A)$

$=Bd(A)$.

Example:

Let X={a, b, c}and A={x, (a, 0.4, 0), (b, 0.7, 0), (c, 0.2, 0)} and B={x, (a, 0.6, 0), (b, 0.9, 0), (c, 0.1, 0)} be fuzzy sets on X. Then $\tau=\{0_X,1_X$ A, B, $A\cup B$, $A\cap B\}$ be fuzzy topology in X.

Let P={ x, (a, 0.4, 0), (b, 0.7, 0), (c, 0.1, 0)} be any fuzzy set in X.

$Int(P)=0_X$.

$Bd(Int(P)) = Bd(0_X)$

$= Cl(0_X) \cap Cl((0_X)^C)$

$= 1_X \cap 0_X$

$= 0_X$

Now

$Cl(P)=1_X$ and $Cl(P^C)=(A\cap B)^C$

$Bd(P)=(A\cap B)^C$

Hence $Bd(Int(P)) \subseteq Bd(P)$.

13. Proposition: Let A be any fuzzy set in fuzzy topological spaces X. $Bd(Cl(A)) \subseteq Bd(A)$.

Proof:

$Bd(Cl(A)) = Cl(Cl(A)) \cap Cl((Cl(A))^C)$

$= Cl(Cl(A)) - (Cl((Cl(A))^C))^C$

$= Cl(Cl(A)) - Int(Cl(A))$

$= Cl(A) - Int(Cl(A))$

$\subseteq Cl(A) - Int(A)$

$= Bd(A)$.

Example:

Let X={a, b, c}and A={x, (a, 0.4, 0), (b, 0.7, 0), (c, 0.2, 0)} and B={x, (a, 0.6, 0), (b, 0.9, 0), (c, 0.1, 0)} be fuzzy sets on X. Then $\tau=\{0_X, 1_X\ A, B, A\cup B, A\cap B\}$ be fuzzy topology in X.

Let Q={ x, (a, 0.6, 0), (b, 0.7, 0), (c, 0.8, 0)} be any fuzzy set in X.

$Cl(Q)=1_X$ and $Bd(Cl(Q))=Bd(1_X)=Cl(1_X) \cap Cl((1_X)^C)=0_X$.

Now $Cl(Q)=1_X$ and $Cl(Q^C)=A^C\cap(A\cap B)^C$

$Bd(Q)= 1_X \cap A^C\cap(A\cap B)^C= A^C\cap(A\cap B)^C$

Hence from this example $Bd(Cl(Q)) \subseteq Bd(Q)$.

14. Proposition: Let A be any fuzzy set in fuzzy topological spaces X. Then $Int(A)=A - Bd(A)$.
Proof:

$$A - Bd(A)= A\cap(Bd(A))^C$$

$$=A\cap(Cl(A) \cap Cl(A^C))^C$$

$$= A\cap((Cl(A))^C \cup (Cl(A^C)^C)$$

$$= A\cap((Cl(A))^C \cup (Int(A))$$

$$= (A\cap(Cl(A))^C) \cup (A\cap Int(A))$$

$$= (A\cap(Cl(A))^C) \cup Int(A)\text{...}(1)$$

Now to calculate $A\cap(Cl(A))^C$, let $A=\{x, \mu(x), 0; x\epsilon X \}$ then $Cl(A)=\{x, \max(\mu_i(x)), 0; x\epsilon X \}$,

where $\mu(x)), \leq\mu_i(x))$, and $(Cl(A))^C=\{x, 1, \max(\mu_i(x)); x\epsilon X \}$.

$$A\cap(Cl(A))^C=\{x, \mu(x), 0; x\epsilon X \}\cap\{x, 1, \max(\mu_i(x)); x\epsilon X \}$$

$$=\{x, \mu(x), \max (\mu_i(x)); x\epsilon X \}.$$

In this case we see three cases

Case-1 when $\mu(x)= \max (\mu_i(x))$, then $A\cap(Cl(A))^C=\phi$.

Case-2 when $\mu(x) \leq \max (\mu_i(x))$, then $A\cap(Cl(A))^C=\phi$.

Case-3 Since $\mu(x) \leq \mu_i(x)$, so it is clear $\mu(x)$ can't be greater than $\mu_i(x)$ hence $\mu(x)$ can't be greater than

$\max (\mu_i(x))$.

Hence $A\cap(Cl(A))^C=\phi$.

Now from (1) we have

$A - Bd(A)= \phi \cup Int(A)$

$= Int(A)$.

Example:

Let X={a, b, c}and A={x, (a, 0.4, 0), (b, 0.7, 0), (c, 0.2, 0)} and B={x, (a, 0.6, 0), (b, 0.9, 0), (c, 0.2, 0)} be fuzzy sets on X. Then $\tau=\{0_X,1_X$ A, B, $A\cup B$, $A\cap B\}$ be fuzzy topology in X.

Let E={ x, (a, 0.6, 0), (b, 0.7, 0), (c, 0.8, 0)} be any fuzzy set in X.

Then $Cl(E)=1_X$, $Cl(E^C)=A^C$. $Bd(E)=A^C$.

$E - Bd(E)=E\cap A=A$.

Also Int(E)=A.

Hence E – Bd(E)=Int(E).

15. Proposition: Let A and B be fuzzy sets in fuzzy topological space X. Then $Bd(A\cup B) \subseteq Bd(A) \cup Bd(B)$.

Proof:

$Bd(A\cup B) =Cl(A\cup B)\cap Cl((A\cup B)^C)$

$\subseteq Cl(A)\cup Cl(B)\cap(Cl(A^C)\cap Cl(B^C))$

$= (Cl(A)\cap(Cl(A^C)\cap Cl(B^C)))\cup(Cl(B)\cap(Cl(A^C)\cap Cl(B^C)))$

$=(Bd(A))\cap Cl(B^C))\cup(Bd(B)\cap Cl(A^C))$

$\subseteq Bd(A) \cup Bd(B)$.

Example:

Let X={a, b, c}and A={x, (a, 0.4, 0), (b, 0.7, 0), (c, 0.2, 0)} and B={x, (a, 0.6, 0), (b, 0.9, 0), (c, 0.1, 0)} be fuzzy sets on X. Then τ={0_X,1_X A, B, A⋃B, A∩B} be fuzzy topology in X.

Let U={x, (a, 0.3, 0), (b, 0.2, 0), (c, 0.1, 0)} and V={x, (a, 0.7, 0), (b, 0.7, 0), (c, 0.8, 0)}

U∪V={x, (a, 0.7, 0), (b, 0.7, 0), (c, 0.8, 0)}

Cl(U∪V)=1_X and Cl($(U\cup V)^C$)=A^C so Bd(U∪V)=A^C

Also Cl(U)=1_X, Cl(U^C)=1_X and Cl(V)=1_X, Cl(v^C)=A^C

Bd(U)= 1_X and Bd(V)=A^C.

Now Bd(U) ∪ Bd(V)=1_X.

Hence Bd(U∪V) ⊆ Bd(U) ∪ Bd(V).

16. Proposition: Let A and B be fuzzy sets in fuzzy topological space X.
Then Bd(A∩B) ⊆ Bd(A) ∪ Bd(B).

Proof: Prove is straightforward.

CONCLUSION

In this article we attempted definition of fuzzy set on the basis of reference function to give definition of fuzzy boundary and to prove some propositions on fuzzy boundary because there are some drawbacks in the existing definition of complement of fuzzy set. It is seen that some propositions of fuzzy boundary satisfy the condition as propositions of boundary of classical topology could satisfy. If we apply existing definition of fuzzy set then all the proposition of fuzzy boundary cannot satisfy as classical boundary could do. Our main aim is to revisit the drawback of the propositions and try to improve the definitions or propositions so that we could get better result not to contradict others definition. We hope our work will help for future work of fuzzy topology.

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