CHAPTER 1

INTRODUCTION

1.1 BRIEF HISTORY:

The concept of fuzzy set was first introduced by Lotfi Zadeh in 1965. After the discovery of Fuzzy set by Zadeh [86], thoughts of the Fuzzy topology have been discussed by Chang [20] who introduced the concept of fuzzy topological spaces as an extension to classical topological spaces. After that many authors studied topological properties under fuzzy settings and suggested different definitions for the same property which leads to different approaches. Also some of the properties, theorems of the fuzzy topological space are not same as classical topological does. Fuzzy topology, fuzzy interior, fuzzy closure, fuzzy point and fuzzy boundary are some basic concepts which play an important role in fuzzy topology.

Fuzzy set theory and fuzzy topology are approached as new generalizations of ordinary set theory and ordinary topology. Chang [20] introduced the concept of a fuzzy topology on a set X as a family $\tau \subset I^X$, where I = [0,1] satisfying the well – known axioms, and he referred to each member of τ as an open set. He also defined fuzzy topology by a membership set on an arbitrary complete and distributive lattice. While analyzing his definition of a fuzzy topology some authors have noticed that fuzziness in the concept of openness of a fuzzy set has not been considered. Keeping this in mind, A. P. Shostak [80] began the study of fuzzy structures of topological type and stated a new fuzzy topology concept. Observing this K. C. Chattopadhyay et al.[21] redefined the Shostak′s fuzzy topology concept and gave the name the gradation of openness by function τ.

But the fundamental concept of fuzzy sets introduced by Zadeh in 1965 [86] has been using in topology and in some other branches of analysis. With this concept many authors have extensively developed the theory of fuzzy sets and applications.

In 1968, C.L. Chang [20] used the fuzzy set theory for defining and introducing fuzzy topological spaces. In 1973, C.K.Wong [85] discussed the covering properties of fuzzy topological spaces.

P.P. Ming and L.Y. Ming [74] in 1980, used fuzzy topology to define neighborhood structure of fuzzy point and Moore - Smith convergence.

1.2 OBJECTIVES OF THE RESEARCH

The main objectives of this research are to bring about an attention of the fact that there are some drawbacks in the existing definition of complementation of fuzzy sets and the results associated with it. By observing these drawbacks, we make an attempt to extend some basic concepts and results, particularly the idea of reference function to fuzzy setting and to study the properties of fuzzy function, fuzzy interior, fuzzy closure, fuzzy points and fuzzy boundary of fuzzy topology. Considering these points the present research has been undertaken with the following objectives:

(i) To implement the new definition of fuzzy function and fuzzy topology on the basis of reference function.

(ii) To define fuzzy interior, fuzzy closure and fuzzy point of fuzzy set on the basis of reference function.

(iii) To propose some properties of fuzzy interior, fuzzy closure and fuzzy point of fuzzy set on the basis of reference function.

(iv) To define fuzzy boundary of fuzzy set on the basis of reference function.

(v) To propose some properties of fuzzy boundary of fuzzy set on the basis of reference function.

(vi) To propose some properties of fuzzy (τ_i, τ_j)-r-boundary of fuzzy bitopological spaces on the basis of reference function.

The main focus of this research is to develop a new methodology to discuss more appropriately the result obtained through conventional methods and to apply the proposed methodology in different areas.

1.3 DISCUSSION ON FUZZINESS

Fuzziness is associated with subjective judgment. In real world, complexity often arises from uncertainty in the form of ambiguity. The theory of probability has been an age old and effective tool to handle uncertainty, but it can be applied only to situations whose characteristics are based on random processes. Uncertainty may also arise due to partial information about the problem, or due to information which is not fully reliable. The mathematics of fuzziness is an excellent mathematical tool to handle the uncertainty arising due to vagueness. From an historical point of view the issue of uncertainty has not always been embraced within the scientific community (Klir and Yuan[39]). The leading theory in quantifying uncertainty in scientific models from the late nineteenth century until the late twentieth century had been probability theory. However, the gradual evolution of the expression of uncertainty using probability theory was challenged, first in 1937 by Max Black [19], an American philosopher, with his studies in vagueness, then with the introduction of fuzzy sets by Lotfi Zadeh [86] in 1965. Since then the application of the theory of fuzzy sets has started. The theory was further developed and refined by Dubois and Prade [31], Kandel and Lee [36] and Kauffman [37].

Fuzzy set theory is applied in any field in which, issues of complexity arise. It is now being applied in almost all branches of knowledge such as management science, information retrieval, process control, clustering, pattern recognition, decision making, biology, medicine, system theory, operations research and so on. In the last three decades, significant progress has been made in the development of fuzzy set and fuzzy logic theory and their use in large varieties of applied topics where uncertainty, vagueness and ambiguity are involved. The idea of fuzziness can be found in many areas of daily life where the human judgment, evaluation and decision are important. By Zimmerman [88], aplications of fuzzyness in different areas have been noticed such as applications to mathematics that is generalizations of traditional mathematics such as topography, graph theory, algebra, logic and so on. Applications to algorithms such as clustering methods, control algorithm, mathematical programming and so on. Application to standard models such as transportation model, inventory control model, maintenance model etc. Application to real world problems of different kinds. The fuzziness has the most promising areas of applications in the field of pattern recognition which extends into the areas of image processing, medical diagnosis, speech and handwriting recognition and many others.

1.3.1 FUZZY SET

The notion of fuzzy sets is an extension of the most fundamental property of sets. Fuzzy sets allow a grading of *to what extent* an element of a set belongs to that specific set. It is defined as the class of objects with a continuum of grades of membership. For a fuzzy set, every value has a membership value, and so is a member to some extent. The membership value defines the extent to which a variable is a member of a fuzzy set, taking value between 0 and 1. Unlike classical or crisp set, where an element either belongs or does not belong to the set, in a fuzzy set there is a gradual assessment of the membership of elements in a set.

If X be a universal set and x be any element particular element of X, then a fuzzy set A, defined on X may be written as a collection of ordered pairs

$A = \{(x, \mu_A(x); x \epsilon X\}$, where $\mu_A(x): X \rightarrow [0, 1]$, is called the membership function or grade of membership of x in A.

Fuzzy set theory plays a very important role in day to day practical life. Different authors have done a lot of research in this field till date (Konda *et al.[41]*; Kumar and Schuhmacher,[42] and many others). They have forwarded their research work by differentiating the concept of randomness and fuzziness separately. However, the essence of this present research work has been based on a result linking fuzziness with randomness. The link between fuzziness and randomness has been initiated by Baruah ([8], [9], [10], [11], [12], [13], [14]) named as *Randomness-Fuzziness Consistency Principle*. According to this principle, every normal fuzzy number can be expressed in terms of two law of randomness.

1.3.2 Membership Function

All information contained in a fuzzy set is described by its membership function. The membership function of a fuzzy set defines how the grade of membership of an element in the set is determined. It is graphical representation of the magnitude of participation of each input. It associates a weighting with each of the inputs that are processed, defines functional overlaps between inputs, and ultimately determines an output response. The following type is commonly used to denote membership function.

The membership function of a fuzzy set A is denoted by $\mu_A(x)$: X $\rightarrow$[0, 1].

In this research work, we used the notion $\mu_A(x)$: X $\rightarrow$[0, 1] to denote membership function.

Generally, the membership functions are triangular in shape, but trapezoidal, bell and exponential shape membership functions are also used.

The *core* of a membership function for a fuzzy set *F* is defined as that region of the universe that is characterized by complete and full membership in the set *F*, i.e. *core* (*F*) = $\{x \in X \mid \mu_F(x) = 1\}$

The *support* of a membership function for a fuzzy set *F* is defined as that region of the universe that is characterized by nonzero membership in the set *F*, i.e.

Support (*F*) = $\{x \in X \mid \mu_F(x) > 0\}$

The *boundaries* of a membership function for a fuzzy set *F* are defined as that region of the universe containing elements that have a nonzero membership but not complete membership, i.e. *Boundary* (F) = $\{x \in X \mid 0 < \mu_F(x) < 1\}$

The *height* of a fuzzy set F is the maximum value of the membership function, i.e.

$hgt\ (F) = \max\ \{\mu_F\ (x)\}$

If the $hgt\ (F) = 1$, the fuzzy set is said to be *normal* fuzzy set and if $hgt\ (F) < 1$, the fuzzy set is said to be *subnormal*.

1.3.3 BASIC OPERATIONS ON FUZZY SETS

Three basic operations on crisp sets- complement, intersection and union can be generalized to fuzzy sets in more than one way. However, one particular generalization which results in operations that are usually referred to as standard fuzzy set operations has a specific significance in fuzzy set theory.

Let us consider two fuzzy sets A and B in the universal set X defined with membership functions

$$A = \{(x, \mu_A(x); x \in X\} \text{ and}$$

$$B = \{(x, \mu_B(x); x \in X\}$$

Then following operations can be observed

(i) Equality: The fuzzy sets A and B are equal denoted by A=B if and only if for every $x \in X$, $\mu_A(x) = \mu_B(x)$.

(ii) Subset: Fuzzy set A is subset of fuzzy set B denoted by $A \subseteq B$ if for every $x \in X$, $\mu_A(x) \leq \mu_B(x)$.

(iii) Complement: The fuzzy set A^C is complement of fuzzy set A if $\mu_A{}^C(x) = 1 - \mu_A(x)$.

(iv) Union: The union of two fuzzy sets A and B denoted by $A \cup B$ is defined by

$$\mu_{A \cup B}(x) = \max\{\mu_A(x), \mu_B(x)\}$$

(v) Intersection: The intersection of two fuzzy sets A and B denoted by $A \cap B$ is defined by $\mu_{A \cap B}(x) = \min\{\mu_A(x), \mu_B(x)\}$

1.3.4 PROPERTIES OF FUZZY SETS

Let A, B and C are fuzzy sets, then some properties of are shown below

(i) Idempotent: $A \cap A = A$ and $A \cup A = A$

(ii) Commutative: $A \cap B = B \cap A$ and $A \cup B = B \cup A$

(iii) Associative: $(A \cap B) \cap A = A \cap (B \cap C)$ and $(A \cup B) \cup C = A \cup (B \cup C)$

(iv) Distributive: $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ and $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(v) Double complement: $(A^C)^C = A$

(vi) DeMorgan's laws: $(A \cap B)^C = A^C \cup B^C$

(vii) Exclusion: $A \cup A^C \neq X$

(viii) Contradiction: $A \cap A^C \neq \phi$

(ix) Identity: $A \cup \phi = A$ and $A \cap X = A$, where ϕ and X are empty and universal set respectively.

It is very important to observe, for fuzzy set the excluded middle laws do not hold.

Now let us discussed about the complement of fuzzy set

Consider first the usual definition of a fuzzy number. Let A be a fuzzy number characterized by $A = \{x, \mu(x); x \in X\}$. Its complement A^C is characterized by $A^C = \{x, (1 - \mu(x)); x \in X\}$. The diagram concerned looks as the one shown in Figure 1. Now A and A^C looks like having something common, that is why it has been accepted that $A \cap A^C \neq \varphi$. Further, as looks obvious from the diagram, $A \cup A^C \neq X$. For these two inequalities, it has been accepted that the fuzzy sets do not form a field.

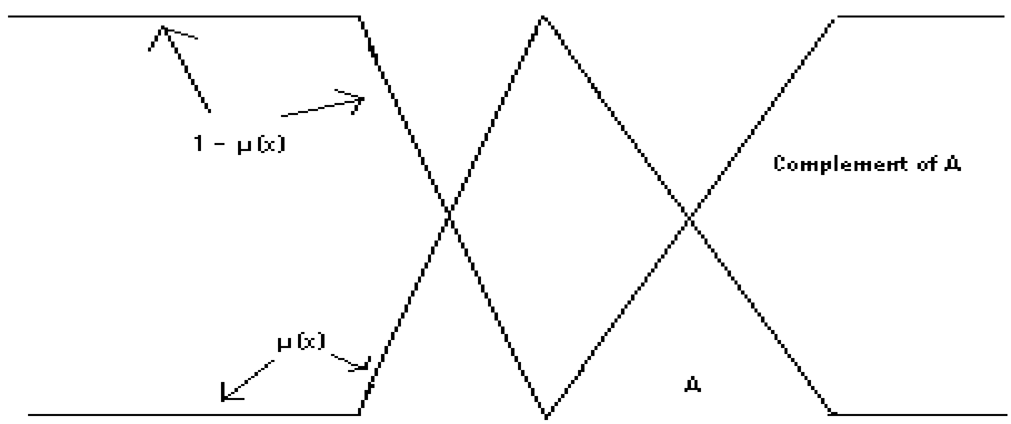

Figure 1 Complement of a Fuzzy Set with existing definition of fuzzy set

Zadeh introduced Fuzzy Set Theory in 1965. The concept of fuzzy set corresponds to the physical situation in which there is no precisely defined criterion for membership, fuzzy sets are having useful and increasing applications in various fields including probability theory, information theory and pattern recognition. Thus the developments in abstract mathematics using the idea of fuzzy sets possess sound footing. In accordance with this like general topology, which was developed using the idea of classical sets, there is another topology, called fuzzy topology which is developed by using the idea of fuzzy sets. It was introduced by Chang [20] in 1968. Thus fuzzy topology is a generalization of topology in classical mathematics. But it also has its own marked characteristics. It can deepen the understanding of basic structure of classical mathematics, offers new methods and results and obtain significant results of classical mathematics. Moreover, it has applications in some important respects of science and technology.

Uncertainty remained an unresolved universal dilemma. Uncertainty intrudes into the plans for the future through ineffective interpretations of the past, and while making precise decisions for the present. The development of measures of uncertainty in mathematical systems has been evolved as a major component of Klir's research undertaken by Klir [39, 40]. This has paved the way to classify uncertainty into two major categories - fuzziness, which deals with information that is indistinct, and ambiguity, which deals with multiplicity.

1.4 FUZZY FUNCTION

Let f: X→Y be a function, A and B be fuzzy sets on X and Y respectively. Then f(A) is a fuzzy set in Y, defined by

$$f(A)=\begin{cases} Sup\{A(x): x\epsilon f^{-1}(y)\} & if \quad f^{-1}(y) \neq \emptyset \\ 0 & if \quad f^{-1}(y) = \emptyset \end{cases}$$

and f^{-1}(B) is a fuzzy set in X, defined by f^{-1}(B)(x)=B(f(x)), xϵX.

1.4.1 SOME PROPERTIES ON FUZZY FUNCTION

1. $f^{-1}(B^C) = f^{-1}(B))^C$, for fuzzy set B in Y.
2. $f(f^{-1}(B)) \subseteq B$, for fuzzy set B in Y.
3. $A \subseteq f^{-1}(f(A))$, for fuzzy set in X.

1.5 FUZZY TOPOLOGY

A fuzzy topology on a nonempty set X is a family τ of fuzzy set in X satisfying the following axioms

(T1) $0_X, 1_X \in \tau$

(T2) $G_1 \cap G_2 \in \tau$, for any $G_1, G_2 \in \tau$

(T3) $\bigcup G_i \in \tau$, for any arbitrary family $\{G_i : G_i \in \tau, i \in I\}$.

In this case the pair (X,τ) is called a fuzzy topological space and any fuzzy set in τ is known as fuzzy open set in X and clearly every element of τ^C is said to be fuzzy closed set.

1.5.1 Interior of a fuzzy set

Let (X, τ) be fuzzy topology also $A=\{x, \mu_A(x), \gamma_A(x)\}$ be fuzzy set on X.

Then interior of a fuzzy set A is defined as union of all open subsets contained in A, denoted it as int(A) and is defined as follows

$\text{Int}(A)=\bigcup \{P : P \text{ is open set in X and } P \subseteq A \}$

1.5.2 Closure of fuzzy set

Let (X, τ) be fuzzy topology and A={x, μ(x), γ(x); xϵX} be fuzzy set in X. Then fuzzy closure of A are defined by

Cl(A)=$\bigcap$ {G: G is fuzzy closed set in X and A$\subseteq$G}

1.6 Fuzzy Boundary: Let A be a fuzzy set in fuzzy topological space X. Then the fuzzy boundary of A is defined as Bd(A)=cl(A)$\cap$cl(A^C).

1.7 Controversies over the fuzzy set theory

There is controversy over the application of fuzzy set theory. Fuzzy set theory is not the panacea for dealing with the world of uncertainty in certain terms, but it is not contender. Many authors mentioned in their research papers that theory of fuzzy sets criticized some concepts of the theory whereas some authors made an attempt to redefine it from their standpoint. Shimoda[61] presented a new and natural interpretation of fuzzy sets and fuzzy relations, but still did not change the fact that it could not satisfy all the properties and formulas of the classical set. Piegat [73] also presented new definition of the fuzzy set but nothing about essential shortcomings and mistakes of Zadeh's fuzzy set theory and how to overcome them completely was discussed.

Shi Gao et. al. [22] found that there is some mistakes Zadeh's fuzzy sets and found that it is incorrect to define the set complement as because it can be shown that set complement may not exist in Zadeh's fuzzy set theory. Since they found some drawbacks in Zadeh's fuzzy set theory, they wanted to move away the drawback and worked towards removing the drawbacks which according to them debarred fuzzy sets to satisfy all the properties of classical set. They introduced a new fuzzy set theory, called C-fuzzy set theory which satisfies all the formulas and properties of classical sets.

Baruah ([8, 9, 10, 11, 12, 13, 14]) seen that there are some drawbacks in the theory of fuzzy sets. He observed that the complementation of fuzzy sets and probability-possibility consistency principles are not well defined. He defined the complementation of fuzzy set on the basis of reference function to make it perfect.

1.8 FUZZY SET ON THE BASIS OF REFERENCE FUNCTION

Let $\mu_1(x)$ and $\mu_2(x)$ be two functions such that $0 \leq \mu_2(x) \leq \mu_1(x) \leq 1$. For fuzzy number denoted by $\{x, \mu_1(x), \mu_2(x); x \epsilon X\}$, we call $\mu_1(x)$ as fuzzy membership function and $\mu_2(x)$ a reference function such that $(\mu_1(x) - \mu_2(x))$ is the fuzzy membership value for any x in X.

1.8.1 Basic operations

Let $A=\{x, \mu_1(x), \mu_2(x); x \epsilon U\}$ and

$B=\{x, \mu_3(x), \mu_4(x); x \epsilon U\}$ be two fuzzy sets defined over the same universe U.

1. $A \subseteq B$ iff $\mu_1(x) \leq \mu_3(x)$ and $\mu_4(x) \leq \mu_2(x)$ for all $x \epsilon U$.

2. $A \cup B = \{x, \max(\mu_1(x), \mu_3(x)), \min(\mu_2(x), \mu_4(x))\}$ for all $x \epsilon U$.

3. $A \cap B = \{x, \min(\mu_1(x), \mu_3(x)), \max(\mu_2(x), \mu_4(x))\}$ for all $x \epsilon U$.

If for some $x \epsilon U$, $\min(\mu_1(x), \mu_3(x)) \leq \max(\mu_2(x), \mu_4(x))\}$, then our conclusion will be $A \cap B = \phi$

4. $A^C = \{x, \mu_1(x), \mu_2(x); x \epsilon U\}^C$

$= \{x, \mu_2(x), 0; x \epsilon U\} \cup \{x, 1, \mu_1(x); x \epsilon U\}$

5. If D = {x, μ(x), 0; x∈U}, then D^C={x, 1, μ(x); x∈U} for all x∈U.

Now we shall discuss some propositions regarding fuzzy topology considering the concepts of reference function, which are as follows:

Now let us see some propositions of fuzzy sets on the basis of reference function

1.8.2 Proposition:

Let τ={A_i : i∈I} be a collection of fuzzy sets over the same universe X. Then

1. $\{\bigcup_i A_i\}^C = \bigcap_i A_i^C$

2. $\{\bigcap_i A_i\}^C = \bigcup_i \{A_i\}^C$

Proof :

1.

Case1- When reference function is zero. Let A_i={x, $\mu_i(x)$, 0; x∈X}.

Then $\{\bigcup_i A_i\}^C$={x, max($\mu_i(x)$), 0; x∈X}^{C}

={x, 1, max($\mu_i(x)$); x∈X}

And A_i^C={x, 1, $\mu_i(x)$; x∈X}.

$\bigcap \{A_i^C\}$ ={x, 1, max($\mu_i(x)$); x∈X}.

Hence, $\{\bigcup_i A_i\}^C = \bigcap_i A_i^C$

Case-2

When reference function is not zero.

Let $A_i=\{x, \mu_i(x), \gamma_i(x); x\epsilon X\}$.

$\{\bigcup_i A_i\}^C=\{x, \max(\mu_i(x)), \min(\gamma_i(x)); x\epsilon X\}^C$

$=\{x, 1, \max(\mu_i(x)); x\epsilon X\} \cup \{x, \min(\gamma_i(x)), 0; x\epsilon X\}$

$\bigcap_i A_i^C=\bigcap_i \{\{x, 1, \max(\mu_i(x)); x\epsilon X\} \cup \{x, \min(\gamma_i(x)), 0; x\epsilon X\}\}$

$=\{\bigcap_i \{x, 1, \max(\mu_i(x)); x\epsilon X\}\} \cup \{\bigcap_i \{x, \min(\gamma_i(x)), 0; x\epsilon X\}\}$

$=\{x, 1, \max(\mu_i(x)); x\epsilon X\} \cup \{x, \min(\gamma_i(x)), 0; x\epsilon X\}$.

Thus $\{\bigcup_i A_i\}^C=\bigcap_i A_i^C$.

2. Proof: Following result proposition 1.8.2.(1) we can prove 1.8.2(2).

Example: Let X={a,b,c} and

A_1={(a, 0.5, 0), (b, 0.2, 0), (c, 0.3, 0)},

A_2={(a, 0.6, 0), (b, 0.3, 0), (c, 0.4, 0)}

A_3={(a, 0.2, 0), (b, 0.7, 0), (c, 0.8, 0)}.

$\bigcup_{i=1}^{3} A_i$={(a, 0.6, 0), (b, 0.7, 0), (c, 0.8, 0)}.

$\{\bigcup_{i=1}^{3} A_i\}^C$={(a, 1, 0.6), (b, 1, 0.7), (c, 1, 0.8)}.

Now, A_1^C=={(a, 1, 0.5), (b,1, 0.2), (c, 1, 0.3)},

A_2^C ={(a, 1, 0.6), (b, 1, 0.3), (c, 1, 0.4)}

A_3^C={(a, 1, 0.2), (b, 1, 0.7), (c, 1, 0.8)}.

$\bigcap_{i=1}^{3} A_i^C$={(a, 1, 0.6), (b, 1, 0.7), (c, 1, 0.8)}

Thus $\{\bigcup_i A_i\}^C = \bigcap_i A_i^C$

1.8.3 Proposition:

For a fuzzy set A={x, μ(x), γ (x); x∈U}.

$$(A^C)^C = A.$$

1.8.4. Proposition:

For a fuzzy set A

1. $A \cap A^C = \phi$
2. $A \cup A^C = U$.

Proof :

1.

Case 1

When reference function is zero. Let A={x, μ(x), 0; xϵU}.

Then A^C={x, 1, μ(x); xϵU}.

$A \cap A^C = \{x, \mu(x), 0; x\epsilon U\} \cap \{x, 1, \mu(x); x\epsilon U\}$

$= \{x, \min(\mu(x), 1), \max(0, \mu(x)); x\epsilon U\}$

$= \{x, \mu(x), \mu(x); x\epsilon U\}$

$= \phi$

Case-2

When reference function is not zero.

Here we shall consider the definition of fuzzy sets as introduced by [6].

Let A={x, μ(x), γ(x); xϵU}.

Then $A^C = \{x, \gamma(x), 0; x\epsilon U\} \cup \{x, 1, \mu(x); x\epsilon U\}$.

$A \cap A^C = \{x, \mu(x), \gamma(x); x\epsilon U\} \cap [\{x, \gamma(x), 0; x\epsilon U\} \cup \{x, 1, \mu(x); x\epsilon U\}]$

$= [\{x, \mu(x), \gamma(x); x\epsilon U\} \cap \{x, \gamma(x), 0; x\epsilon U\}] \cup [\{x, \mu(x), \gamma(x); x\epsilon U\} \cap \{x, 1, \mu(x); x\epsilon U\}]$

$= \{x, \min(\mu(x), \gamma(x)), \max(\gamma(x), 0); x\epsilon U\} \cup \{x, \min(\mu(x), 1), \max(\gamma(x), \mu(x)); x\epsilon U\}$

$=\{x, \gamma(x), \gamma(x); x\epsilon U\} \cup \{x, \mu(x), \mu(x); x\epsilon U\}$

$=\phi \cup \phi$

$=\phi$.

Proof: 2

Case1

When reference function is zero.

Let $A=\{x, \mu(x), 0; x\epsilon U\}$.

Then $A^C=\{x, 1, \mu(x); x\epsilon U\}$.

$A \cup A^C= \{x, \mu(x), 0; x\epsilon U\} \cup \{x, 1, \mu(x); x\epsilon U\}$

$=\{x, \max(\mu(x), 1), \min(0, \mu(x)); x\epsilon U\}$

$=\{x, 1, 0; x\epsilon U\}$

$=U$.

Case-2

When reference function is not zero. Let $A=\{x, \mu(x), \gamma(x); x\epsilon U\}$.

Then $A^C=\{x, \gamma(x), 0; x\epsilon U\} \cup \{x, 1, \mu(x); x\epsilon U\}$.

$A \cup A^C=\{x, \mu(x), \gamma(x); x\epsilon U\} \cup [\{x, \gamma(x), 0; x\epsilon U\} \cup \{x, 1, \mu(x); x\epsilon U\}]$

$= [\{x, \mu(x), \gamma(x); x\epsilon U\} \cup \{x, \gamma(x), 0; x\epsilon U\}] \cup \{x, 1, \mu(x); x\epsilon U\}$

$=\{x, \max(\mu(x), \gamma(x)), \min(\gamma(x), 0); x\epsilon U\} \cup \{x, 1, \mu(x); x\epsilon U\}$

$=\{x, \mu(x), 0; x\epsilon U\} \cup \{x, 1, \mu(x); x\epsilon U\}$

$=\{x, \max(\mu(x), 1), \min(0, \mu(x)); x\epsilon U\}$

$=\{x, 1, 0; x\epsilon U\}$

$=U.$

Example:

1. Let $U=\{a, b, c\}$

$A=\{(a, 0.1, 0), (b, 0.3, 0), (c, 0.7, 0)\}$

$A^C=\{(a, 1, 0.1), (b, 1, 0.3), (c, 1, 0.7)\}$

$A \cup A^C=\{(a, \max(0.1, 1), \min(0, 0.1)), (b, \max(0.3, 1), \min(0, 0.3)), (c, \max(0.7, 1), \min(0, 0.7))\}$

$=\{(a, 1, 0), (b, 1, 0), (c, 1, 0)\}$

$=U$

Also,

$A \cap A^C=\{(a, \min(0.1, 1), \max(0, 0.1)),$

$(b, \min(0.3, 1), \max(0, 0.3)),$

(c, min(0.7, 1), max(0, 0.7))}

={(a, 0.1, 0.1), (b, 0.3, 0.3), (c, 0.7, 0.7)}

=ϕ

ORGANIZATION OF THE THESIS

This thesis has been organized into five chapters each consisting of several subsections.

Chapter 1: Introduction.

Chapter 2: Methodology.

Chapter 3: Fuzzy set and Fuzzy Topology.

Chapter 4: Fuzzy closure and Fuzzy Point.

Chapter 5: Fuzzy Boundary.

Chapter 6: Fuzzy (τ_i, τ_j)-r-Boundary of fuzzy bitopological spaces on the basis of reference function.

Now briefly discussions of our research topics are as follows:

Chapter 1: Generally it is an introductory chapter presents some basic concepts o fuzzy set theory and fuzzy topology and its importance. In this chapter, literature survey of the various approaches such as fuzzy set, fuzzy topology, fuzzy interior, fuzzy closure, fuzzy point and fuzzy boundary are discussed. It also deals with the structure of the thesis.

Chapter 2: In this chapter we are presenting the proposed methodologies of this research work. Here, new definition of complementation of fuzzy set on the basis of reference function is discussed. Also, a theoretical background of constructing the membership function of a fuzzy number is discussed. These are the methods on which our works on the subsequent chapters are carried out.

Chapter 3: It covers the expression of fuzzy sets with examples and fuzzy topology. In this chapter we have proposed new definition fuzzy topology on the basis of reference function also example is given. New definition of fuzzy function is given with new definition of fuzzy set. Also some propositions of fuzzy functions are discussed on the basis of reference function.

Chapter 4: It deals with fuzzy closure and fuzzy point on the basis of reference function. In this chapter new definition of fuzzy closure and fuzzy point is proposed. Some propositions of fuzzy closure and fuzzy point are proved on the basis of reference function.

Chapter 5: This chapter is about fuzzy boundary here we proposed new definition of fuzzy boundary. Also some propositions of fuzzy boundary are discussed in connection with classical boundary of classical topology. In this chapter it is seen that all the propositions of fuzzy boundary are satisfies as proposition of classical boundary could satisfy if we express fuzzy set on the basis of reference function.

Chapter 6: This chapter is about fuzzy (τ_i, τ_j)-r-Boundary of fuzzy bitopological spaces. Here we proposed new definition of fuzzy (τ_i, τ_j)-r-closed set and fuzzy (τ_i, τ_j)-r-boundary. Also some propositions of fuzzy (τ_i, τ_j)-r-boundary are discussed in connection with classical (τ_i, τ_j)-boundary of classical bitopology.

References: A details reference of the relevant literature is given in the last part of this thesis.