Chapter 1

General Introduction

1.1 Introduction

As the human being is considered mostly developed and changeable creature in-universe so this is our moral responsibility to discover the unknown things of the whole universe like the origin, evolution and ultimate fate etc. of the universe. Being a member of Mathematics family it can be efficiently done by constructing mathematical models of our universe, by using various theories of Relativity especially by using Einstein's theory of gravitation and other modified theories of gravitation. The results of the model, thus, formulated may be compared with the various observational findings of present day about the origin, evolution, shape, size, physics etc. of the universe. Motivated from this, we have considered the investigations in this thesis entitled **"STUDY OF SOME COSMOLOGICAL MODELS IN LYRA GEOMETRY"**. This thesis comprises of 9 (Nine) chapters and deals with the study of some Bianchi type I and III cosmological models with 4 and 5 dimensions in Lyra's Geometry, which is a modified theory of general relativity.

This Introduction chapter is organized as follows:

In section 1.2 we have tried to present a brief idea about the universe according to various astrophysicists of the different time and cosmological models of the universe. Section 1.3 deals with the brief review of Einstein's general theory of relativity. In section 1.4 we have discussed Weyl's Geometry. Section 1.5 is devoted to Lyra's Geometry. In section 1.6 deals with the derivation of the field equations in Lyra's Geometry. In section 1.7 we have presented about some aspects of the work related to Lyra's Geometry. In section 1.8 contains a brief introduction to Bianchi type space-times. In Section 1.9 we deal with some aspects of the work related to Bianchi Space-times. Section 1.10 deals with discussion of strings and string cosmology in general relativity. Brief discussions about higher dimensional cosmology, dark energy are discussed in sections 1.11 and 1.12 respectively. In section 1.13, we discussed about Perfect Fluid & Energy-momentum tensor. Section 1.14 and 1.15 contains the brief study about Hubble's law & Hubble's constant and deceleration parameter respectively.

1.2 History of Cosmology and Cosmological Models

The term cosmology originated from the Greek word "Kosmos" and the meaning of which is "Something revealed in the beauty". The branch of science in which the large-scale properties of the universe as a whole are studied in order to understand the origin, evolution and ultimate fate of it is known as Cosmology. Cosmology deals with the various theories of the formation of the universe and makes a hypothesis for specific predictions of different phenomena for observations regarding the universe. These theories may be accepted or rejected or may be revised or extended to accommodate the observational data after detailed verification.

Presently the "Big Bang Theory" is the most accepted theory about the origin and ultimate fate of the universe. The main theme of this theory is that the universe is expanding and so it implies that in the distant past there was enormous dense and hot. Cosmology deals also with the problems of the understandings the formation of galaxies and clusters of galaxies together with the determination of the nature of their masses.

Before the big-bang theory, many cosmologists from their studies and investigations gave an idea about the shape, size, nature, and formation of the universe. Out of these some of the chief contributors were Aristotle, Ptolemy, Nicholas Copernicus, Johannes Kepler and Galileo Galilei, Newton, Richard Bentley, Heinrich Olbers, St. Augustine, Einstein, Weyl, Lyra, Friedmann, Edwin Hubble etc.

In the early days, human believe that the earth was not spherical rather it was flat in the beginning. This may be expected because the curvature of the surface of our planet is not perceptible at once. But this idea of assuming the earth to be flat creates a problem that the earth must end somewhere unless we imagine it to extend infinitely. Also, the human being of that era was unknown about actually what lies beyond these boundaries and became an open challenge to all for speculation. People from various parts of the world tried to explain their cosmogonies. From history, it is seen that the Greeks were first suggested that our planet is not flat rather it is spherical. Around 340 B.C., Greek philosopher Aristotle, in his book "On the Heavens" first suggested that our planet was not like a flat plate rather than it was spherical. He was the first person to realize that eclipses of the moon occur when the earth comes in between the moon and the sun. He observed that the earth's shadow on the moon was always round that would be true only if the earth was spherical. Even, Aristotle was able to measure the distance around the earth as 4, 00,000 stadia approximately. He also thought that the earth was stationary and the sun, the moon, the planets and the stars were moving around the earth in circular orbits. This idea of the spherical earth did not get importance until Ptolemy (Claudius Ptolemaeus) elaborated it into a complete Mathematical description of the universe in around 150 A.D. and it became the most important cosmology up until the 16^{th} century. This system was later adopted by the Christian church and became the dominant cosmology until the 16th century. As the time progresses this mesmerizing subject was developed by so many cosmologists. Out of them some of the chief contributors are Pythagoras, Aristotle, Nicholas Copernicus, Johannes Kepler, Galileo Galilee, Newton, Richard Bentley, Heinrich Olbers, St. Agustine, Einstein, Weyl, Friedmann, Edwin Hubble etc.

Some important cosmological developments are mentioned briefly.

During the period 500BC to 300 BC, Pythagoras believed that the earth was in motion whereas Aristotle thought that the earth was stationary and the sun, the moon, the planets and the stars were moving around the stationary Earth in circular orbits. Greek Philosophers also estimated the distance to the Moon and calculated the size of the finite universe. Aristotle was also able to measure the distance around the earth as 4, 00,000 stadia approximately. From 300BC to 210 BC, Greek Mathematician Aristarchus was first the person to propose a scientific heliocentric model of the solar system and placing the Sun at the center of the universe. He has also established the order of planets from the Sun. In 200AD, Ptolemy proposed about an Earth-centered universe, with Sun & planets are revolving around the Earth.

From 1401AD to 1464 AD, Nicholas de Cusa suggested that the Earth is a nearly spherical shape that revolve around the Sun and that each star is itself a distant Sun.

Around 1500AD, Many Astronomers like European mathematician Copernicus and Indian Mathematicians, the Great Aryabhata & Bhaskara I, proposed model universes with the sun at its center.

In 1576AD, Thomas Digges modified the Copernican system and proposed a model universe containing a multitude of stars extending to infinity.

In 1584AD, non-hierarchical cosmology was proposed by Giordano Bruno, with the assumption that the universe had its center everywhere and its circumference nowhere.

In 1600AD, Tycho proposed a system in which the planets other than Earth orbited the Sun while the Sun orbited the Earth.

In 1609AD, Johannes Kepler used the dark night sky to argue for a finite universe. He told that planets moved in an ellipse and not in perfect circles, about the Sun, known as the law of planetary motion. Newton later explained it by his inverse square law for the gravitational force. Galileo observed Moon of Jupiter in support of the heliocentric model.

In 1687AD, Newton established Laws of motion.

In 1791AD, Erasmus Darwin gave the first description of cyclic expanding and contracting universe.

In 1848AD, Edgar Allan Poe offers a solution to Olbers paradox in an essay that also suggests the expansion and collapse of the universe.

In 1905AD, Albert Einstein published the Special Theory of Relativity pointing that space and time are not separate continuums.

In 1916AD, Einstein published the General Theory of Relativity (GTR).

In 1922AD, The Russian Mathematician Friedmann realized that Einstein's equations could describe an expanding universe and published his paper entitled "*Ü*ber die Kr*ü*mmung des Raumes" (English Translation is "On the curvature of Space"). Einstein was reluctant in believing about static (non-expanding) universe.

In 1927 AD, Georges Lemaitre presented his idea of an expanding universe. He also derived Hubble's law and provided the first observational estimation of the Hubble's constant. Through the research contribution, Lemaitre has added a completely new feature to the discussion of cosmology and he proposed that the universe began as a single lump of matter, a primeval atom that radioactively decays in an outrushing explosion. Due to this, he may be called as 'father of the Big-Bang.

In 1929AD, The American Astronomer Hubble established that some nebulae were indeed distant galaxies comparable in size to our own Milky Way. Hubble was the first person to give the concept of expanding universe & cosmological constant.

In 1950AD, The British astronomer Fred Hoyle change the phrase "Big-Bang", which means that the universe was born at about ten thousand million years ago and the galaxies are still receding away from us after that initial rupture.

1965AD, Penzias & Wilson discovered a cosmic microwave background radiation (CMB).

In 1970AD, Cosmologists have accepted the Hot Big Bang model.

Dr. Milo Woltt, in 1986, discovered the wave structure of matter.

1.3 Einstein's Theory of Gravitation:

Newton, in the year 1687, established the Laws of motion. In the inverse square for the gravitational forces, he formulated a mathematical description about the motion of the bodies in space (and time), which is also known as the Newtonian theory of Gravitation. Later, in the year 1905, Einstein formulated the special theory of relativity, in which he discussed the relativity of uniform translatory motion in a region of free space where the gravitational effects are to be neglected. The mathematician Herman Minkowski (1864-1909) showed that space and time coordinates in the special theory of relativity are to be considered as a single entity space-time which seems to satisfy the Lorentz Transformation. Special theory of relativity is applicable only to study the relative motion in the inertial frame of references but in an accelerated frame of references, the special theory of relativity fails to study the relative motions. In the perspective of these limitations of the special theory of relativity, Einstein made a huge effort to resolve them and ultimately in the year 1915, he suggested that gravity is not only a force like other forces (as had been assumed previously) but is a consequence of the fact that the space-time is not flat, rather it is curved by the distribution of energy and mass in it. After having this concept, Einstein generalized the special theory of relativity and formulated the Einstein's Theory of Gravitation which is also known as General Theory of Relativity. This theory of Gravitation represents a theory of space-time and so it describes a theory of Dynamics of the Universe. Einstein's describes gravitation more accurately in a comprehensive manner in his Theory of Gravitation than Newton's theory of gravitation. The basic difference between the Einstein's Theory of Gravitation and Newton's Theory of Gravitation is that the geometry plays an active role in the General Theory of Relativity but it has no role in the latter. Einstein's Theory of Gravitation or General Theory of Relativity also supports the observational data and is treated as the foundation of other geometric theories of gravitation.

Einstein's General Theory of Relativity is based on the Riemannian metric tensor gij that describes both geometry and gravitational field. In the process of developing General Theory of Relativity, Einstein's vision was guided by three basic principles, which are:

i. Principle of Covariance

ii. Principle of Equivalence and

iii. Mach's Principle

Principle of Covariance:

According to the Special theory of relativity, the law describing any physical phenomena in free space must be independent of the velocity of a particular observer who makes measurements and must have the same form and contents; when referred to a different set of Cartesian axes which are in uniform relative translatory motion. The principle of covariance states that the physical laws must be expressed in covariant form so that their form remains unaltered in all coordinate system i.e. the physical laws must be expressible in a form which is independent of the particular space-time coordinate system chosen or the laws of nature remain invariant with respect to any space-time coordinate system. This implies that the laws should be expressed in tensor form and hence the line element of special relativity

$$ds^2 = -dx^2 - dy^2 - dz^2 + c^2 dt^2 \quad , \tag{1.1}$$

which is not invariant under a general coordinate transform, is replaced by

$$ds^2 = g_{ij} dx^i dx^j \quad ; \quad i,\ j = 1,\ 2,\ 3,\ 4 \tag{1.2}$$

which is valid in any coordinate system. The fundamental tensor g_{ij} is a symmetric tensor of rank two satisfying transformation law

$$g'_{ij} = \frac{\partial x^a}{\partial x'^i}\frac{\partial x^b}{\partial x'^j}g_{ab} \tag{1.3}$$

where dashed ($\prime$) quantities belong to the new coordinate system x'^i .

Principle of Equivalence:

The actual hypothesis by which the gravitational considerations are introduced into the development of general relativity is known as the principle of equivalence. According to Newtonian theory, the principle of equivalence states that the gravitational mass and inertial mass are always equal, which is also known as weak equivalence principle. But, according to Einstein, the principle of equivalence can be stated as 'In the neighbourhood of any given point, one can distinguish between the gravitational field produced by the attraction of masses and the field produced by accelerating a frame of reference' and it is known as the strong equivalence principle.

Mach's Principle:

Mach Principle states that inertial properties of a body are determined by the distribution of matter in the universe. Since the gravitational field interacts with all matters, one could hope to see the Mach principle relationship between inertial and distant matter described in terms of the gravitational field. In order to state this in a way independent of units, the ratio of the inertial mass of a body to its active gravitational mass is considered. Einstein included extended remarks on the Mach principle and the issue of inertia in GRT in the series of the lecture delivered by him in 1921, on GRT at Princeton (The Meaning of Relativity, 5^{th} ed., Princeton University Press, Princeton, 1955, pp. 99-108) as (in his words):

i. If ponderable masses are piled up in the neighbourhood of the body then the inertia of a body must increase.

ii. If the neighbouring masses of a body are accelerated then the body must experience an accelerating force, and, actually, the direction of the force must be same with that of acceleration.

iii. A "Coriolis field" ought to be generated inside of a rotating hollow body that deflects moving bodies in the sense of the rotation, and a radial centrifugal field as well.

The space-time in Einstein's General Theory of Relativity is described by the pseudo-Riemannian metric as

$$ds^2 = g_{ij}dx^i dx^j \quad ; \quad i\,,j = 1,2,3 \textit{ and } 4 \tag{1.4}$$

where the symmetric metric tensor g_{ij} act as gravitational potential. The gravitational field equation or, simply the field equations in Einstein's General Theory of Relativity are given by

$$G_{ij} = R_{ij} - \frac{1}{2}Rg_{ij} = -\frac{8\pi G}{c^4}T_{ij} \tag{1.5}$$

where G_{ij} is the Einstein tensor, R_{ij} is the Ricci tensor, R is the Ricci scalar (Scalar Curvature) and T_{ij} is the energy-momentum tensor due to matter and Λ is the cosmological constant. In order to study Static cosmological model, Einstein modified his field equation by introducing another term as

$$G_{ij} = R_{ij} - \frac{1}{2}Rg_{ij} + \Lambda g_{ij} = -8\pi T_{ij} \tag{1.6}$$

where Λ is the cosmological constant. Later, he later dropped this term by saying that "greatest blunder" of his life. In both the equations (1.5) and (1.6), left hand sides represent the geometry of space and the right hand sides represent the matter.

1.4 Weyl's Geometry:

In order to describe the general theory of relativity, Einstein (in the year 1916) used the Riemannian geometry, in which the affine connections Γ^i_{jk} are taken as symmetric functions with respect to the two lower suffixes i.e. $\Gamma^i_{jk} = \Gamma^i_{kj}$ and the metric tensors g_{ij} are transformed under coordinate transformation but the length of a vector v^i is not changed/remains same when it (i.e. the vector) undergoes parallel transform. Therefore the affine connections of Riemannian geometry are uniquely determined by the metric tensors g_{ij} through the Christoffel symbols of the second kind as $\Gamma^i_{jk} = \left\{ \begin{matrix} \mathrm{i} \\ j \quad k \end{matrix} \right\}$. In Riemannian geometry, the connections are both torsion-free and metric preserving. In the year 1918, Weyl geometrized both gravitation and electromagnetism in his unified field theory that is treated as the first so-called unified field theory. But in Weyl's geometry, unlike Riemannian geometry, the length of a vector v^i under parallel transfer (infinitesimal) is not conserved rather it is assumed that the increment in length of the vector is proportional to its length and is taken as the homogeneous function of the displacement vector dx^i so that a gauge vector $\phi_i(x)$ is introduced into the affine connection.

In the year 1918, Weyl assumed the existence of both "coordinate transformation" and "gauge transformations". Therefore when the metric tensor g_{ik} in the coordinate system x^i is transformed to g'_{ik} in the coordinate system x'^i then

$$g'_{ik} = g_{ab}\frac{\partial x^a}{\partial x'^i}\frac{\partial x^b}{\partial x'^k} \tag{1.7}$$

and when g_{ik} is transformed to g'_{ik} under a gauge transformation then

$$g'_{ik} = \lambda(x) g_{ik} \tag{1.8}$$

where λ is an arbitrary function of coordinates. Therefore the transformation of infinitesimal metric $ds \to ds'$ is given by

$$ds' = \lambda^{\frac{1}{2}} ds \tag{1.9}$$

In the case of the vector v^i, if we assume that under the gauge transformation the components of v^i remain unchanged, the length

$$v^2 = g_{ik} v^i v^k \tag{1.10}$$

changes, $v \to v'$, with

$$v' = \lambda^{\frac{1}{2}} dv \tag{1.11}$$

The parallel displacement of the vector v^i from a point $P(x^i)$ to $Q(x^i + dx^i)$ is given by

$$dv^i = -v^j \Gamma^i_{jk} dx^k \tag{1.12}$$

where Γ^{i}_{jk} is an affine connection.

The length v of the vector v^i changes according to the relation

$$dv = v\phi_i dx^i \tag{1.13}$$

where ϕ_i is a given vector which, together with g_{ik}, characterizes the geometry. If ϕ_i is the gradient of a scalar function, then it follows from (1.13) that the change in the length dv of v^i is going from one point to another is independent of the path followed.

Therefore applying the gauge transformation, it can be obtained a relation

$$dv' = v'\phi'_i dx^i \tag{1.14}$$

If one makes use of (1.11) and (1.14), one finds

$$\phi'_i = \phi_{,i} + \frac{1}{2}\lambda^{-1}\lambda_{\ ,i} \tag{1.15}$$

As the gauge transformation for ϕ_i , where $\lambda_{,i} = \frac{\partial\lambda}{\partial x^i}$.

In order for (1.13) to be a consequence of (1.10) and (1.12), for arbitrary v^i and dx^k , one finds that

$$g_{ij}\Gamma^{j}_{ka} + g_{kj}\Gamma^{j}_{ia} = g_{ik,a} - 2g_{ik}\phi_a \ . \tag{1.16}$$

If we assume that

$$\Gamma^{a}_{ik} = \Gamma^{a}_{ki} \tag{1.17}$$

we get

$$\Gamma^{b}_{ik} = \left\{ \begin{matrix} & b & \\ i & & k \end{matrix} \right\} + g_{ik}\phi^b - \delta^b_i\phi_k - \delta^b_k\phi_i \ . \tag{1.18}$$

Carrying out the gauge transformation (1.8) gives

$$\left\{ \begin{matrix} & b & \\ i & & k \end{matrix} \right\}' = \frac{1}{2}\lambda^{-1}\left[\delta_i^b \lambda_{,k} + \delta_k^b \lambda_{,i} - g_{ik}\lambda_{,a} g^{ab}\right] + \left\{ \begin{matrix} & b & \\ i & & k \end{matrix} \right\}. \tag{1.19}$$

If one takes into account the gauge transformation (1.15) for ϕ_i , one can see that Γ^b_{ik} is invariant under the gauge transformation (Rosen 1982).

Thus the Weyl's geometry is characterized by the two independent quantities g_{ij} and ϕ_i so that the affine connection Γ^i_{jk} is given by

$$\Gamma^i_{jk} = \left\{ \begin{matrix} & i & \\ j & & k \end{matrix} \right\} + S^i_{jk}$$

where

$$S^i_{jk} = \frac{1}{2}\left(\delta^i_j \phi_k + \delta^i_k \phi_j - g_{jk}\phi^i\right) \tag{1.20}$$

and

$$\phi^i = g^{ij}\phi_j \tag{1.21}$$

1.5 Lyra's Geometry:

After geometrizing gravitation and electromagnetism by Weyl in his unified theory, Lyra (1951) suggested another modification of Riemannian geometry. Lyra defined a displacement vector $\overrightarrow{PQ}$ between two neighbouring points $P(x^i)$ and $Q(x^i + dx^i)$ by its components $f dx^i$, where $f = f(x^i)$ is a non-zero gauge function. Lyra used the reference system $\left(f, x^i\right)$ that include both gauge function $f(x^i)$ and the coordinate system x^i [i.e. the gauge function $f(x^i)$ and the coordinate system x^i together form a reference system $\left(f, x^i\right)$]. The transformation to a new reference system $\left(\bar{f}, \bar{x}^i\right)$ is defined by the following functions

$$\bar{f} = \bar{f}\left(f, x^i\right) \quad , \quad \bar{x}^i = \bar{x}^i(x^i) \tag{1.22}$$

with

$$\frac{\partial \bar{f}}{\partial f} \neq 0 \quad and \quad Jacobian \left| \frac{\partial \bar{x}^i}{\partial x^i} \right| \neq 0. \tag{1.23}$$

Lyra (1951) and Sen (1957,1958) shown that in any general reference system the coefficients of the affine connection are determined by the independent quantities Γ^i_{jk} and ϕ_i, where the displacement vector field quantities ϕ_i arise as a natural consequence of the formal introduction of the gauge function $f(x^i)$ into the structure-less manifold. The symmetric affine connections Γ^i_{jk} in the Lyra's manifold is given by

$$\Gamma^i_{jk} = \frac{1}{f} \left\{ \begin{matrix} i \\ j \quad k \end{matrix} \right\} + \frac{1}{2} \left(\delta^i_j \phi_k + \delta^i_k \phi_j - g_{jk} \phi^i \right) \tag{1.24}$$

The infinitesimal parallel transfer of a vector v^i is given by

$$\delta v^i = -\widetilde{\Gamma}^i_{jk} v^j f dx^k \tag{1.25}$$

where

$$\widetilde{\Gamma}^i_{jk} = \Gamma^i_{jk} - \frac{1}{2} \delta^i_j \phi_k \tag{1.26}$$

is not symmetric with respect to j and k, but the Lyra connection Γ^i_{jk} is symmetric with respect to the two lower suffixes i.e. $\Gamma^i_{jk} = \Gamma^i_{kj}$.

In Lyra's geometry, the metric given by in Lyra's geometry is invariant under both coordinate and gauge transformations and is given by

$$ds^2 = g_{ij} \left(f dx^i \right) \left(f dx^j \right)$$

i.e.

$$ds^2 = f^2 g_{ij} dx^i dx^j \ , \tag{1.27}$$

is invariant under both coordinate and gauge transformations, where g_{ij} is a second rank fundamental symmetric tensor. As in Riemannian geometry, the length of a vector is conserved under parallel transport, so the parallel transfer of a vector is integrable in Lyra's geometry.

A geodesic is defined as a curve given by $x^i = x^i(s)$ whose tangent vector $v^i = f\left(\frac{dx^i}{dS}\right)$ is transferred parallel to itself. The equation for the geodesic becomes

$$\begin{aligned} f\frac{d^2x^k}{dS^2} + \left[f^{-1}\left\{\begin{matrix} k \\ i \quad j \end{matrix}\right\} + \frac{1}{2}\left(\delta_i^k\phi_j + \delta_j^k\phi_i - g_{ij}\phi^k\right)\right] f^2\frac{dx^i}{dS}\frac{dx^j}{dS} \\ = \frac{1}{2}\left(\phi_i - \widetilde{\phi}_i\right) f^2\frac{dx^i}{dS}\frac{dx^k}{dS} \end{aligned} \tag{1.28}$$

where

$$\widetilde{\phi}_i = f^{-1}\frac{\partial\left[logf^2\right]}{\partial x^i}. \tag{1.29}$$

is the displacement vector in Lyra geometry. Therefore, in general, a geodesic is not a curve of extremal length given by $\int ds = 0$, in contrast to the Riemannian geometry. However, a sufficient condition for the coincidence of these two types of curves have shown by Sen and Dunn (1971) as

$$\phi_i = \widetilde{\phi}_i\ . \tag{1.30}$$

Since $\widetilde{\phi}_i$ transforms exactly as ϕ_i whenever $f \to f'$, the above condition is invariant under gauge transformations. Therefore the Lyra's geometry is characterized by the two fundamental quantities, the scalar f and the entities g_{ij} .

In Lyra's geometry, the curvature tensor $\widetilde{R}^k_{hij}$ is defined in the same manner as defined in the Riemannian geometry and is given by

$$\widetilde{R}^k_{hij} = \frac{1}{f^2}\left[\frac{\partial}{\partial x^i}\left(f\widetilde{\Gamma}^k_{hj}\right) - \frac{\partial}{\partial x^j}\left(f\widetilde{\Gamma}^k_{hi}\right) + f\Gamma^k_{ai}\, f\widetilde{\Gamma}^a_{hj} - f\Gamma^k_{aj}\, f\widetilde{\Gamma}^a_{hi}\right]. \tag{1.31}$$

Then the curvature scalar of Lyra's geometry is

$$\widetilde{R} = f^{-2}R + 3f^{-1}\phi^i_{;i} + \frac{3}{2}\phi_i\phi^i + 2\widetilde{\phi}_i\phi^i \tag{1.32}$$

where R is the Riemannian curvature scalar. If we now choose the normal gauge $f = 1$, then equation (1.32) reduces to

$$\widetilde{R} = R + 3\phi^i_{;i} + \frac{3}{2}\phi_i\phi^i \tag{1.33}$$

This curvature scalar of Lyra geometry is identical with that of (the curvature scalar of) Weyl's geometry.

1.6 Field Equations in Lyra's Geometry:

The invariant volume integral in Lyra's geometry is given by

$$I = \int L(-g)^{\frac{1}{2}} x^0 dx^1 x^0 dx^2 x^0 dx^3 x^0 dx^4 \tag{1.34}$$

where L is a scalar and is an absolute invariant in this geometry.

If we now use the normal gauge $x^0 = 1$ (Sen 1957) and following Halford (1970) let $L = *R$ the volume integral (1.34) becomes

$$I = \int *R(-g)^{\frac{1}{2}} d^4x \tag{1.35}$$

where d^4x is the element of volume in the four-dimensional space.

The field equations may be obtained from the variational principle

$$\delta\,(I + J) = 0 \tag{1.36}$$

where

$$J = \int \mathscr{L}(-g)^{\frac{1}{2}} d^4x \tag{1.37}$$

where $\mathscr{L}$ is the Lagrangian density of matter. The well-known method (Landau and Lifshitz 1962) gives the field equations

$$R_{ij} - \frac{1}{2}Rg_{ij} + \frac{3}{2}\phi_i\phi_j - \frac{3}{4}g_{ij}\phi^k\phi_k = -\chi T_{ij} \tag{1.38}$$

Jeavons et al. (1975), in their study of "A Correction to the Sen and Dunn Gravitational Field Equations", showed that the field equations formulated by Sen and Dunn (1971) cannot be derived from the normal variational principle and they suggested the modified field equations as

$$R_{ij} - \frac{1}{2}Rg_{ij} + \phi^{-1}(\phi_{i;j} - g_{ij}\Box\phi) - \omega\phi^{-2}(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,k}\phi^{,k}) = -\phi T_{ij} \tag{1.39}$$

where R_{ij} is the Ricci tensor, R is the Ricci scalar (Riemann curvature scalar), $\omega = constant = \frac{3}{2}$, and T_{ij} is the material energy-momentum tensor (in our units $c = 8\pi G = 1$). But Singh and Rai (1983) have indicated that, even though the original field equations formulated by Sen and Dunn (1971) are not derivable from the usual variational principle, they are still prove to be heuristically useful. In Lyra's geometry, the energy momentum tensor T^{ij} is not conserved. In normal gauge, with a constant gauge function, this theory is equivalent to Hoyle's creation field theory (Soleng 1987).

1.7 Some Aspects of the Work Related to Lyra's Geometry:

Our investigation pertains to cosmological models in Lyra's Geometry, a modified theory of gravitation as such we have described some relevant work. Mainly in this section, we have presented some of the relevant work carried out by various authors related to Lyra's Geometry in different contexts.

Lyra (1951) suggested a modification of Riemannian geometry, which is also regarded as a modification of Weyl's geometry, by introducing a Gauge function (or scale function) into the structure less manifold which removes the non-integrability condition of the length of a vector under parallel transport (i.e. the metricity condition is restored) and a cosmological constant is naturally introduced from the geometry. Lyra (1951) and Scheibe (1952) completed the study of this geometry, which is known as Lyra's Geometry. In Lyra's geometry, the connection is metric preserving as Riemannian geometry, and length transfers as integrable in contrast to Weyl's geometry. This alternating theory of Lyra's geometry is of interest since it produces effects similar to Einstein's theory.

On the basis of Lyra's geometry, Sen (1957) studied a static cosmological model universe similar to the Einstein's static model, which had a finite density and showing a red-shift. He also showed that the red-shift of spectral lines from extragalactic nebulae was nothing but an outcome of an intrinsic geometrical property of the model independent of expansion. Also, he obtained the field equations in normal gauge as

$$R_{ij} - \frac{1}{2}g_{ij}R + \frac{3}{2}g_{ij}\phi_i\phi_j - \frac{3}{4}g_{ij}\phi_k\phi^k = -\kappa T_{ij} \tag{1.40}$$

where R_{ij} is the Ricci tensor; R is the Ricci scalar, g_{ij} is a metric tensor, ϕ_i is a displacement field and T_{ij} is the energy-momentum tensor.

Sen (1960) and Sen and Dunn (1971) showed that, unlike Riemannian geometry, the auto parallels associated with the affine connection in Lyra geometry did not coincident with the geodesics arises from the metric. In the Lyra's geometry, they also constructed a new scalar-tensor theory where both the scalar and tensor field had natural geometrical significance.

Sen and Vanstone (1972), in their paper "On Weyl and Lyra Manifolds", showed that the Lyra's geometry and Weyl's geometry are special cases of manifolds with more general connections. Also, they showed the relationship between Lyra's geometry and Weyl's geometry and obtained the relationship of them with Riemannian geometry by giving a global formulation of Lyra's geometry.

Halford (1970) designed a cosmological theory within the framework of Lyra's geometry and showed that the constant displacement vector field in Lyra's geometry plays the role of the cosmological constant in the normal general relativistic study. Also, Halford (1972) obtained a closed-form exact solution of the field equations corresponding to a scalar-tensor theory similar to the B-D theory and showed that the scalar-tensor treatment based on Lyra's geometry predicts the same effect, within observational limits, as far as the classical solar system test are concerned (as in the Einstein's theory of relativity).

Bhamra (1974) obtained a spherically symmetric cosmological model of class-one in the framework of Lyra's geometry and showed that the static universe is physically unrealistic whereas the non-static universe is similar to Lemaitre's model in Riemannian geometry in which the mass-energy conservation law did not hold.

Jeavons et al. (1975), in their study of "A Correction to the Sen and Dunn Gravitational Field

Equations", showed that the field equations formulated by Sen and Dunn (1971) cannot be derived from the normal variational principle and they suggested the modified field equations as

$$R_{ij} - \frac{1}{2}Rg_{ij} + \phi^{-1}(\phi_{i;j} - g_{ij}\Box\phi) - \omega\phi^{-2}(\phi_i\phi_j - \frac{1}{2}g_{ij}\phi_k\phi^k) = -\phi T_{ij} \tag{1.41}$$

where R_{ij} is the Ricci tensor, R is the Ricci scalar (Riemann curvature scalar), $\omega = constant = \frac{3}{2}$, and T_{ij} is the material energy-momentum tensor (in our units $c = 8\pi G = 1$).

Reddy (1973, 1977) investigated the Birkhoff's Theorem of general relativity both in the Brans-Dicke theory and in the scalar-tensor theory suggested by Sen and Dunn (1971). Reddy (1973) showed that the Birkhoff's Theorem of general relativity is hold good in the scalar-tensor theory suggested by Sen and Dunn (1971) for all scalar field irrespective of nature of the scalar field. But in the Brans-Dicke theory, Birkhoff's Theorem is valid only for the scalar field which is independent of time. Considering time-independent scalar field in the scalar-tensor theory suggested by Sen and Dunn, Reddy (1977) showed that the Birkhoff's Theorem of general relativity is also valid in presence of electromagnetic field. So, he suggested that the scalar-tensor theory of Sen and Dunn (1971) may be considered as a superior version of the Brans-Dicke theory.

Karade and Borikar (1978) studied the effects of the thermodynamic equilibrium of a gravitating fluid sphere in Lyra's Geometry and obtained a static model universe with a zero red-shift in it.

Singh and Rai (1979) investigated the Birkhoff's Theorem of general relativity in the scalar-tensor theory suggested by Jeavons et al. (1975) and showed that when the scalar field is independent time then in presence of electromagnetic fields in the scalar-tensor theory suggested by Jeavons et al. (1975), the spherically symmetric gravitational and electromagnetic fields turn out to be static.

Kalyanshetti and Waghmode (1982) obtained a static cosmological model in Einstein- Cartan theory in the framework of Lyra's geometry. Assuming the spin of each fluid particle along the radial direction, he observed that only constant spin has existed in his Einstein's static model universe that can be expressed in terms of central density.

Considering a metric described by a scale constant associated with the size of the universe, Rosen (1983) modified the Weyl-Dirac theory of gravitation and electromagnetism.

Reddy and Innaiah (1985) formulated an anisotropic and spatially homogeneous Bianchi type-I cosmological model in Lyra's manifold with perfect fluid as a source of gravitational field by considering energy density equal to pressure.

Reddy and Innaiah (1986) constructed a plane-symmetric cosmological model in Lyra manifold with perfect fluid as a source of gravitational field by taking energy density equal to pressure.

Beesham (1986) obtained Vacuum FRW Cosmological models in the framework of Lyra's geometry and a number of new solutions are discussed in the de Sitter universe.

In the study of "Cosmologies Based on Lyra's Geometry", Soleng (1987) discussed that the Lyra Geometry together with gauge vector ϕ_i will play either the role of cosmological constant or the creation field (equal to the Hoyle's creation field [Hoyle (1948), Hoyle and Narlikar (1963, 1964)]). He also showed that the solutions in the first case are equal to the solutions in general relativistic cosmologies with a cosmological term.

Considering Friedmann-Lemaitre-Robertson-Walker (FLRW), Beesham (1988) formulated cosmological models in Lyra's manifold with time-dependent displacement field. In this model, not only he solved the existing problems like singularity, horizon and entropy in the standard cosmological models based on Riemannian geometry but also studied the asymptotic behavior of the models.

Singh and his co-authors (1991a,b,c,d; 1992a,b; 1993b; 1997) studied Bianchi types I, II, III, V, VI_0, $VIII$, IX, Kantowski-Sachs, and a new class of cosmological model universes with and without time-dependent displacement field in the framework of Lyra geometry. Comparative study of the Cosmological theory based on Lyra's geometry and the Friedmann-Robertson-Walker (FRW) model universes with a constant deceleration parameter in the Einstein's theory of relativity were also made by them.

Khadekar and Nagpure (2001) studied a Higher Dimensional Static conformally flat spherically symmetric Cosmological Model in Lyra Geometry in presence of perfect fluid and observed that in Lyra's manifold, the displacement vector plays the role of the spin density.

Rahaman et al. (2002) investigated an Inhomogeneous Cosmological Model in Lyra Geometry and obtained the exact solutions of the field equations. He has got an anisotropic model universe where the displacement vector is always non-zero, so the concept of Lyra geometry exists even after infinite time.

Rahaman (2003) discussed a five-dimensional spherically symmetric metric in presence of a homogeneous perfect fluid in the framework of Lyra geometry and obtained a cosmological model for vacuum energy type universe together with matter filled-universe for dust case, Zeldovich fluid and stiff fluid.

Considering a time-dependent displacement field, Pradhan and Vishwakarma (2004) investigated a locally rotationally symmetric Bianchi type-I metric and a new class of exact solutions of the field equations in the framework of Lyra geometry is obtained for constant deceleration parameter. Also, they studied the characteristics of the energy density and displacement field in the power law expansion and exponential expansion of both flat and non-flat universe.

Rahaman et al. (2005) obtained two model universes namely axially symmetric Bianchi type-I and Kantowski-Sach cosmological models with negative constant deceleration parameter based on Lyra geometry.

Casana et al. (2005) studied the coupling of the curved and torsioned Lyra manifold with the electromagnetic field and showed that the coupling between torsion and the massless electromagnetic field was related to scale transformations in Lyra setting. Also, they showed that the suitable choice of the connection of gauge transformations with scale invariance in Lyra manifold would remove the problem of breaking the local gauge invariance connected with this coupling.

Casana et al. (2006) discussed the Dirac field in Lyra geometry and obtained the equation of motions and conservation laws for spin and energy-momentum. They, also, obtained the scale relation, which is a fundamental property of matter fields in Lyra geometry, connecting the spin tensor and energy-momentum tensor.

Studying five-dimensional LRS Bianchi type-I space-time in presence of bulk viscous fluid, Mohanty et al. (2007) constructed a higher dimensional string cosmological model in Lyra

Manifold for time-dependent displacement field and constant coefficient of bulk viscosity. This model had no initial singularity.

In a scalar-tensor theory of Sen (1957) based on Lyra manifold, Rao and Vijaya Santhi (2008a) formulated a Bianchi type-V cosmological model in presence of perfect fluid for a constant displacement vector. Also, when displacement vector is a function of cosmic time then by using negative constant deceleration parameter they had shown that this model exists only for radiation universe. Kumar and Singh (2008) investigated a spatially homogeneous and anisotropic Bianchi type-I in presence of perfect fluid and obtained a cosmological model universe based on Lyra geometry. Using the special law of Hubble's parameter that gives a constant deceleration parameter, they had obtained the exact solutions of the field equations which are consistent with the recent observational data from supernovae type Ia.

Considering five-dimensional plane symmetric metric, Mohanty et al. (2009b) attempted to obtain a string cosmological model universe both in Riemannian geometry and in Lyra geometry. But they had observed that, in both the theories, the string cosmological models were not survived. Accordingly, they had formulated the vacuum cosmological models and discussed their properties.

Investigating plane-symmetric metric under the influence of perfect fluid, Yadav (2010) obtained an inhomogeneous cosmological model universe with electromagnetic field based on Lyra geometry and the exact solutions of the field equations for this model are consistent with the recent observational data from supernovae type Ia.

In the framework of Lyra geometry, Gad (2011) obtained a new class of axially symmetric cosmological model universes in presence of the mesonic stiff fluid with time-dependent displacement field which are expanding, shearing and non-rotating.

Adhav (2011) obtained an anisotropic dark energy model based on Lyra geometry by examining a LRS Bianchi type-I metric under the influence of anisotropic fluid. Considering exponential volumetric expansion, exact solutions of the field equations were determined for constant and time-dependent displacement field and isotropic properties of the space and fluid were examined.

In the framework of Lyra geometry, Mahanta and Biswal (2012) obtained cosmological model universes for both string cloud and domain walls with quark matter by solving the

Einstein's field equations using anisotropy property of the universe, time-dependent displacement field and special law for Hubble's parameter that gives the constant value of deceleration parameter.

Shchigolev (2013) obtained a Cosmological model within the framework of Lyra's geometry with an effective Λ-Term in the field equations that appeared due to the interaction of the displacement vector field with an auxiliary Λ-Term.

In a cosmological model in the framework of Lyra's geometry, Hova (2013) established a relationship between the displacement vector field, the energy density of matter and Hubble's parameter through an arbitrary function $\alpha(t)$ and obtained an effective equation of state parameter ω_{eff} in terms of $\alpha(t)$ and constant equation of state ω_m . The effective equation of state parameter ω_{eff} was completely determined for pressure-less matter by $\alpha(t)$. Consequently, he had obtained exact solutions for the models in Lyra's geometry that yield the ΛCDM and Power-Law Expansion.

Studying an inhomogeneous Bianchi type-I metric in presence of an electromagnetic field, Megied et al. (2014) obtained a cosmological model in the framework of Lyra geometry. Assuming the metric potentials and displacement field as functions of coordinates x ant t, they had obtained a class of exact solutions of the Einstein's field equations.

In the framework of Lyra's geometry, Darabi et al. (2015) studied about the existence of the Einstein's static universe for homogeneous scalar perturbations together with the stability condition and obtained the stability condition in terms of the equation of state parameter ω as $\omega = \frac{p}{\rho}$. Also, they had studied the stability conditions for tensor and vector perturbations. They showed that, in the framework of Lyra's geometry, Einstein's static universe can be obtained for appropriate values of physical parameters.

In order to describe the evolution of the universe, Saadat (2016) formulated a new cosmological model based on extended Chaplygin gas with varying Λ-term in the context of Lyra geometry where extended Chaplygin gas is taken as dark matter and quintessence scalar field is considered as dark energy.

1.8 Bianchi Space-Times:

The cosmological principle is supposed to be deduced from the Copernican principle. Further the analogy of the assumption may be stated as: the world line of our galaxy is not special because if our universe is isotropic about our world line then it is also isotropic about the world line of other galaxies. In the year 2016 (Bull, P.et al., 2016), showed that Einstein's general theory of relativity is a perfect theory to describe our expanding universe by using Friedmann - Robertson-Walker (FRW) space-time which describes isotropic, homogeneous on large scale. But the isotropic Cosmic Microwave Background (CMB) radiation is not certainly able to explain about isotropic space-time. Considering these limitations, it does not suit us to curb ourselves, only in the study of isotropic homogeneous cosmological models; rather it leads to study more conveniently anisotropic and inhomogeneous cosmological models. Since the solution of the Einstein's field equation for study of inhomogeneous cosmological model universes appears to be more complicated and sometimes becomes very hard nut to crack for the researchers to solve, hence, many cosmologists opt to undergo the spatially homogeneous and anisotropic Bianchi type models instead of inhomogeneous models. Generally, Bianchi type models represent a mid way between the FRW model and inhomogeneous & anisotropic universes, hence its important role is found in modern cosmology. In order to understand the properties and structure of the space of all cosmological solutions of the Einstein's field equations, a spatially homogeneous cosmological model plays a vital role (McCallum, 1979).

From the study of different texts of various authors, it has been found that Bianchi type cosmological models are homogeneous and anisotropic, but the isotropization process of these model universes are studied with the passage of time. Also, in the theoretical perspective, the anisotropic universes have possessed a greater generality than the isotropic model universes. In addition, due to the simplicity of the field equations in the Bianchi space-times, the Bianchi Type cosmological models have an added importance in the construction of the anisotropic and spatially homogeneous cosmological model universe. Thus it can be said that Bianchi type cosmological models play a crucial role in the description of the model universes. Di Pietro and Demaret (1999) classified the 9 (nine) different types of Bianchi models into 2 (two) groups 1 and 2. They placed Bianchi types I, II, VI_0, VII_0, $VIII$ and IX models are in group 1 whereas the Bianchi type models III, IV, V, $VI_{h\neq -1}$ and $VII_{h\neq 0}$ are in groups 2.

Generally, the metric for the Bianchi type space-time is characterized as

$$ds^2 = dl^2 - dt^2 \quad where \quad dl^2 = g_{ab}dx^a dx^b \tag{1.42}$$

where dl^2 is the three/four dimensional line element. In order to understand the structure and properties of space of all cosmological solutions of Einstein's field equations, spatially homogeneous cosmological model have taken a vital role [McCallum (1979)].

1.9 Some Aspects of the Work Related to Bianchi Space-Times:

In this thesis our investigation is pertained to Bianchi Type metric as the line element of the problem as such we have described some relevant work. Mainly in this section we have presented some of the relevant work carried out by various authors related to Bianchi type metric.

After obtaining the structure constants of the nine different types of Bianchi type I-IX space-time by Bianchi (1918), Taub (1951) and Petrov (1961), many authors investigated Bianchi type cosmological models. Some of the Bianchi cosmological models are mentioned below.

Rosen (1964) obtained two spatially homogeneous solutions of the Einstein-Maxwell equations for pure-magnetic case. By investigating Bianchi type metric, Zel'dovich (1964) obtained the general solution of the Einstein's field equation for the dust case and the singularity bahaviour of the solution was also explained by him. By investigating homogeneous axially-symmetric model, Kompaneets and Chernov (1964) developed the solutions of the Einstein's field equations for dust and ultra relativistic gas and the singularity of the solutions were also discussed for radiation case.

Zel'dovich (1965) showed that a primordial homogeneous magnetic field is existed in the universe and examined the characteristics of the corresponding solution.

Hawking and Taylor (1966) demonstrated that initially the helium production increases with the increase of the anisotropy parameter but when the anisotropy parameter is large enough then production of helium decreases again.

Misner (1967, 1968), investigated anisotropic Bianchi type I cosmological models and obtained the solutions for large anisotropy in general and small anisotropy in both dust dominated and radiation dominated cosmology. He also used his results, in order to calculate an upper limit to the temperature anisotropy as 2.7^0 k of the cosmic microwave radiation.

Jacob, K. C., (1968), investigated Bianchi type I spatially homogeneous anisotropic cosmological models with uniform magnetic field and obtained general solution with no magnetic field for perfect fluid (with barotropic equation of state). Also, a number of partially realistic cosmological models were obtained by using the solution and studied the anisotropic effects.

By considering Bianchi type I metric, Jacob, K. C., (1969), studied the effects of a primordial uniform magnetic field on spatially homogeneous anisotropic model universe and solutions were achieved in different sub cases like Dust-Magnetic, hard Magnetic, Pure-Magnetic and Zel'dovich-Magnetic.

In the paper entitled with "A Class of Homogeneous Cosmological Models", Ellis and McCallum (1969) studied the solutions of the Einstein's field equations in two different classes', one by assuming the source of the gravitational field as a perfect fluid with barotropic equation of state and the other by assuming that there exist only a transitive group of motions on three surfaces orthogonal to the flow vector of fluid. By considering positive, zero and negative cosmological constants, they also discussed the general dust solutions for these cases.

Investigating Bianchi type IX line element, Khalatnikov and Lifshitz (1970) obtained cosmological models by solving Einstein's equations.

Collins and Hawking (1972) showed that the Bianchi type VII models are the most general uniform cosmological models which are infinite in all three spatial directions. These models have wide class initial conditions providing maximum possible number of arbitrary constants. He also showed that all the initial anisotropic universes do not approaches to isotropy but the subclass of universe having escape velocity would approach to isotropy.

Singh (1975) obtained cylindrically symmetric solutions in a scalar-tensor theory of gravitation of which one solution is nonsingular and others special solutions are reduced to Misra & Radhakrishna's time dependent solutions and Levi-Civita solution .

Kibble (1976), in his paper " Topology of Cosmic Domains and Strings ", recognized the stable topological defects that occurred throughout the phase transition as strings. Also, he showed that the topological structure of the domain structure depend on the homogenity group of the manifold of degenerate vacua.

A new class of exact solutions of Einstein's equations without big-bang singularity was obtained by Tomimura (1978) and Szafron and Collins (1979) where the solutions represent inhomogeneous, cylindrically symmetric cosmological model with perfect fluid. Considering Bianchi type-VIII metric, Collins and Glass (1980) investigated perfect fluid spatially homogeneous cosmological models and obtained a new class of exact solutions. Also, he classified and discussed the new solutions in many ways.

Banerjee and Santos (1981) investigated a Bianchi type-I universe in order to obtain spatially homogeneous cosmological models in general scalar-tensor theory. From the solutions obtained in his model, he observed that the universe expands from zero volume with initial singularity and then retrace back. Therefore the singularity in his model had significant difference with that in the analogous Kasner (1921) model universe in Einstein's theory.

In the paper entitled with "Inhomogeneous cosmology; Gravitational Radiation in Bianchi Background", Adams et al. (1982) used Bianchi types I-IX metric, in order to explain cosmological models and developed an exact formalism where gravitational radiation of arbitrary polarization can be studied. They had also shown how this formalism is used to an empty Bianchi type-I for the transformation of the z-dependent chaotic singularity structure to the propagation of gravitational radiation along the z-axis.

Accioly et al. (1983) studied Bianchi type-I metric and obtained exact solutions for two different cosmological models involving non-minimal coupling between gravitation and other fields and discussed their physical properties in details.

Reddy and Innaiah (1985) obtained a spatially homogeneous, anisotropic Bianchi type-I cosmological model in Lyra's manifold by taking perfect fluid as a source of the gravitational field, assuming pressure equal to energy density.

By considering the linear relationship among fluid density, expansion scalar and shear scalar, exact solutions of a spatially homogeneous LRS Bianchi type-II model universe in presence of shear and bulk-viscosity with barotropic equation of state were obtained by Banerjee et al. (1986).

Nayak and Bhuyan (1987) obtained a Bianchi type-V perfect fluid model with source-free electromagnetic fields by solving exactly the Einstein's-Maxwell equation for the non-locally rotationally symmetric case.

Berman and Gomide (1988) have discussed very simple Bianchi type-I anisotropic cosmological models with constant deceleration parameter. The simplicity of the model and easy way of defining deceleration parameter and Hubble's constant were the key features of this model.

Considering cylindrically symmetric Bianchi type-I metric in presence of perfect fluid with an electromagnetic field, Bali and Tyagi (1989) studied an inhomogeneous cosmological model and discussed some properties together with the behavior of the electromagnetic field tensors. In order to solve the field equations, they used the relation between the metric potential as $A = (BC)^n$ where n is a constant and A, B, C are functions of x and t.

In Barber's self-creation cosmology, Venketeswarlu and Reddy (1990) obtained a cosmological model by considering Bianchi type-I metric in presence of perfect fluid and discussed also some properties in the physical and geometrical sense.

Investing a Bianchi VI_0 Space-Time Chakraborty (1991b) obtained a class of cosmological models of strings both in presence of a magnetic field and without magnetic field.

In the zero-mass scalar field, Rao and Sanyasiraju (1992) obtained the exact Bianchi type-VIII and IX cosmological models in presence of perfect fluid with $p = \rho$ and showed that both the Bianchi type models are expanding, irrotational and shearing.

Romano (1993) investigated a plane-symmetric inflationary Bianchi type-I model universe with the help of causal thermodynamics where the initial anisotropy extinguished quickly.

Latifi et al. (1994) showed that not only the Bianchi type-IX cosmological models are non-integrable in the Painleve sense but also they have no vacuum solutions.

In four dimensional space-times, Demaret and Querella (1995) examined the classical behaviors of Bianchi type cosmological models by using the Hamiltonian formalism of quadratic gravity of Boulware. For the two Bianchi type models I and IX, they also suggested the super-Hamiltonian constraints in the explicit forms.

Investigating Bianchi type I metric in presence of the string cloud, Yavuz and Tarhan

(1996) obtained exact solutions of the Einstein's field equations. They have also studied the significance of the effects of strings on the anisotropic properties.

Considering Anisotropic Bianchi type metric in presence of perfect fluid, Barrow (1997) examined the development of the source of matter with small anisotropic pressure in the various field. Also, he had calculated the evolution of them all over the dust and radiation eras of an almost isotropic universe.

In Barber's Second self-creation theory of gravitation, Ram and Singh (1998) obtained completely anisotropic Bianchi-II type cosmological solutions both in presence of stiff-matter and in the vacuum.

Kawai and Soda (1999) showed that the anisotropic non-singular Bianchi Type-I Cosmological Solutions that obtained from the effective action with a superstring were subsequently super-inflate and then smoothly continue to either Friedmann-type or Kasner-type solutions.

Considering 4-dimensional Bianchi type-I metric, Chakraborty and Ghosh (2000) have obtained a cosmological model in the generalized scalar-tensor theory of gravitation and in presence of bulk viscous fluid; both power law and exponential solutions were discussed.

Assuming string tension density equal to the rest energy density for a cloud of string, Bali and Dave (2001) obtained a deterministic solution for a Bianchi-IX string universe in general relativity.

Pradhan and Vishwakarma (2002) derived a new class of LRS Bianchi-I model universes in presence of perfect fluid with constant deceleration parameter. Exact solutions for Dust universe, radiating universe and false vacuum universe wes also obtained.

Sahu and Panigrahi (2003) investigated a spatially homogeneous anisotropic Bianchi Type-I model universe with perfect fluid in Barber's second theory of gravitation. They have also shown that the vacuum model degenerated to Kasner model and the general fluid distribution would reduce to isotropic vacuum model.

Saha and Boyadjiev (2004) obtained a Bianchi Type-I Cosmology in presence of perfect fluid with interacting spinor field and scalar field. He has shown that the positive and negative values of Λ term correspond to an oscillatory model and non-exponential mode of evolution respectively.

Saha (2005) derived anisotropic cosmological models with perfect fluid and dark energy by considering a Bianchi-I type space-time with a binary mixture of perfect fluid and quintessence type dark energy. He has obtained the exact solutions of the Einstein's field equations and formulated a closed universe with a space-time singularity and a regular, oscillatory type open universe which became infinite in time.

Studying Bianchitype-III space-time, Xiang (2006) obtained a string model universe in presence of both bulk viscosity and magnetic field which was found to be inflationary, shearing and non-rotating.

Considering an anisotropic and homogeneous space-time in five dimensions, Mohanty and Mohanta (2007) derived two Zeldovich fluid models in Barber's self-creation theory of gravitation. He has shown that the extra dimension in one of the models remain constant during the evolution whereas in other model it was contracted.

Kumar and Singh (2008) constructed an exact Bianchi type-I cosmological model in Lyra's manifold. In order to solve the field equations, he used constant deceleration parameter that obtained from special law for variation of Hubble's parameter and the solutions are consistent with the observational data from supernovae type Ia.

In the paper entitled with "Geometrical Behaviors of LRS Bianchi Type-I Cosmological Model", Amirhashchi, Zainuddin and Dezfouli (2009) investigated a LRS Bianchi-I type Space-time in empty space and showed that initially when $t \to 0$ then the vacuum model do not posses singularity whenever the scale factors A and B are equal and equal to $exp(t)$.

Akarsu and Kilinc (2010a,b) constructed two Bianchi models entitled with "LRS Bianchi Type-I Models with Anisotropic Dark Energy and Constant Deceleration Parameter" and "Bianchi Type-III Models with Anisotropic Dark Energy". In the first paper, they have assumed a special law of variation of average Hubble's parameter that produced a constant value of the deceleration parameter for obtaining the solution of the field equations, whereas in the second paper, they have obtained a general anisotropy parameter of the expansion for Bianchi-III type space time with imperfect fluid (single diagonal).

Pradhan, Amirhashchi and Saha (2011a) formulated a Bianchi type-I anisotropic dark energy model with constant deceleration parameter where the equation of state parameter was dependent on cosmic time. Using a variation law for Hubble's parameter, they have obtained two types of solutions (exponential form and power-law form) for the Einstein's field equations that satisfy the recent observational data.

Investigating a LRS Bianchi type-I space time in presence of perfect fluid with constant deceleration parameter, Adhav (2012) derived a spatially homogeneous anisotropic model universe in $f(R,T)$ theory of gravity. Also, he showed that the LRS Bianchi-I type cosmology in $f(R,T)$ theory of gravity may be reconstructed with the suitable choice of the function $f(T)$.

Sahu and Kumar (2013) obtained a Tilted Bianchi type-I cosmological model with stiff fluid for time varying displacement vector field in Lyra Geometry, where they had shown the importance of time varying displacement vector in the dynamics of our universe.

Ladke (2014) formulated five dimensional spatially homogeneous and anisotropic Bianchi type-I cosmological models with variable gravitational and cosmological constant in Kasner form. Using a time dependent deceleration parameter due to a law of variation of scale factor, the exact solutions of the field equations were obtained.

Recently, authors like Mishra et al. (2015a), Sahoo & Mishra (2015), singh et al. (2016a) sdudied Bianchi type cosmological models in general relativity and modified theories of gravity in different contexts.

1.10 Strings and String Cosmology:

Despite the vicissitudes of literary tastes and temperament, the Big-Bang theory is the most prevailing theory of the formation of our universe, however, till today we are not in a state to provide an exactly clear statement about the origin and evolution of our universe that is why the origin of our universe is treated as one of the greatest mysteries amongst the cosmologists. Therefore the study of the exact physical situation of the universe at the early stage of its evolution becomes a matter of concern. In order to describe the events at the early stages of the evolution of our universe, cosmologists develop the concept of string theory, where cosmic string is one of the most important objects of study. In order to study about the period before the creation of the particle in the universe, string theory is used by cosmologists. At the early stages of the evolution of our universe, it might have passage through a number of phase transitions just after the big-bang as it was cooled down from the hot initial state (*i. e. the universe passage through its critical temperatures*). The symmetry of the universe might have been broken spontaneously during the phase transition.

During the phase transitions at the early stage of our universe, the symmetry of the universe might have been broken spontaneously and as a result, various topological defects like cosmic string, domain walls, monopole and textures [Kibble (1976); Mermin (1979)] were formed. (A defect is nothing but a discontinuity in the vacuum depending on the topology of the vacuum it could be string, domain walls, monopole and textures). In the year 1998, Pando and his co-authors (Pando et al., 1998) suggested that the topological defects were responsible for the formation of the structure of the universe. Vilenkin (1985) and Vilenkin & Shellard (1994) showed that amongst the all topological defects, only string can explain to very interesting cosmological consequences like galaxy formation and double quasar problem. Different works of literatures reveal that the string theory is also treated as one of the important contenders for the unification of all forces. Present configurations of our universe are also agreed by the presence of large-scale network of strings in the early stage of it. Strings are also treated as one of the main causes of density perturbations which are necessary for the configuration of the large-scale structure in our universe. Due to the stress-energy possessed by the strings they could have produced a gravitational field. Therefore the gravitational effects arising from the strings are also treated as an interesting topic of study. Letelier (1979, 1983) and Stachel (1980) were the first who introduced the strings into the general theory of relativity.

Considering that the massive strings were formed from the massless geometric strings with particle attached along its extension, Letelier (1979) formulated the equation of energy-momentum tensor for a cloud of massive strings as

$$T_{ij} = \rho u_i u_j - \lambda x_i x_j$$

where ρ is the rest energy density for a cloud of string with particle attached to them and λ is the string cloud density and

$$\rho = \rho_p + \lambda$$

in which ρ_p is the particle density of the configuration.

Many authors like Zeldovich et al. (1974), Kibble (1976), Letelier (1979), Stachel (1980) showed that existence of the string in the early universe may be explained by using GUT (grand unified theories). Brief discussion of the number of the important chronological achievements in string theory was presented by Schwarz (2001). Recently strings and string cosmological models have got increasing importance in the cosmological society. Various authors like, Vilenkin (1981), Litelier (1983), Gott (1985), Krori et al. (1990), Banerjee et al. (1990), Chakraborty (1991a), Tikeker & Patel (1992), Tikeker et al. (1994), Tikeker & Patel (1994), Roy & Banerjee (1995), Ram & Singh (1995), Yavus & Tarhan (1996), Brustein & Hadad (1998), Singh & Singh (1999), Bali & Dave (2001), Bhattacharjee & Baruah (2001), Baysal et al. (2001), Bali et al. (2006), Pradhan (2007), Rao et al. (2008b, c), Rao & Vijaya Santhi (2012b) studied string cosmological models in general relativity and modified theories of relativity in different contexts.

Recently, authors like Venkateswarlu et al. (2013); Das & Ali (2014); Ram & Priyanka (2014); Venkateswarlu & Satish (2014); Singh & Beesham (2014); Korpinar & Unluturk (2015); Bali & Singh (2015); Sahoo et al. (2016); Patil & Bhojne (2016); Kar et al. (2016); Behal & Shukla (2017); Mete et al. (2017); Azevedo & Martins (2017); Kanakavalli et al. (2017); Bena & Grana (2017); Sahoo et al. (2017) studied various string cosmological models in different contexts.

1.11 Higher Dimensional Cosmology:

In the general relativistic physics, our present universe seems to be four-dimensional of which three are used to denote usual spatial dimensions and the fourth dimension represents time. But many researchers established their theories about the universe in higher-dimensional space-time mainly due to the significant achievement in solving long-standing problems relating to the stability of the results in general relativity and quantum mechanics. Before Einstein, two mathematicians namely, Herman Weyl (1918) and Theodor Kaluza (1921) attempted to unify gravity with the electromagnetic force. In the standard four-dimensional space-times, the first unified theory was suggested by Herman Weyl on the basis of generalizing the Riemannian geometry. But in the five-dimensional space-times, a unified theory of gravitation and electromagnetic force was established the first time by the mathematician Kaluza. Also in the year 1926, Oskar Klein, Swedish physicist, suggested the unification law of the gravitational force and the electromagnetic force by using the fifth dimension. This theory is known as Kaluza-Klein theory. Later on, it was established that their approaches were to some extent erroneous, but this theory provides a basis to the researchers for further investigation over the last few decades. Einstein (1927), later on, showed that in general relativity, the Kaluza's idea gives a rational foundation for Maxwell's electromagnetic equations and combines them with gravitational equations to a formal whole.

1.12 Dark Energy:

It has been established and believed that our universe is accelerating at the present epoch instead of showing down as predicted by the Big Bang theory Silk (1989). This concept of accelerated expansion is one of the excellent achievements in cosmology. However, till today we are not in a state to provide an exactly clear statement about the origin and evolution of our universe. From the study of different literatures and philosophical point of views in this regard, we found that different mines provide different opinions about our universe. Actually, the universe is full of mysterious elements and numerous effects of interactions which cannot be detected and difficult to explain even with advanced technology. So the most challenging problem in Astrophysics and modem cosmology is to understand the late time acceleration of the universe. In recent years many researchers and Scientists are putting huge effort to explain the dynamics of the universe and to understand the future evolution of the universe with the attention in the context of dark energy and modified theories of gravity. Also there are lots of outstanding results of the cosmological observations such as Riess et al. (1998) ; Perlmutter et al. (1999) ; Bennett et al. (2003) ; Seljak et al. (2005) ; Astier et al. (2006) ; Daniel et al. (2008) ; Amanullah et al. (2010) ; Suzuki et al. (2012) ; Nishizawa (2014) are perceived to us for the cosmic acceleration with direct and indirect evidence. The cosmological results and data sets like Atacama Cosmology Telescope (ACT) (Sievers et al. 2013), Planck 2015 results- XIII (Ade et al. 2015), are also suggested about late time inflation of the universe and allows the researchers to determine cosmological parameters such as the Hubble constant H and the deceleration parameter q. Studying the constraints given by the data from CMBR (Cosmic Microwave Background Radiation) investigations [Bernardis et al. (2000) ; Spergel et al. (2003)], WMAP (Wilkinson Microwave Anisotropy Probe) (Hinshaw et al. 2013), observations of clusters of galaxies at low red shift [Chaboyer et al. (1998) ; Salaris & Weiss (1998)] etc., it can be deduced that the universe is dominated by some mysterious components. This mysterious component of the energy is called dark energy which has negative pressure and positive energy density (giving negative EoS parameter). In the energy budget of the universe, it has been estimated that about 73% of our universe is Dark energy, about 23% is occupied by Dark matter and the usual baryonic matter occupies about 4%. Therefore the study of the nature of dark energy has become one of the most important topics in the field of fundamental physics [Sahni et al. (2000) ; Padmanabhan (2003) ; Li et al. (2011) ; Bamba et al. (2012) ; Bahrehbakhsh & Farhoudi, (2013) ; Wang et al. (2016) ; El-Nabulsi (2015a,b)]. Einstein's cosmological constant Λ is the simplest candidate for dark energy and physically it is equivalent to the quantum vacuum energy. The cosmological model with Λ and cold dark matter (CDM) is usually called the ΛCDM model.

Recent literatures and findings suggested that dark energy dominates the universe with positive energy density and negative pressure, responsible to produce sufficient acceleration in the late time evolution of the Universe. So the study of dark energy becomes a major outstanding issue in physics and cosmology today. There are a number of useful reviews of dark energy that are focus on theory [Yoo & Watanabe (2012); Ziaeepour (2014); Copeland et al. (2006) ; Linder et al. (2016) ; Tsujikawa (2010 , 2011b)], on probes of dark energy [Frieman et al. (2008)] and on the cosmological constant [Carroll (2001) ; Martin (2012)]. Also, many prominent researchers like Yoo & Watanabe (2012), Bahrehbakhsh et al. (2013) , Joyce et al. (2016), Pasechnik (2016), Wang et al. (2016), El-Nabulsi (2011d, 2013a,b,c,d , 2015a,b , 2016a,b) , Motloch et al. (2015), Silbergleit (2016), ; Linder (2015), Josset et al. (2016), Nojiri et al. (2006a,b), Nojiri & Odintsov (2007, 2014a), Felice & Tsujikawa (2010) , Faraoni (2008), Boehmer et al. (2012) discussed different models about the dark energy in the different context with the comparisons of observational findings. Some of the important claimants of dark energy are tachyons [Padmanabhan et al. (2002)], chaplygin gas [Elmardi et al. (2016)], phantom [Dabrowski (2008)], k-essence and quintessence [Armendariz-Pecon et al. (2000a,b) ; de Putter et al. (2007)] along with other four elements i.e. dark matter, baryons, radiation and neutrinos. But so far there is no direct detection of such exotic fluids. Although the literature is now flooded with hundreds of model for dark energy what we lack is precise cosmological data coming from a variety of observations involving both background and the inhomogeneous universe that can discriminate among these models. In this connection, if we accept that Einstein was correct with his general relativity theory to explain accelerated expansion of the universe could also be explained by negative pressure working against gravity. The belief of Einstein to the static universe made him to think about negative pressure which will stop the attraction of the gravity. However, it is known that our universe is not only non-static but also have cosmic acceleration. According to the above observational data analysis, the amount of the negative pressure in our universe can be estimated, which we call it as dark energy. The simple question about the nature of the dark energy is still one of the intriguing questions and left free space for new speculations.

In recent time, Bianchi type dark energy models are studied by some of the authors like, Akarsu and Kilinc (2010b); Adhav (2012); Katore & Shaikh (2014); Sahoo & Mishra (2014); Ali et al. (2016) in General relativity and $f(R,T)$ gravity, whereas Sahu & Kumar (2013); Samanta (2013); Singh & Sharma (2014) studied Bianchi type dark energy models in Lyra geometry.

1.13 Perfect Fluid and Energy-Momentum Tensor:

A perfect fluid is a frictionless homogeneous and incompressible fluid which is incapable of sustaining any tangential stress or action in the form of a shear but the normal force acts between the adjoining layers of fluid. The pressure at every point of a perfect fluid is equal in all directions, whether the fluid be at rest or in motion. It can be completely characterized by its rest frame energy stresses, viscosity and heat conduction.

The energy-momentum tensors describing matter is given by

$$T^{ij} = \rho u^i u^j + S^{ij} \tag{1.43}$$

where ρ is the mass density, u^i is the four-velocity $u^i = \frac{dx^i}{ds}$ of the individual particles, and S^{ij} is the stress tensor where the speed of the light $c = 1$. If the matter consists of perfect fluid, namely, one whose pressure is isotropic the stress tensor can be expressed as

$$S^{ij} = p\left(u^i u^j - g^{ij}\right) \tag{1.44}$$

where p is the pressure.

Thus the energy-momentum tensors becomes

$$T^{ij} = (\rho + p)\, u^i u^j - p g^{ij} \tag{1.45}$$

The only stress they can sustain is the isotropic pressure p ; ρ is the mass density. It is generally agreed that except in the early universe, the pressure of the sources can be neglected. A perfect fluid with zero pressure is technically referred to as dust such that it is still on the substratum. Since any random motion would constitute a pressure. In the early universe, however uniform radiation is thought to have predominated. This does have pressure, its equation of state is

$$p = \frac{1}{3}\rho \tag{1.46}$$

1.14 Hubble's Law and Hubble's Constant:

Hubble discovered that the galaxies recede from the earth with a velocity proportional to their distance i.e. the recession velocity is proportional to the distance.

Therefore, Recessional velocity = Hubble's constant times distance.

i.e. $V = HD$

where V is the observed velocity of the galaxy away from us, usually in km/sec.

H is Hubble's constant in km/sec/MPc.

D is the distance of the galaxies in MPc.

The basic entity in this law is, however, the Hubble's constant H which has to be first calibrated accurately before the law can be used. The calibration of H, however, contains some inherent uncertainties in it. One has to derive by independent methods the distances to galaxies for which the red-shifts are significant. The recessional speed must largely supersede the random speed of the galaxy. For the purpose, the Virgo Cluster of galaxies has so far been considered the most suitably situated. It contains a large number of bright galaxies and it distances $(m - M \cong 31)$, the photometric method of distance measurement is applicable on the one hand, and on the other hand, red-shifts are significant. But unfortunately, the random velocities of the individual member galaxies of the Virgo Cluster about the center of mean recessional motion are of the same order as the mean recessional motion itself.

From the above relation, we know that the Hubble parameter or Hubble constant H defines the rate of cosmic expansion. The recession velocity of V of an object situated at a distance D given by $H = V/D$. Also, it is the logarithmic derivative of the scale factor $R(t)$

$$i.e. \quad H = \frac{\dot{R}(t)}{R(t)} \tag{1.47}$$

Since the mean recessional speed of the cluster is computed from the motions of these individual members, themselves having a large random motion, large uncertainty may be introduced in the computation of the mean recessional speed. When this speed is used to compute the value of H, that value should be accepted with a reservation have devotedly worked for many decades for the correct evaluation of H.

In 1929 Hubble projected the value of the expansion factor, which is now termed as the Hubble constant about 500 km/sec/MPc. For many decades the controversy rests between two groups of astronomers. Alan Sandage and his co-authors claim on the basis of their observation that $H = 50kmS^{-}MPc^{-1}$, on the other hand, G. de Vaucauleurs and his co-authors claim that the value should be around $10kmS^{-1}MPc^{-1}$. But many outsiders thought the geometric mean of their value $H = 71kmS^{-}MPc^{-1}$ was a good compromise. The controversy persists while authors often work with some intermediate value of H. Much work has been done in the sixties and seventies with $H = 75kmS^{-}MPc^{-1}$ considering various aspects of the problem and inherent uncertainties in the determination. A Dressler has suggested that $H = 70kmS^{-}MPc^{-1}$ should be a better acceptable value. Many authors are however currently working with the value $H = 50kmS^{-}MPc^{-1}$. Bret from the latest source the Hubble space telescope key project team came up with the answer.

H = 75+/-8 kmS-1MPc-1.

And finally, WMAP came up with

H = 71+/-3.5 kmS-1MPc-1.

where $1MPc = 3.26$ million light years.

1.15 Deceleration Parameter:

In Relativity and Cosmology, the dimensionless parameter that is used to measure the cosmic acceleration of the expansion of our universe is known as deceleration parameter, which is generally denoted by q and is defined by

$$q = -\frac{R\ddot{R}}{\dot{R}^2} \tag{1.48}$$

where R is the average scale factor of the universe and the overhead dots denote the derivatives with respect to proper time. The value of q is negative or positive according as $\ddot{R}$ is positive or negative, so the universe is expanding with acceleration whenever q is negative. At the time of defining deceleration parameter q, it was supposed that the value of then q is to be positive. But from the recent literatures and observational findings, it has been established that our universe is accelerating at the present epoch instead of showing down as predicted by the Big Bang theory Silk (1989). It is believed that dark energy dominates the universe with positive energy density and negative pressure which is responsible to produce sufficient acceleration in the late time evolution of the universe.

Also in terms of the Hubble's parameter H, the deceleration parameter q is defined as

$$q = -\frac{d}{dt}\left(\frac{1}{H}\right) - 1 \tag{1.49}$$

The team, Riess et al. (1998) was the first to suggest about the accelerating universe. For all suggested form of matter, the deceleration parameter q is found to be greater than -1 but the phantom dark energy violets all the energy conditions. Hence for the expanding universe, the Hubble's parameter H should be decreasing so that the local expansion of space is always slowing.