Chapter 5

Higher Dimensional Cosmological Model Universe with Special Law of Hubble's Parameter in Lyra Geometry

5.1 Introduction:

In spite of the various efforts made by the cosmological and astrophysical scientists about the future evolution of the universe together with the comprehension of the present and past state of the universe, we are not able to ascertain a final statement about the origin and evolution of the universe. But from the various results of the observational experiments from different sources like high red-shift supernovae Type Ia [Riess et al. (1988, 2004); Perlmutter et al. (1999); Astier et al. (2006); Amanullah et al. (2010); Suzuki et al. (2012)], CMB (Cosmic Microwave Background) anisotropies [Bennett et al. (2003); Spergel et al. (2007)], the large-scale galaxies structures of universe [Daniel et al. (2008); Allen et al. (2004)], WMAP (Wilkinson Microwave Anisotropy Probe) [Hinshaw et al. (2013)], Atacama Cosmology Telescope (ACT) [Sievers et al. (2013)], Sachs-Wolfe effects [Nishizawa (2014)] and SDSS [McCarthy et al. (2008)], Planck [Ade et al. (2014, 2016)], it can be said with direct and indirect evidence about the cosmic acceleration.

The unidentified component of energy usually known as dark energy with positive energy density and negative pressure is regarded as the prime cause of outstanding transformation of accelerating the expansion of the universe in the cosmic history. Several approaches have been made to understand the dark energy. In present days, the investigation for the source of negative pressure or the best fitted dark energy candidate has become one of the centers of research in cosmology. Though cosmological constant is supposed to be the simplest candidate of dark energy, in recent years many cosmologists suggested many alternative models in different directions like inclusion of cosmological constant (Peebles and Ratra 2003; Sahni and Starobinsky 2000), modified theories of gravity (Bamba et al. 2012), Lyra Geometry (Scheibe 1952; Sen 1957), models with different equations of state (Rao et al. 2015; Reddy et al. 2015), scalar fields (Nordvedt 1970; Barker 1978), higher dimensional theories (Kaluza, 1921; Witten, 1995), dynamical models (Copeland et al., 2006) to unfold this mystery.

Some of the frequently discussed important modified theories of gravitation are Kaluza-Klein theory of higher dimensions (Kaluza, 1921; Klein, 1926; Witten, 1984), Brans-Dicke theory (Brans and Dicke, 1961), Bimetric theory (Rosen, 1973), Scalar-tensor theory (Barker, B. M. 1978), F(R) Gravity (Buchdahl, 1970; Nojiri et al., 2003; Singh, Bishi and Sahoo, 2016a,b), Mimetic Gravity (Chamseddine et al., 2013), Mimetic F(R) gravity (Nojiri et al., 2014b), Lyra geometry (Scheibe 1952) and many more. Among all these modified theories of gravitation, here we will consider about the Lyra geometry, which is nothing but a modification of Riemannian geometry. Introducing a gauge function into the structure-less manifold, Lyra (1951) modified the Riemannian geometry, which removes the non-integrability condition of the length of a vector under parallel transport. Lyra geometry together with constant gauge vector ϕ_k will either play the role of the cosmological constant or creation field (equal to Hoyle's creation field [Hoyle (1948); Hoyle and Narlikar (1963); Hoyle and Narlikar (1964)]) which is discussed by Soleng (1987). Halford (1970, 1972) suggested that the constant displacement vector field ϕ_k in Lyra's geometry plays the role of cosmological constant in the normal general relativistic treatment and the scalar-tensor treatment in the framework of Lyra's geometry predict the same effect, within observational limits as in the Einstein's theory of relativity.

Different works of literature reveal (it has been observed) that the anisotropic Bianchi type model universes do not isotropize adequately as they evolve into future. But, it has been established from the latest observations like cosmic microwave background radiation (Bennett et al. 2003) and large-scale structure (Tegmak et al. 2004) that our universe is isotropic and highly homogeneous on large scale. The anisotropy problem of Bianchi type model can be solved by inflation. Wald (1983) showed that, like de Sitter model, the Bianchi type models in presence of a positive cosmological constant approach to isotropic and spatially homogeneous model universes.

Quadratic equation of state plays a crucial role, in order to study the dark energy and general relativistic dynamics for various cosmological models. Considering different equations of state, authors like, Capozziello et al. (2006a), Nojiri and Odintsov (2004, 2005a), Nojiri et al. (2005b) and Bamba et al. (2012) studied the dark energy universe and showed that the Quadratic equation of state may possibly describe dark energy or unified dark matter. The general form of the quadratic equation of state is given by

$$p = p_0 + \alpha\rho + \beta\rho^2 \ ,$$

where p_0, α, β are the parameters, which is nothing but the first term of the Taylor's expansion of any equation of state of the form $p = p(\rho)$ about $\rho = 0$.

Ananda and Bruni (2006) studied the general relativistic dynamics of Robertson-Walker models with a non-linear equation of state (EoS), focusing on the quadratic equation of state $p = p_0 + \alpha\rho + \beta\rho^2$. They have shown that the behavior of the anisotropy at the singularity found in the brane scenario can be recreated in the general relativistic context. Also by considering quadratic equation of state of the form

$$p = \alpha\rho + \frac{\rho^2}{\rho_c} \ ,$$

they have discussed the anisotropic homogeneous and inhomogeneous cosmological models in general relativity and tried to isotropize the universe at early times when the initial singularity is approached. In our present study, we have considered the quadratic equation of state of the form

$$p = \alpha\rho^2 - \rho \ ,$$

where $\alpha \neq 0$ is a constant quantity but we can take $p_0 = 0$ to avoid complexities in our calculations. This will not affect the quadratic nature of the equation of state.

Again Chavanis (2013) studied a four dimensional Friedmann-Lemaitre-Roberston-Walker (FLRW) cosmological model based on a quadratic equation of state in the form $\frac{p}{c^2} = -\frac{4\rho^2}{3\rho_p} + \frac{\rho}{3} - \frac{4\rho_\Lambda}{3}$ unifying vacuum energy, radiation and dark energy. Also considering a quadratic equation of state, Chavanis (2015) formulated a cosmological model that describe the early inflation, the intermediate decelerating expansion, and the late-time accelerating expansion of the universe.

Sharma and Ratanpal (2013) suggested a class of solution that describes the interior of a static spherically symmetric compact anisotropic star and proved that the model admits a quadratic equation of state. Considering quadratic equation of state and anisotropic matter distribution, Malaver (2014) investigated the behavior of the compact relativistic objects and obtained new solutions to the system of Einstein-Maxwell equations in terms of elementary functions. Many authors like Rahaman et al. (2009a); Feroze and Siddiqui (2011), Maharaj et al. (2012) studied cosmological models with a quadratic equation of state under different circumstances. Recently Reddy et al. (2015), Adhav et al. (2015), Rao et al. (2015) studied Kaluza-Klein Space-time cosmological models with a quadratic equation of state in general and modified theories of relativity.

Motivated from the above mentioned research, here we have investigated a higher dimensional cosmological model universe in the framework of Lyra geometry with special law of Hubble's parameter producing constant value of deceleration parameter. Physical and geometrical properties of the model are also discussed.

5.2 Field Equations and Their Solutions:

Let us consider a five dimensional Locally Rotationally Symmetric (LRS) Bianchi type-I axially symmetric metric in the form

$$ds^2 = A^2dx^2 + B^2(dy^2 + dz^2) + C^2d\psi^2 - dt^2 \tag{5.1}$$

where the scale factors A, B and C are functions of cosmic time t only in which the extra coordinate is taken to be space like.

The Einstein's field equations in normal gauge for Lyra's Geometry as obtained by Sen (1957) and Sen and Dunn (1971) are given by -

$$R_{ij} - \frac{1}{2}g_{ij}R + \frac{3}{2}\phi_i\phi_j - \frac{3}{4}g_{ij}\phi^k\phi_k = -8\pi T_{ij} \tag{5.2}$$

where we use the units in which $\frac{8\pi G}{c^4} = 1$ [Wesson 1992; Baysal et al. 2001; Bali and Dev 2002], R_{ij} is the Ricci tensor, R is the Ricci scalar, g_{ij} is the metric tensor and ϕ_i is the displacement vector given by-

$$\phi_i = (0, 0, 0, 0, \beta(t)) \tag{5.3}$$

The energy momentum tensor T_{ij} for the perfect fluid is given by

$$T_{ij} = (\rho + p)u_i u_j + p g_{ij} \tag{5.4}$$

where, ρ is the energy density, p is the pressure and u^i is the five velocity vector given by

$$u^i = (0,0,0,0,1) \tag{5.5}$$

which satisfies

$$g_{ij}u^i u^j = u^i u_i = -1 \tag{5.6}$$

In comoving coordinate system, we have from (5.4)

$$T_1^1 = T_2^2 = T_3^3 = T_4^4 = -p \;\; ; \;\; T_5^5 = \rho \;\; and \;\; T_j^i = 0 \;\; for \;\; all \;\; i \neq j \tag{5.7}$$

The conservation of R.H.S of (5.2) leads to

$$\left(R_j^i - \frac{1}{2}g_j^i R\right)_{;i} + \frac{3}{2}\left(\phi^i \phi_j\right)_{;i} - \frac{3}{4}\left(g_j^i \phi^k \phi_k\right)_{;i} = 0 \tag{5.8}$$

After simplification, the equation (5.8) can be written as

$$\frac{3}{2}\phi_j\left[\frac{\partial \phi^i}{\partial x^i} + \phi^l \Gamma_{li}^i\right] + \frac{3}{2}\phi^i\left[\frac{\partial \phi_j}{\partial x^i} - \phi_l \Gamma_{ij}^l\right] - \frac{3}{4}g_j^i \phi_k\left[\frac{\partial \phi^k}{\partial x^i} + \phi^l \Gamma_{lj}^k\right] - \frac{3}{4}g_j^i \phi^k\left[\frac{\partial \phi_k}{\partial x^i} - \phi_l \Gamma_{kj}^l\right] = 0 \tag{5.9}$$

This equation (5.9) identically satisfied for $j = 1,2,3,4$.

But for $j = 5$, this equation reduces to

$$\frac{3}{2}\beta\dot{\beta} + \frac{3}{2}\beta^2\left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = 0 \tag{5.10}$$

In commoving coordinate system, the Einstein's field equations (5.2) for the metric (5.1) with the help of equation (5.3)-(5.7) reduces to

$$2\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+2\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{B}^2}{B^2}+\frac{3}{4}\beta^2=-p \tag{5.11}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{A}\dot{C}}{AC}+\frac{3}{4}\beta^2=-p \tag{5.12}$$

$$\frac{\ddot{A}}{A}+2\frac{\ddot{B}}{B}+2\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}^2}{B^2}+\frac{3}{4}\beta^2=-p \tag{5.13}$$

$$2\frac{\dot{A}\dot{B}}{AB}+2\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{A}\dot{C}}{AC}+\frac{\dot{B}^2}{B^2}-\frac{3}{4}\beta^2=-\rho \tag{5.14}$$

where, the overhead dot $(.)$ denote the derivative with respect to time t.

The physical quantities like Volume V, average Scale factor R , Expansion Scalar θ , Hubble's parameter H , Shear Scalar σ , Anisotropy Parameter Δ and Deceleration parameter q have observational interest in cosmology are defined for the metric (5.1) as

$$V=R^4(t)=AB^2C \tag{5.15}$$

$$\theta=\frac{\dot{A}}{A}+2\frac{\dot{B}}{B}+\frac{\dot{C}}{C} \tag{5.16}$$

$$H=\frac{\dot{R}}{R}=\frac{1}{4}\left(\frac{\dot{A}}{A}+2\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right) \tag{5.17}$$

$$\sigma^2=\frac{1}{2}\left(\frac{\dot{A}^2}{A^2}+2\frac{\dot{B}^2}{B^2}+\frac{\dot{C}^2}{C^2}-\frac{\theta^2}{4}\right) \tag{5.18}$$

$$\Delta=\frac{1}{4}\left[\sum_{i=1}^{4}\left(\frac{H_i-H}{H}\right)^2\right] \tag{5.19}$$

and

$$q=\frac{d}{dt}\left(\frac{1}{H}\right)-1 \tag{5.20}$$

where $H_i\,; i=1,2,3,4$ represents the directional Hubble's parameter in the directions of x, y, z and ψ respectively, which are given by

$$H_x=\frac{\dot{A}}{A}\ ,\ H_y=H_z=\frac{\dot{B}}{B}\ ,\ H_\psi=\frac{\dot{C}}{C}$$

Since the equations (5.11)-(5.14) together with equation (5.10) represents a set of five independent equations involving six unknown parameters viz. A, B, C, p, ρ and β, so in order to obtain deterministic solutions of the above system of equations, it is required one more physical relations involving them. Therefore, it is assumed like Berman (1983); Berman and Gomide (1988) and Ram et al. (2010) [By using this type of relation, Berman (1983); Berman and Gomide (1988) solved FRW models whereas Ram et al. (2010) solved Bianchi Type V cosmological models in Lyra's Geometry] that the Hubble parameter H is related to the average scale factor R by the relation

$$H = aR^{-m} \tag{5.21}$$

where $a > 0$ and $m \geq 0$ are constants.

From equations (5.15) and (5.17) we have

$$H = \frac{1}{4}\frac{\dot{V}}{V} = \frac{\dot{R}}{R} = \frac{1}{4}\left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) \tag{5.22}$$

Using equation (5.21) in equation (5.22), it may be obtained

$$\dot{R} = aR^{1-m} \tag{5.23}$$

From equations (5.11)-(5.15) we have

$$\frac{A}{B} = D_4 e^{D_1 \int \frac{1}{V} dt} \tag{5.24}$$

$$\frac{A}{C} = D_5 e^{D_2 \int \frac{1}{V} dt} \tag{5.25}$$

$$\frac{B}{C} = D_6 e^{D_3 \int \frac{1}{V} dt} \tag{5.26}$$

where, D_1, D_2, D_3, D_4, D_5 and D_6 are constants satisfying

$$D_2 = D_1 + D_3 \quad and \quad D_5 = D_4 D_6 \tag{5.27}$$

Again from equation (5.20), the deceleration parameter (q) is obtained as

$$q = m - 1 \tag{5.28}$$

From equation (5.28) it is seen that the deceleration parameter (q) obtained under the law of variation of Hubble's parameter (H) given by equation (5.21) is a constant. Again from the study of various literatures it is found that the sign of deceleration parameter (q) determines whether a model is inflationary or not inflationary. The negative sign of q corresponds an inflationary model universe whereas the positive sign of it indicates a standard decelerating model. From equation (5.28) it is observed that whenever $0 \leq m < 1$ then $-1 \leq q < 0$ but if $m > 1$ then $q > 0$. Therefore, $m \leq 1$ will describe an accelerating/inflationary model universe and $m > 1$ will give us a decelerating model. [Or, From equation (5.28) it is found that the whenever $m > 1$ then $q > 0$, therefore the model represent a decelerating model whereas for $m \leq 1$ we get $-1 \leq q < 0$ which describes an accelerating model of universe.]

Integrating equation (5.23), the value of average scale factor (R) is obtained as

$$R = (amt + b)^{\frac{1}{m}} \qquad whenever \qquad m \neq 0 \tag{5.29}$$

$$R = ce^{at} \qquad whenever \qquad m \neq 0 \tag{5.30}$$

where a, b and c are constants.

Case-I: Whenever $m \neq 0$:

From equations (5.22) and (5.29), the volume V can be obtained as

$$V = V_0(amt + b)^{\frac{4}{m}} \tag{5.31}$$

where V_0 is an integrating constant. Without loss of generality, if we choose $V_0 = 1$ then the equation (5.31) reduces to

$$V = (amt + b)^{\frac{4}{m}} \tag{5.32}$$

Using this value of V in equations (5.24)-(5.26), the scale factors A, B and C are found as

$$A = a_1 X^{\frac{1}{m}} e^{a_2 X^{1-\frac{4}{m}}} \tag{5.33}$$

$$B = b_1 X^{\frac{1}{m}} e^{b_2 X^{1-\frac{4}{m}}} \tag{5.34}$$

$$C = c_1 X^{\frac{1}{m}} e^{c_2 X^{1-\frac{4}{m}}} \tag{5.35}$$

where

$$X = amt + b \tag{5.36}$$

and

$$\begin{aligned} a_1 &= \left(D_3^4 D_6\right)^{\frac{1}{4}} \quad ; \quad b_1 = \left(\frac{D_6}{D_4}\right)^{\frac{1}{4}} \quad ; \quad c_1 = (D_4 D_6)^{-\frac{1}{4}} \\ a_2 &= \frac{2D_1 + D_3}{4am(1-\frac{4}{m})} \quad ; \quad b_2 = \frac{D_3 - D_1}{4am(1-\frac{4}{m})} \quad ; \quad c_2 = -\frac{D_1 + 3D_3}{4am(1-\frac{4}{m})} \end{aligned} \tag{5.37}$$

are constants.

Therefore the metric (5.1) can be written as

$$ds^2 = a_1^2 X^{\frac{2}{m}} e^{2a_2 X^{1-\frac{4}{m}}} dx^2 + b_1^2 X^{\frac{2}{m}} e^{2b_2 X^{1-\frac{4}{m}}} (dy^2 + dz^2) + c_1^2 X^{\frac{2}{m}} e^{2c_2 X^{1-\frac{4}{m}}} d\psi^2 - dt^2 \tag{5.38}$$

The equation (5.38) represents Bianchi type I cosmological model universe with special law of Hubble's parameter of the form $H = aR^{-m}$, where $m > 0$.

From equation (5.10) the displacement vector β is found as follows

$$\beta = \beta_0 X^{-\frac{4}{m}} \tag{5.39}$$

where $\beta_0 > 0$ is a constant and X is given by equation (5.36).

Again the energy density and pressure are obtained from equation (5.14) and any one of (5.11)-(5.13) as

$$\rho = \left[6a^2 - \frac{1}{8}(3D_1^2 + 2D_1 D_3 + 3D_3^2) X^{2(1-\frac{4}{m})}\right] \frac{1}{X^2} - \frac{3}{4}\beta_0^2 X^{-\frac{8}{m}} \tag{5.40}$$

and

$$p = -\left[3(2-m)a^2 + \frac{1}{8}(3D_1^2 + 2D_1D_3 + 3D_3^2)X^{2(1-\frac{4}{m})}\right]\frac{1}{X^2} - \frac{3}{4}\beta_0^2 X^{-\frac{8}{m}} \tag{5.41}$$

The expressions for physical quantities such as Average Scale Factor R, expansion scalar θ, Hubble's parameter H, Shear scalar σ, anisotropy parameter Δ and Deceleration parameter q having observational interest in cosmology may be obtained from equations (5.15)-(5.20) as

$$R = (amt + b)^{\frac{1}{m}} \tag{5.42}$$

$$\theta = \frac{4a}{amt + b} \tag{5.43}$$

$$H = \frac{a}{amt + b} \tag{5.44}$$

$$\sigma^2 = \frac{1}{8}(3D_1^2 + 2D_1D_3 + 3D_3^2)(amt + b)^{-\frac{8}{m}} \tag{5.45}$$

$$\frac{\sigma^2}{\theta^2} = \frac{1}{128a^2}(3D_1^2 + 2D_1D_3 + 3D_3^2)(amt + b)^{2(1-\frac{4}{m})} \tag{5.46}$$

Therefore

$$\lim_{n\to\infty} \frac{\sigma^2}{\theta^2} = 0 \tag{5.47}$$

$$\Delta = \frac{1}{16a^2}(3D_1^2 + 2D_1D_3 + 3D_3^2)(amt + b)^{2(1-\frac{4}{m})} \tag{5.48}$$

and

$$q = m - 1 \qquad for \quad m \neq 0 \quad i.e. \quad m > 0 \tag{5.49}$$

The variations of some parameters with respect to time of the **case-I** are shown in **Figs. 1-6**.

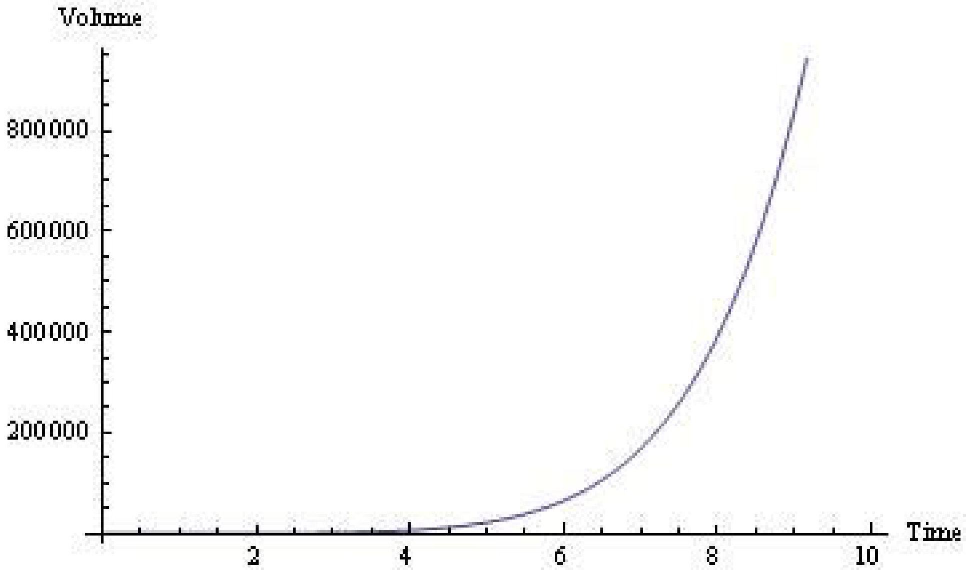

Figure-5.1 : Variation of volume V vs. time t , whenever $a = b = 1; D_1 = D_4 = D_6 = 1; D_3 = 2; m = 0.5$.

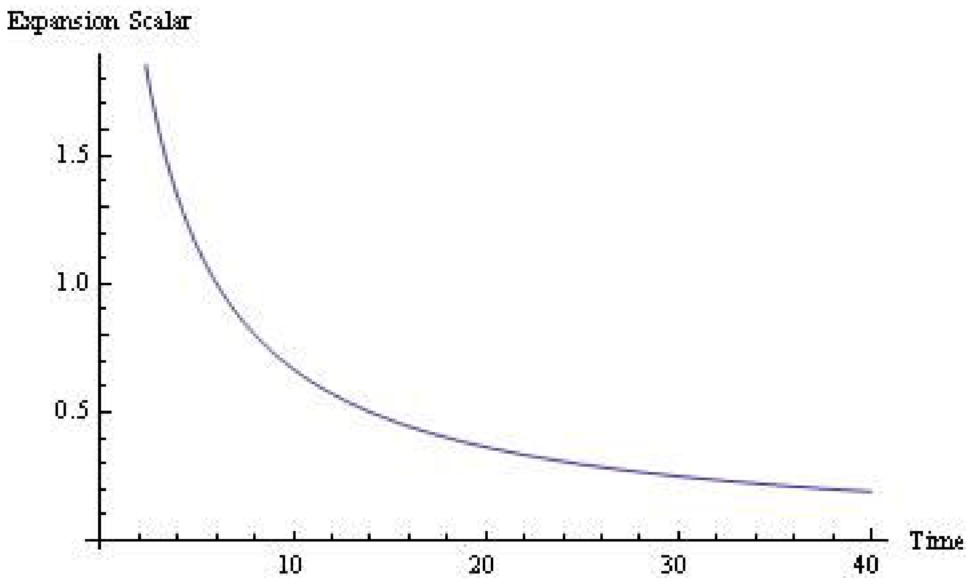

Figure-5.2 : Variation of Expansion scalar θ vs. Time t , whenever $a = b = 1; D_1 = D_4 = D_6 = 1; D_3 = 2; m = 0.5$.

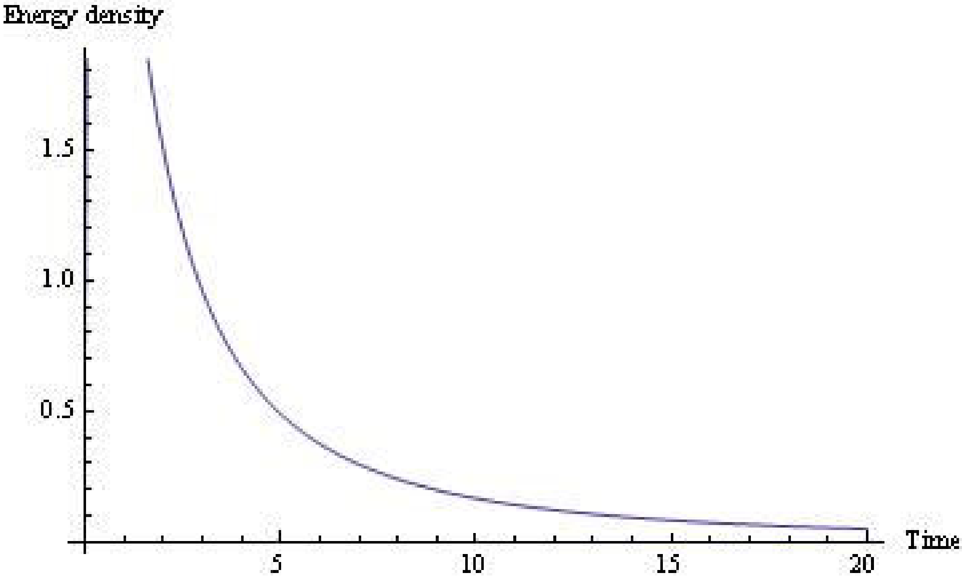

Figure-5.3 : Variation of energy density ρ vs. Time t , whenever $a = b = 1; m = 0.5$.

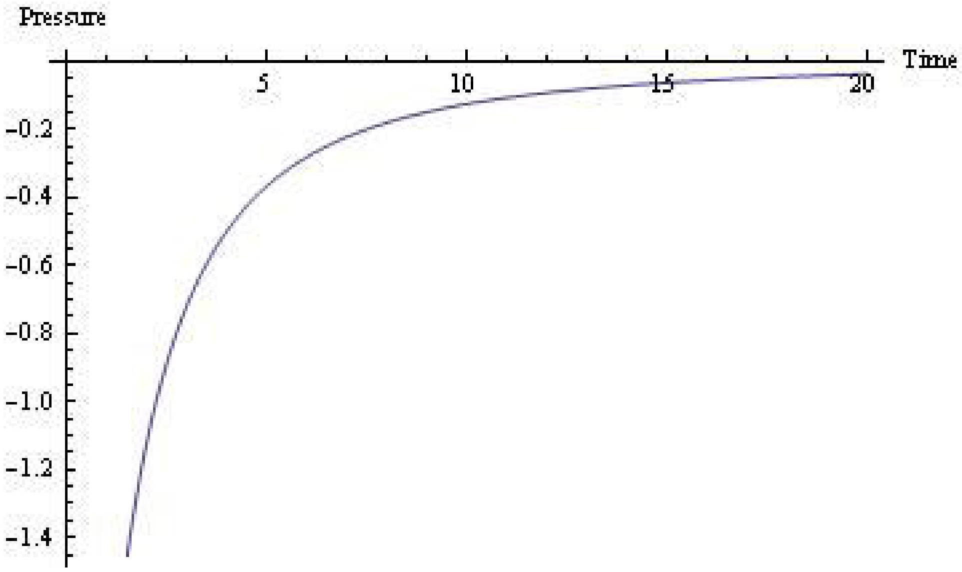

Figure-5.4 : Variation of pressure p vs. Time t , $a = b = 1; D_1 = D_4 = D_6 = 1; D_3 = 2; m = 0.5$.

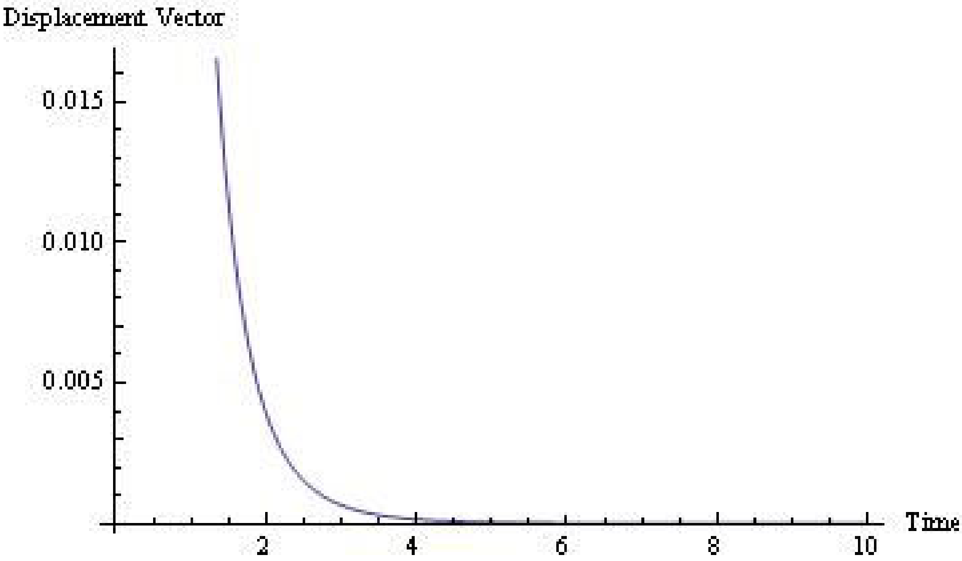

Figure-5.5 : Variation of displacement vector β vs. Time t , $a = b = 1; m = 0.5; \beta_0 = 1$.

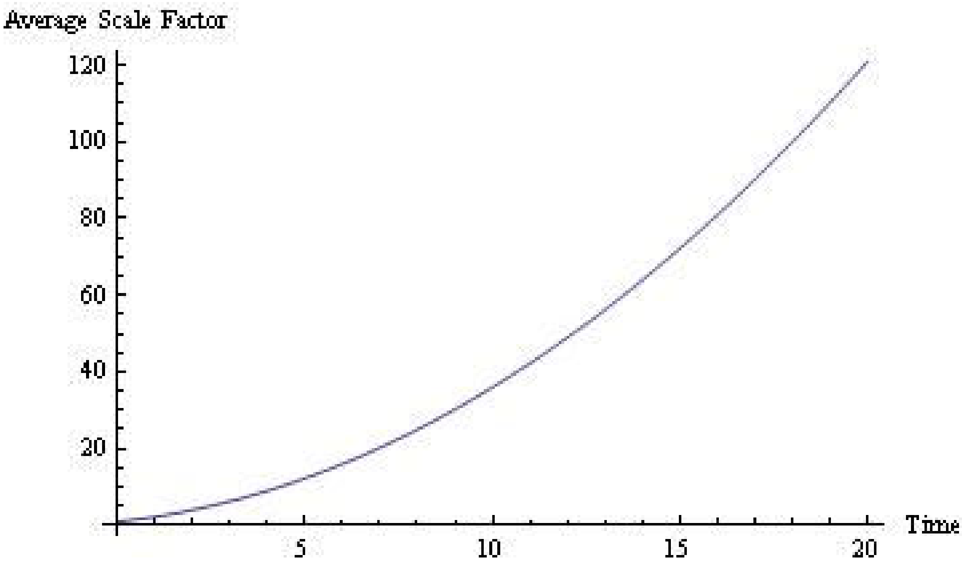

Figure-5.6 : Variation of Average Scale factor R vs. Time t , $a = b = 1; m = 0.5$.

Case-II: Whenever $m = 0$:

When $m = 0$ the from equations (5.22) and (5.30) we have the volume V as

$$V = V_1 e^{4at} \tag{5.50}$$

where V_1 is a constant of integration. Without loss of generality, if we choose $V_1 = 1$ then from equation (5.50) we have

$$V = e^{4at} \tag{5.51}$$

From equations (5.24)-(5.26) and (5.51), the scale factors A, B and C are found as

$$A = a_1 e^{at} e^{a_3 e^{-4at}} \tag{5.52}$$

$$B = b_1 e^{at} e^{b_3 e^{-4at}} \tag{5.53}$$

$$C = c_1 e^{at} e^{c_3 e^{-4at}} \tag{5.54}$$

where, $a_3 = -\frac{3D_1+D_3}{16a}$, $b_3 = \frac{D_1-D_3}{16a}$ and $c_3 = -\frac{D_1+3D_3}{16a}$ are constants.

Therefore the metric (5.1) can be written as

$$ds^2 = a_1^2 e^{2at} e^{2a_3 e^{-4at}} dx^2 + b_1^2 e^{2at} e^{2b_3 e^{-4at}} (dy^2 + dz^2) + c_1^2 e^{2at} e^{2c_3 e^{-4at}} - dt^2 \tag{5.55}$$

which represents a Bianchi type I cosmological model universe under the Hubble's expansion law given by equation (5.21) when $m = 0$.

From equation (5.10) the displacement vector β is found as follows

$$\beta = \beta_0 e^{-4at} \tag{5.56}$$

where $\beta_0 > 0$ is a constant.

From equations (5.14) and any one of (5.11)-(5.13), the energy density and pressure are obtained as

$$\rho = 6a^2 - \frac{1}{8}(3D_1^2 + 2D_1D_3 + 3D_3^2)e^{-8at} - \frac{3}{4}\beta_0^2 e^{-8at} \tag{5.57}$$

and

$$p = -\left[6a^2 + \frac{1}{8}(3D_1^2 + 2D_1D_3 + 3D_3^2)e^{-8at} + \frac{3}{4}\beta_0^2 e^{-8at}\right] \tag{5.58}$$

From equations (5.15)-(5.20), the expressions for kinematical parameters like Average Scale Factor R, expansion scalar θ, Hubble's parameter H, Shear scalar σ, anisotropy parameter Δ and Deceleration parameter q are obtained as

$$R = e^{at} \tag{5.59}$$

$$\theta = 4a \tag{5.60}$$

$$H = a \tag{5.61}$$

$$\sigma^2 = \frac{1}{8}(3D_1^2 + 2D_1D_3 + 3D_3^2)e^{-8at} \tag{5.62}$$

$$\frac{\sigma^2}{\theta^2} = \frac{1}{128a^2}(3D_1^2 + 2D_1D_3 + 3D_3^2)e^{-8at} \tag{5.63}$$

Therefore

$$\lim_{n\to\infty} \frac{\sigma^2}{\theta^2} = 0 \tag{5.64}$$

$$\Delta = \frac{1}{16a^2}(3D_1^2 + 2D_1D_3 + 3D_3^2)e^{-8at} \tag{5.65}$$

and

$$q = -1 \tag{5.66}$$

The variations of some parameters with respect to time of the **case-II** are shown in **Figs. 7-15**.

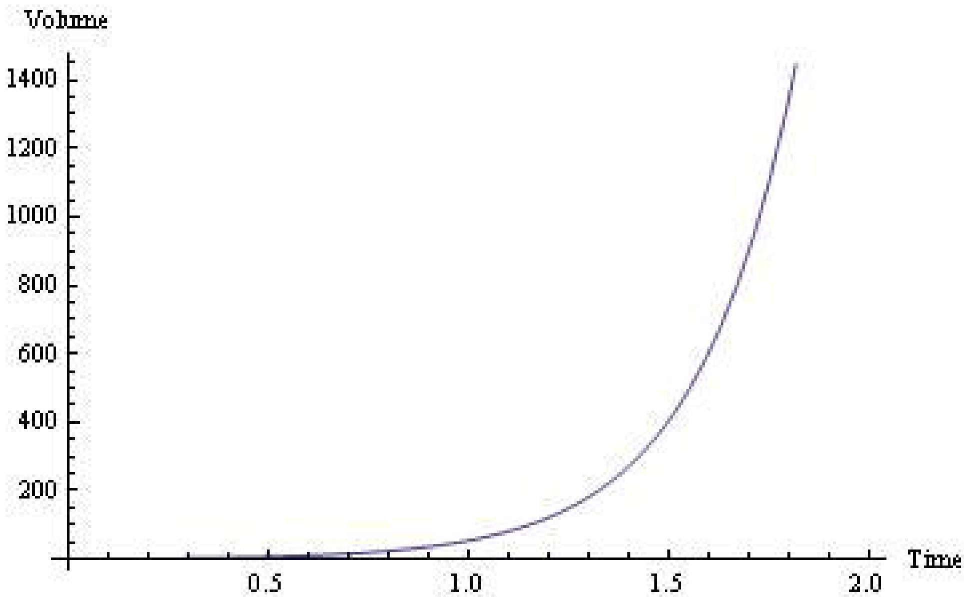

Figure-5.7 : Variation of Volume V vs. Time t , whenever $a = 1$.

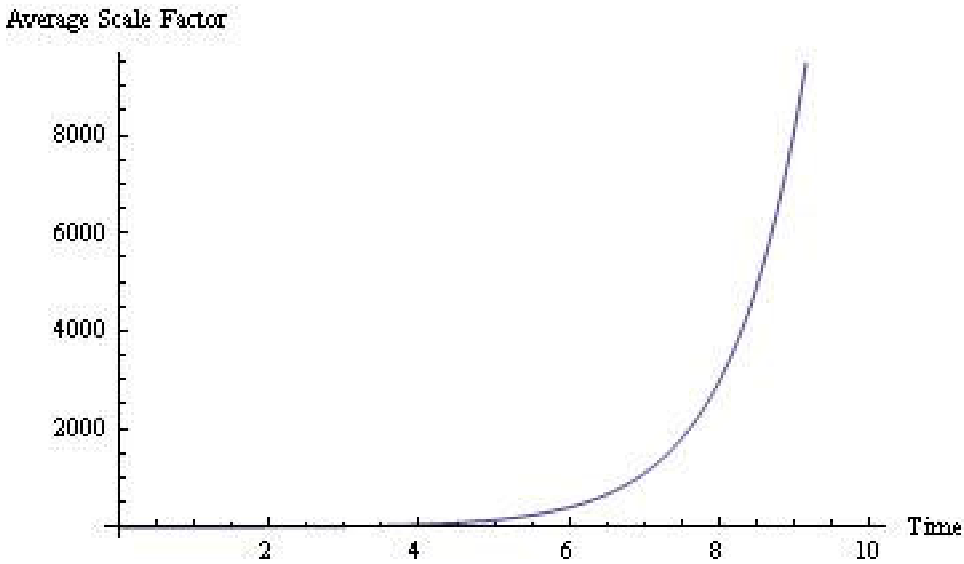

Figure-5.8 : Variation of Average Scale Factor R vs. Time t , whenever $a = 1$.

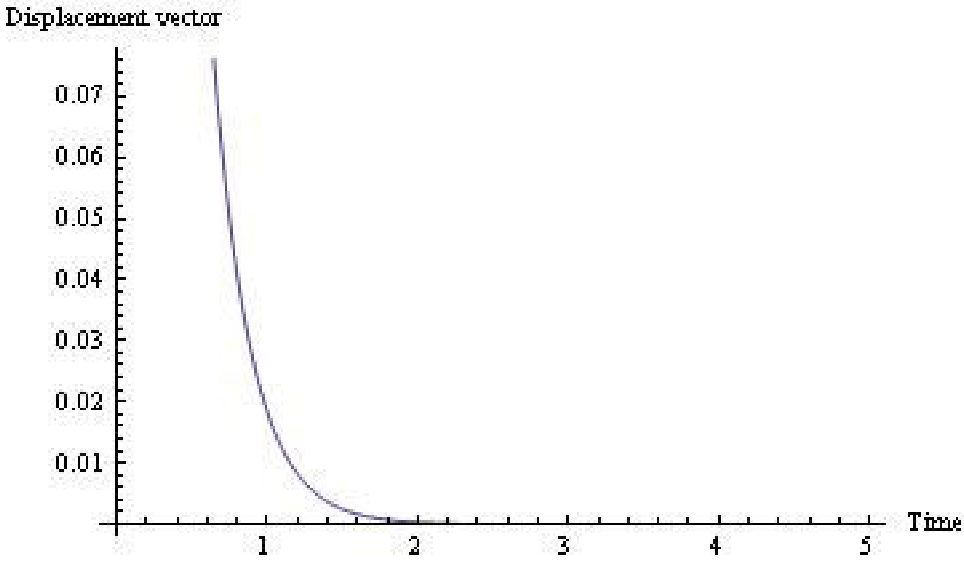

Figure-5.9 : Variation of Displacement Vector β vs. Time t , whenever $a = \beta_0 = 1$.

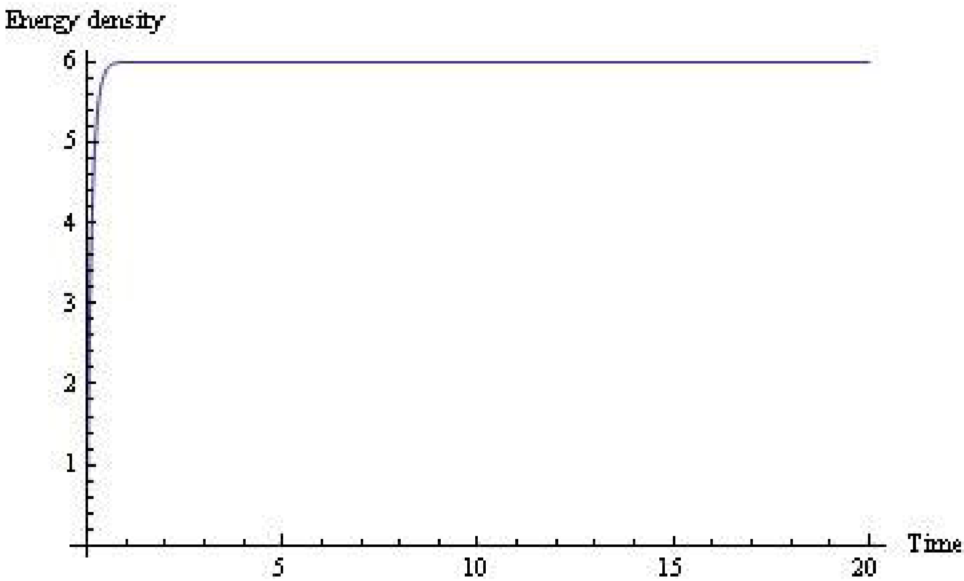

Figure-5.10 : Variation of Energy density ρ vs. Time t , $a = \beta_0 = 1; D_1 = D_4 = D_6 = 1; D_3 = 2$.

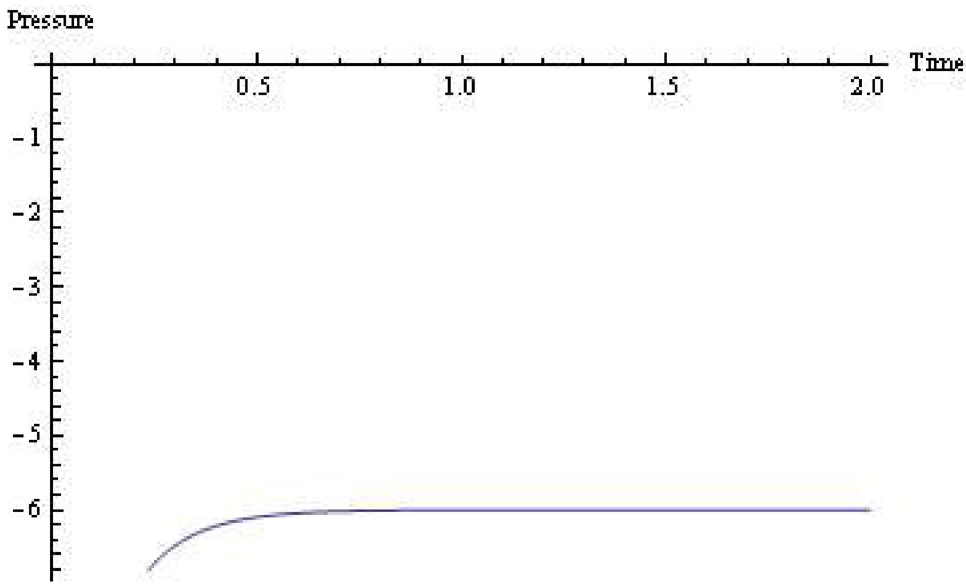

Figure-5.11 : Variation of Pressure p vs. Time t , $a = \beta_0 = 1; D_1 = D_4 = D_6 = 1; D_3 = 2$.

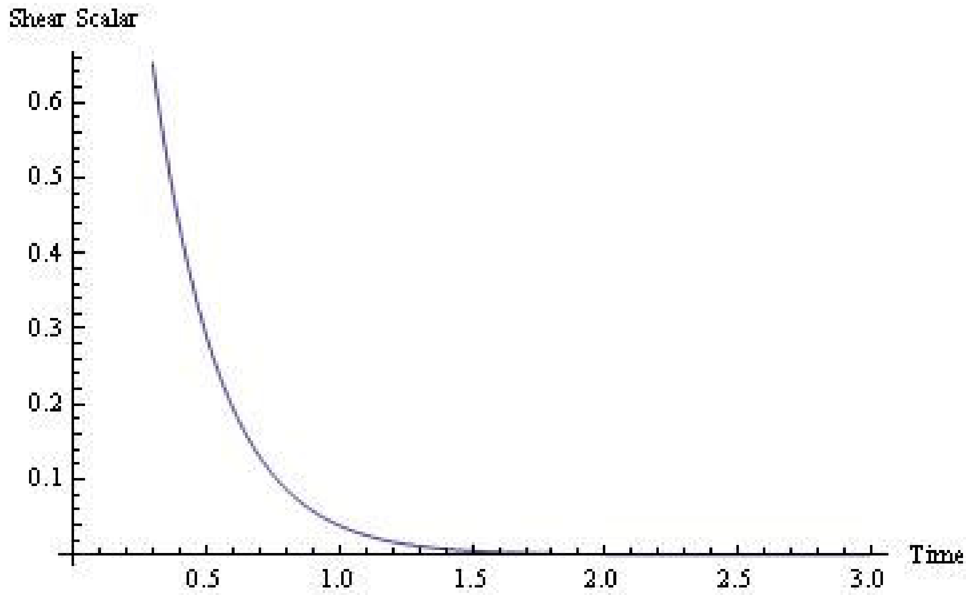

Figure-5.12 : Variation of Shear Scalar σ vs. Time t , whenever $a = 1; D_1 = D_4 = D_6 = 1; D_3 = 2$.

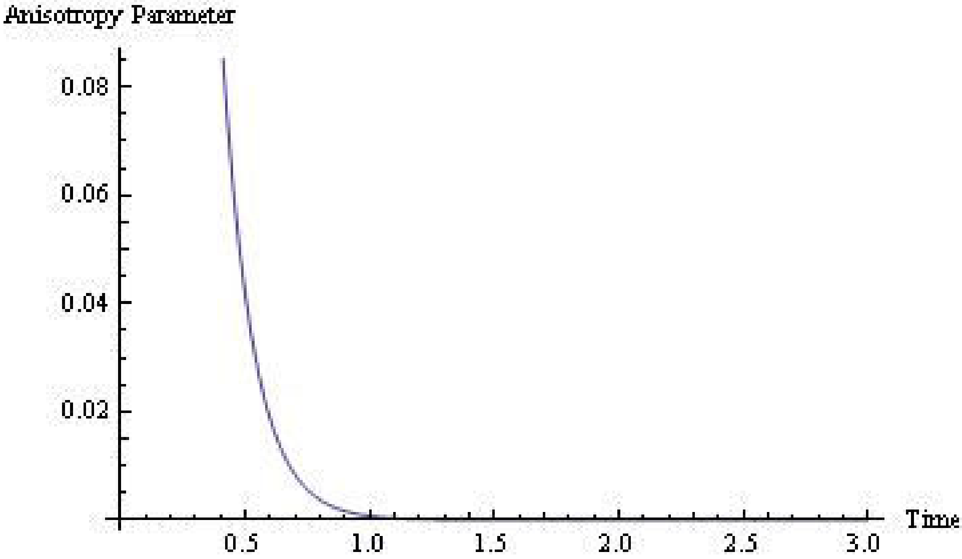

Figure-5.13 : Variation of Anisotropy Parameter Δ vs. Time t , whenever $a = 1; D_1 = D_4 = D_6 = 1; D_3 = 2$.

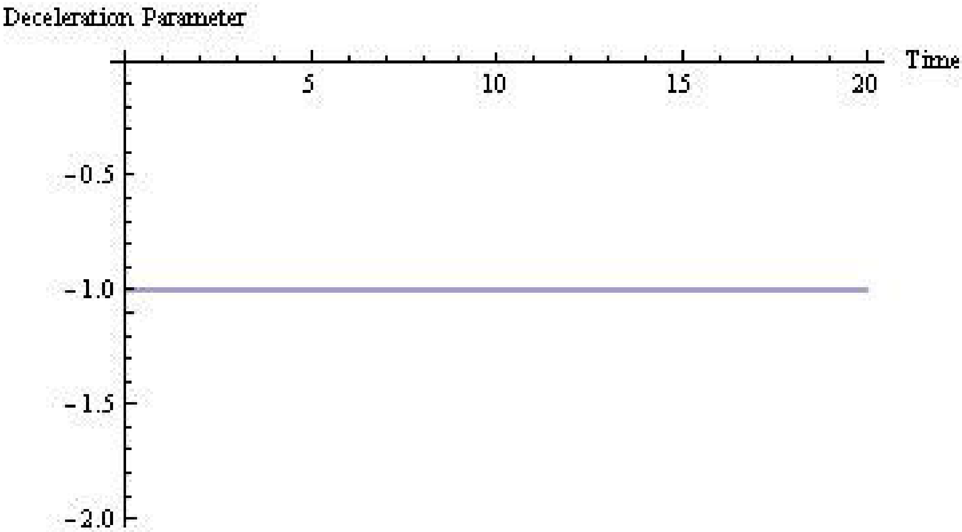

Figure-5.14 : Variation of Deceleration Parameter q vs. Time t.

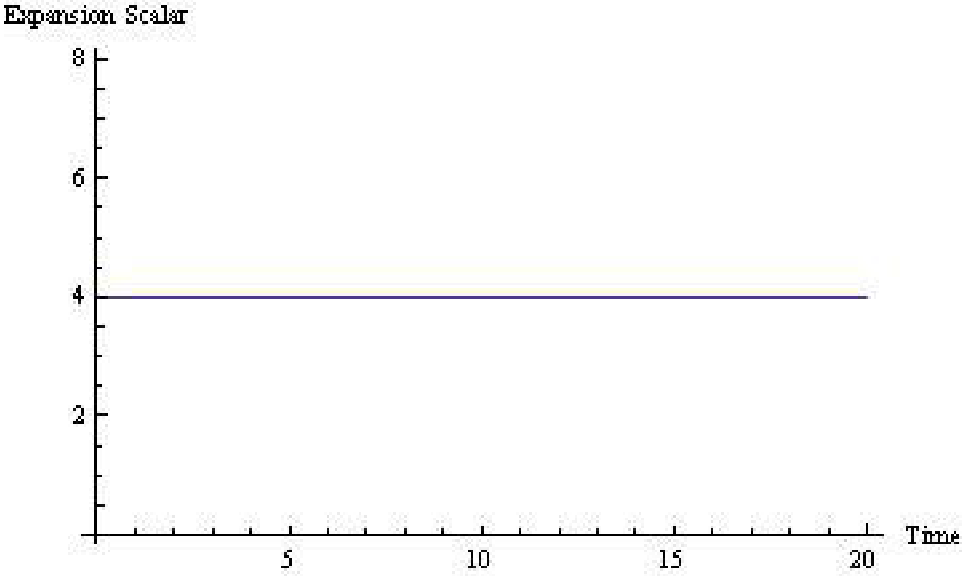

Figure-5.15 : Variation of Expansion Scalar θ vs. Time t , $a = 1$.

5.3 Physical and Geometrical Properties of the Solutions:

Case-I:

From equation (5.32) it has been observed that the spatial volume V is zero at $t = -\frac{b}{am}$ and it becomes infinite as $t \to \infty$. This behavior of volume V is shown in **Figure 5.1**. Again from the expression of the average scale factor R obtained in equation (5.41), it is seen that R has a constant value at $t = -\frac{b}{am}$ and it becomes infinite as $t \to \infty$. The evolution of expansion scalar θ has been shown in **Figure 5.2** corresponding to the equation (5.44) and it is observed that the expansion scalar θ starts with infinite value at initial epoch of cosmic time $t = -\frac{b}{am}$ but as time t progresses it decreases and becomes constant after some finite time that explains the Big-Bang scenario. Also from the expression of Hubble's expansion factor H given by equation (5.44), we see that dH/dt is negative. These show that our model represents an expanding universe that expands with an accelerated rate.

The expression (5.40) for the energy density $\rho(t)$ shows that ρ is a decreasing function of cosmic time t that tend to zero as $t \to \infty$ and $\rho > 0$ for all values of t . Also from equation (5.41) it is seen that the pressure $p(t)$ is negative and is an increasing function of time t that tend to zero as $t \to \infty$. These behaviors of energy density ρ and pressure p are shown in **Figure 5.3** and **Figure 5.4** respectively. Therefore our model represents a dark energy model.

Again from equation (5.39), it has been observed that the displacement vector $\beta(t)$ is a decreasing function of time and it approaches a small positive value with increase in time. **Figure 5.5** shows this behavior of β. Thus the nature of β in our derived model of the universe is consistent with recent observations [Perlmutter et al. (1997, 1999), Garnavich et al. (1998), Schmidt et al. (1998), Riess et al. (2004)]. The equation (5.47) shows that the $\lim_{n\to\infty}\frac{\sigma^2}{\theta^2}=0$. Thus our inflationary model universe eventually approaches isotropy for large values of t.

Case-II:

From the equations (5.51) and (5.59), it has been observed that initially at $t=0$ both the spatial volume V and average scale factor R have values 1 and increases exponentially to infinite value whenever $t\to\infty$. The variations of V and R are shown respectively in **Figure 5.7** and **Figure 5.8**. In this case the expansion scalar θ obtained in equation (5.60) is found to be a constant. Also from equation (5.61), the Hubble's expansion factor H is found to be constant. These show that our model universe starts evolving with unit volume at $t=0$ and is expanding with acceleration with time.

Figure 5.9 of equation (5.56) showing the behavior of the displacement vector β describes that β is always positive which tend to zero 0 as $t\to\infty$. Also from equation (5.62), it has been observed that the shear scalar σ is a nonzero positive quantity for all values of comic time t that tends to zero as $t\to\infty$. The behavior of shear scalar is shown in **Figure 5.12**. Also we find that $\lim_{n\to\infty}\frac{\sigma^2}{\theta^2}=0$. This means that after a long time our model approach to an isotropic model universe.

From equation (5.57), it can be noticed that the energy density $\rho(t)$ is positive for all values of time t that approaches to a finite quantity with the increase of time. This behavior of energy density is shown in **Figure 5.10**. Again from equation (5.58) and **Figure 5.11**, it is seen that the pressure p is always negative. Thus our model essentially represents a dark energy model universe that is full of matter.

5.4 Conclusion:

In this paper we studied five dimensional LRS Bianchi type I cosmological model in the framework of Lyra's geometry in presence of perfect fluid, by using quadratic equation of state given by the equation (5.30) which is an inflationary model. Our work analyze the general feature of LRS Bianchi Type-I cosmological model with time dependent displacement vector so the concept of Lyra geometry is still exist even after the infinite times with different ideas and concepts. So it will be interesting to study the different properties of different topological defects within the framework of Lyra geometry and beneficial for further study to investigate the different models of our universe.