# CHAPTER 5

# CHAPTER 5

# PLITHOGENIC NEUTROSOPHIC HYPERSOFT ALMOST TOPOLOGICAL GROUP

*In this chapter, the concept of the neutrosophic hypersoft topological group is studied. Moreover, some definitions related to the NHTG are introduced, and the PNHATG and its related propositions are studied.*

**Definition 5.0.1**
*Let NHS* $(\mathcal{U}_{\mathcal{N}}, E) = \mathcal{N}$ *be the family of all NHS over* $\mathcal{U}_{\mathcal{N}}$ *via attributes in* $E$ *and* $\urcorner_{\mathcal{U}_{\mathcal{N}}} \subseteq NHS(\mathcal{U}_{\mathcal{N}}, E)$. *Then* $\urcorner_{\mathcal{U}_{\mathcal{N}}}$ *is said to be NHT on* $\mathcal{N}$ *if the following conditions are hold:*

*(a)* $\phi_{\mathcal{U}_{\mathcal{N}}}, 1_{\mathcal{U}_{\mathcal{N}}} \in \urcorner_{\mathcal{U}_{\mathcal{N}}}$.

*(b) The intersection of any finite number of members of* $\urcorner_{\mathcal{U}_{\mathcal{N}}}$ *also belongs to* $\urcorner_{\mathcal{U}_{\mathcal{N}}}$.

*(c) The union of any collection of members of* $\urcorner_{\mathcal{U}_{\mathcal{N}}}$ *belongs to* $\urcorner_{\mathcal{U}_{\mathcal{N}}}$.

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The results discussed in this chapter has published in the journal,
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*Then $(N, \urcorner_{\mathcal{U}_{\mathcal{N}}})$ is said to be NHTS. Every member of $\urcorner_{\mathcal{U}_{\mathcal{N}}}$ is called $\urcorner_{\mathcal{U}_{\mathcal{N}}}$- open neutrosophic hypersoft set. An NHS is called $\urcorner_{\mathcal{U}_{\mathcal{N}}}$ - closed if and only if its complement is $\urcorner_{\mathcal{U}_{\mathcal{N}}}$ - open.*

**Definition 5.0.2**

*Let the pair $(F, E_\alpha) = H$ be a NHG of a crisp group $\mathcal{U}$. Let $\urcorner_{\mathcal{U}_G}$ be the NHT on $H$ then $(H, \urcorner_{\mathcal{U}_G})$ is said to be NHTG if the following conditions are satisfied:*

*(a) The mapping $\psi : (H, \urcorner_{\mathcal{U}_G}) \times (H, \urcorner_{\mathcal{U}_G}) \to (H, \urcorner_{\mathcal{U}_G})$ such that $\psi(x, y) = xy$, for all $x, y \in H = (F, E_\alpha)$, is relatively neutrosophic hypersoft continuous.*

*(b) The mapping $\mu : (H, \urcorner_{\mathcal{U}_G}) \to (H, \urcorner_{\mathcal{U}_G})$ such that $\mu(x) = x^{-1}$, for all $x \in H = (F, E_\alpha)$, is relatively neutrosophic hypersoft continuous.*

*where $x = (b_1, r_1)$ and $y = (b_2, r_2)$. Then the pair $(H, \urcorner_{\mathcal{U}_G})$ is known as NHTG.*

**Definition 5.0.3**

*Let the pair $(F, E_\alpha) = H$ be a NHG of a crisp group $\mathcal{U}$. Let $\urcorner_{\mathcal{U}_G}$ be the NHTG on $H$. Then for fixed $\sigma = (a_1, a_2) \in H$, the left translation $l_\sigma : (H, \urcorner_{\mathcal{U}_G}) \to (H, \urcorner_{\mathcal{U}_G})$ is defined by $l_\sigma(x) = \sigma x, \forall\, x \in H$,*

$$\sigma x = \left\{ \langle \sigma, T_{\mathcal{U}_G}(\sigma x), I_{\mathcal{U}_G}(\sigma x), F_{\mathcal{U}_G}(\sigma x) \rangle : x \in H = (F, E_\alpha) \right\}.$$

*Similarly, the right translation $r_\sigma : (H, \urcorner_{\mathcal{U}_G}) \to (H, \urcorner_{\mathcal{U}_G})$ is defined by $r_\sigma(x) = x\sigma, \forall\, x \in H$,*

$$x\sigma = \left\{ \langle \sigma, T_{\mathcal{U}_G}(x\sigma), I_{\mathcal{U}_G}(x\sigma), F_{\mathcal{U}_G}(x\sigma) \rangle : x \in H = (F, E_\alpha) \right\}.$$

**Lemma 5.0.1**

*Suppose $(F, E_\alpha) = H$ be a NHG of a crisp group $\mathcal{U}$. Let $\urcorner_{\mathcal{U}_G}$ be an*

*NHTG in $H$. Then for each $\sigma = (a_1, a_2) \in \mathcal{G}_e$, the translations $l_\sigma$ and $r_\sigma$ respectively neutrosophic hypersoft homomorphism of $(H, \urcorner_{\mathcal{U}_G})$ into itself.*

**Proof**

From the Proposition 3.11 [67], we have $l_\sigma[H] = \mathcal{G}$ and $r_\sigma[H] = H$, for all $\sigma \in H_e$ and let $\pi : (H, \urcorner_{\mathcal{U}_G}) \to (H, \urcorner_{\mathcal{U}_G}) \times (H, \urcorner_{\mathcal{U}_G})$ defined by $\pi(x) = (\sigma, x)$ for each $x \in H$. Then $r_\sigma : \beta \circ \pi$. Since $\sigma \in H_e, T_{\mathcal{U}_G}(\sigma)$ $= T_{\mathcal{U}_G}(e), I_{\mathcal{U}_G}(\sigma) = I_{\mathcal{U}_G}(e)$ and $F_{\mathcal{U}_G}(\sigma) = F_{\mathcal{U}_G}(e)$. Thus, $T_{\mathcal{U}_G}(\sigma) \supseteq T_{\mathcal{U}_G}(x), I_{\mathcal{U}_G}(\sigma) \supseteq I_{\mathcal{U}_G}(x)$ and $F_{\mathcal{U}_G}(\sigma) \subseteq F_{\mathcal{U}_G}(x)$, for each $x \in H$. It follows from Proposition 3.34 [68] that $\pi : (H, \urcorner_{\mathcal{U}_G}) \to (H, \urcorner_{\mathcal{U}_G}) \times (H, \urcorner_{\mathcal{U}_G})$ is relatively neutrosophic hypersoft continuous. By the hypothesis $\beta$ is relatively neutrosophic hypersoft continuous. So, $r_\sigma$ is relatively neutrosophic hypersoft continuous. Moreover ${r_\sigma}^{-1} = r_{\sigma^{-1}}$. Similarly, we have shown the relatively neutrosophic hypersoft continuous of ${l_\sigma}^{-1} = l_{\sigma^{-1}}$.

**Definition 5.0.4**

*Let PNHS $(\mathcal{U}_\mathcal{P}, E) = P$ be the family of all PNHS over $\mathcal{U}_\mathcal{P}$ via attributes in $E$ and $\urcorner_{\mathcal{U}_\mathcal{P}} \subseteq$ PNHS$(\mathcal{U}_\mathcal{P}, E)$. Then $\urcorner_{\mathcal{U}_\mathcal{P}}$ is said to be PNHT on $P$ if the following conditions are satisfied:*

*(a)* $\phi_{\mathcal{U}_\mathcal{P}}, 1_{\mathcal{U}_\mathcal{P}} \in \urcorner_{\mathcal{U}_\mathcal{P}}$.

*(b) The intersection of any two NHSs in $\urcorner_{\mathcal{U}_\mathcal{P}}$ belongs to $\urcorner_{\mathcal{U}_\mathcal{P}}$.*

*(c) The union of NHSs in $\urcorner_{\mathcal{U}_\mathcal{P}}$ belongs to $\urcorner_{\mathcal{U}_\mathcal{P}}$.*

*Then $(P, \urcorner_{\mathcal{U}_\mathcal{P}})$ is said to be PNHTS.*

**Definition 5.0.5**

*The complement $\mathcal{A}^c$ of a PNHOS in a NHTS $(P, \urcorner_{\mathcal{U}_\mathcal{P}})$ is said to be PNHCoS in $(P, \urcorner_{\mathcal{U}_\mathcal{P}})$.*

**Definition 5.0.6**

*Let the pair* $(F, E_\alpha) = \mathcal{M}$ *be a PNHS of a crisp group* $\mathcal{U}$. *Let* $\urcorner_{\mathcal{U}_G}$ *[from definition 5.0.2] be the PNHT on* $\mathcal{M}$ *then* $(\mathcal{M}, \urcorner_{\mathcal{U}_G})$ *is said to be PNHTG if the following conditions are satisfied:*

*(a) The mapping* $\psi : (\mathcal{M}, \urcorner_{\mathcal{U}_G}) \times (\mathcal{M}, \urcorner_{\mathcal{U}_G}) \to (\mathcal{M}, \urcorner_{\mathcal{U}_G})$ *such that* $\psi(x, y) = xy$, *for all* $x, y \in \mathcal{M} = (F, E_\alpha)$, *is relatively plithogenic neutrosophic hypersoft continuous.*

*(b) The mapping* $\mu : (\mathcal{M}, \urcorner_{\mathcal{U}_G}) \to (\mathcal{M}, \urcorner_{\mathcal{U}_G})$ *such that* $\mu(x) = x^{-1}$, *for all* $x \in \mathcal{M} = (F, E_\alpha)$, *is relatively plithogenic neutrosophic hypersoft continuous.*

*where* $x = (b_1, r_1)$ *and* $y = (b_2, r_2)$. *Then the pair* $(\mathcal{M}, \urcorner_{\mathcal{U}_G})$ *is called a PNHTG.*

**Definition 5.0.7**

*Let the pair* $(F, E_\alpha)$ *be a PNHS of a crisp group* $\mathcal{U}$, *where* $E_\alpha = \mathcal{A}_1 \times \mathcal{A}_2 \times \ldots \times \mathcal{A}_n$ *and* $\mathcal{A}_i, i = \{1, 2, \ldots, n\}$ *are crisp groups. Let* $U, V$ *be two PNHS in* $(F, E_\alpha)$. *We define the product of* $UV$ *PNHS* $U, V$ *and* $V^{-1}$ *of* $V$ *as follows:*

$$UV(z) = \Big\{\langle z, T_{UV}(z), I_{UV}(z), F_{UV}(z)\rangle : z = (b, r) \in (F, E_\alpha)\Big\}$$

*where*

$$T_{UV}(z) = sup\big\{min\{T_U(x), T_V(y)\}\big\},$$
$$I_{UV}(z) = sup\big\{min\{I_U(x), I_V(y)\}\big\},$$
$$F_{UV}(z) = sup\big\{min\{F_U(x), F_V(y)\}\big\},$$

*where* $z = x.y$ *and* $x = (b_1, r_1); y = (b_2, r_2)$ *and for* $V = \Big\{\langle z, T_V(z), I_V(z), F_V(z)\rangle : z = (b, r) \in (F, E_\alpha)\Big\}$, we have $V^{-1} = \Big\{\langle z, T_V(z^{-1}), I_V(z^{-1}), F_V(z^{-1})\rangle : z = (b,r) \in (F, E_\alpha)\Big\}$.

**Definition 5.0.8**

*Let the ordered pair $(F, E_\alpha)$ be a PNHS, where $E_\alpha = \mathcal{A}_1 \times \mathcal{A}_2 \times \ldots \times \mathcal{A}_n$. Let $(P, \urcorner_{\mathcal{U}_\mathcal{P}})$ be a PNHTS and $\mathcal{A} = \Big\{\langle x, T_\mathcal{A}(x),\ I_\mathcal{A}(x),\ F_\mathcal{A}(x)\rangle : x \in (F, E_\alpha)\Big\}$ be a PNHS in $(P, \urcorner_{\mathcal{U}_\mathcal{P}})$, then the plithogenic neutrosophic hypersoft interior of $\mathcal{A}$ is defined as*

$$\text{PNH} - int(\mathcal{A}) = \bigcup \Big\{G : G \text{ is PNHOS and } G \subseteq \mathcal{A}\Big\}.$$

**Definition 5.0.9**

*Let the ordered pair $(F, E_\alpha)$ be a PNHS, where $E_\alpha = \mathcal{A}_1 \times \mathcal{A}_2 \times \ldots \times \mathcal{A}_n$. Let $(P, \urcorner_{\mathcal{U}_\mathcal{P}})$ be a PNHTS and $\mathcal{A} = \Big\{\langle x, T_\mathcal{A}(x),\ I_\mathcal{A}(x),\ F_\mathcal{A}(x)\rangle : x \in (F, E_\alpha)\Big\}$ be a PNHS in $(P, \urcorner_{\mathcal{U}_\mathcal{P}})$, then the plithogenic neutrosophic hypersoft closure of $\mathcal{A}$ is defined as*

$$\text{PNH} - cl(\mathcal{A}) = \bigcap \Big\{K : K \text{ is PNHCoS and } K \supseteq \mathcal{A}\Big\}.$$

**Definition 5.0.10**

*A mapping $\phi : (P, \urcorner_{\mathcal{U}_{\mathcal{P}1}}) \to (K, \urcorner_{\mathcal{U}_{\mathcal{P}2}})$ is a plithogenic neutrosophic hypersoft continuous if the pre-image of each PNHOS in $(K, \urcorner_{\mathcal{U}_{\mathcal{P}2}})$ is PNHOS in $(P, \urcorner_{\mathcal{U}_{\mathcal{P}1}})$.*

**Definition 5.0.11**

*Let $\mathcal{A}$ be a PNHS of a PNHTS $(P, \urcorner_{\mathcal{U}_\mathcal{P}})$, then $\mathcal{A}$ is called a PNHSOS of $(P, \urcorner_{\mathcal{U}_\mathcal{P}})$ if there exists a $\mathcal{B} \in \urcorner_{\mathcal{U}_\mathcal{P}}$ such that $\mathcal{A} \subseteq \text{PNH} - cl(\mathcal{B})$.*

**Definition 5.0.12**

*Let $\mathcal{A}$ be a PNHS of a PNHTS $(P, \urcorner_{\mathcal{U}_\mathcal{P}})$, then $\mathcal{A}$ is called a PNHSCoS of $(P, \urcorner_{\mathcal{U}_\mathcal{P}})$ if there exists a $\mathcal{B}^c \in \urcorner_{\mathcal{U}_\mathcal{P}}$ such that $\text{PNH} - Int(\mathcal{B}) \subseteq \mathcal{A}$.*

**Definition 5.0.13**

*A PNHS $\mathcal{A}$ of a PNHTS $(P, \urcorner_{\mathcal{U}_{\mathcal{P}}})$ is said to be a PNHROS of $(P, \urcorner_{\mathcal{U}_{\mathcal{P}}})$ if PNH $-$ $int\big($PNH $-$ $cl(\mathcal{A})\big) = \mathcal{A}$.*

**Definition 5.0.14**

*A PNHS $\mathcal{A}$ of a PNHTS $(P, \urcorner_{\mathcal{U}_{\mathcal{P}}})$ is said to be a PNHRCoS of $(P, \urcorner_{\mathcal{U}_{\mathcal{P}}})$ if PNH $-$ $cl\big($PNH $-$ $int(\mathcal{A})\big) = \mathcal{A}$.*

**Theorem 5.0.1**

*(i) The intersection of any two PNHROSs is a PNHROS, and*

*(ii) The union of any two PNHRCoSs is a PNHRCoS.*

**Proof**

(i) Let $\mathcal{A}_1$ and $\mathcal{A}_2$ be any two PNHROSs of a PNHTS $(P, \urcorner_{\mathcal{U}_{\mathcal{P}}})$. Since $\mathcal{A}_1 \bigcap \mathcal{A}_2$ is PNHOS, we have $\mathcal{A}_1 \bigcap \mathcal{A}_2 \subseteq$ PNH $-$ $int\Big($PNH $-$ $cl(\mathcal{A}_1 \bigcap \mathcal{A}_2)\Big)$. Now, PNH$-int($PNH$-cl\Big(\mathcal{A}_1 \bigcap \mathcal{A}_2)\Big) \subseteq$ PNH$-int\Big($PNH$-cl(\mathcal{A}_1)\Big) = \mathcal{A}_1$ and PNH$-int\Big($PNH$-cl(\mathcal{A}_1 \bigcap \mathcal{A}_2)\Big) \subseteq$ PNH$-int\Big($PNH$-cl(\mathcal{A}_2)\Big) = \mathcal{A}_2$ implies that PNH$-int\Big($PNH$-cl(\mathcal{A}_1 \bigcap \mathcal{A}_2)\Big) \subseteq \mathcal{A}_1 \bigcap \mathcal{A}_2$. Hence the theorem.

(ii) Let $\mathcal{A}_1$ and $\mathcal{A}_2$ be any two PNHROSs of a PNHTS $(P, \tau_{\mathcal{U}_{\mathcal{P}}})$ . Since $\mathcal{A}_1 \bigcup \mathcal{A}_2$ is PNHOS, we have $\mathcal{A}_1 \bigcup \mathcal{A}_2 \supseteq$ PNH$-cl\Big($PNH$-int(\mathcal{A}_1 \bigcup \mathcal{A}_2)\Big)$. Now, PNH$-cl\Big($PNH$-int(\mathcal{A}_1 \bigcup \mathcal{A}_2)\Big) \supseteq$ PNH$-cl\Big($PNH$-int(\mathcal{A}_1)\Big) = \mathcal{A}_1$ and PNH$-cl\Big($PNH$-int(\mathcal{A}_1 \bigcup \mathcal{A}_2)\Big) \supseteq$ PNH $-$ $cl\Big($PNH $-$ $int(\mathcal{A}_2)\Big) = \mathcal{A}_2$ implies that $\mathcal{A}_1 \bigcup \mathcal{A}_2 \subseteq$ PNH $-$ $cl\Big($PNH $-$ $int(\mathcal{A}_1 \bigcup \mathcal{A}_2)\Big)$. Hence the theorem.

**Definition 5.0.15**

*Let $\phi : (P, \urcorner_{\mathcal{U}_{\mathcal{P}1}}) \rightarrow (K, \urcorner_{\mathcal{U}_{\mathcal{P}2}})$ be a mapping from a PNHTS $(P, \urcorner_{\mathcal{U}_{\mathcal{P}1}})$*

*to another PNHTS $(K, \daleth_{\mathcal{U}_{\mathcal{P}2}})$, then $\phi$ is called a PNHCM, if $\phi^{-1}(\mathcal{A}) \in \daleth_{\mathcal{U}_{\mathcal{P}1}}$ for each $\mathcal{A} \in \daleth_{\mathcal{U}_{\mathcal{P}2}}$; or equivalently $\phi^{-1}(\mathcal{B})$ is a PNHCoS of $(P, \daleth_{\mathcal{U}_{\mathcal{P}1}})$ for each PNHCoS $\mathcal{B}$ of $(K, \daleth_{\mathcal{U}_{\mathcal{P}2}})$.*

**Definition 5.0.16**

*Let $\phi : (P, \daleth_{\mathcal{U}_{\mathcal{P}1}}) \to (K, \daleth_{\mathcal{U}_{\mathcal{P}2}})$ be a mapping from a PNHTS $(P, \daleth_{\mathcal{U}_{\mathcal{P}1}})$ to another PNHTS $(K, \daleth_{\mathcal{U}_{\mathcal{P}2}})$, then $\phi$ is called a PNHOM if $\phi(\mathcal{A}) \in \daleth_{\mathcal{U}_{\mathcal{P}2}}$ for each $\mathcal{A} \in \daleth_{\mathcal{U}_{\mathcal{P}1}}$.*

**Definition 5.0.17**

*Let $\phi : (P, \daleth_{\mathcal{U}_{\mathcal{P}1}}) \to (K, \daleth_{\mathcal{U}_{\mathcal{P}2}})$ be a mapping from a PNHTS $(P, \daleth_{\mathcal{U}_{\mathcal{P}1}})$ to another PNHTS $(K, \tau_{\mathcal{U}_{\mathcal{P}2}})$, then $\phi$ is called a PNHCoM if $\phi(\mathcal{B})$ is a PNHCoS of $(K, \daleth_{\mathcal{U}_{\mathcal{P}2}})$ for each PNHCoS $\mathcal{B}$ of $(P, \daleth_{\mathcal{U}_{\mathcal{P}1}})$.*

**Definition 5.0.18**

*Let $\phi : (H, \daleth_{\mathcal{U}_{\mathcal{P}1}}) \to (K, \daleth_{\mathcal{U}_{\mathcal{P}2}})$ be a mapping from a PNHTS $(H, \daleth_{\mathcal{U}_{\mathcal{P}1}})$ to another PNHTS $(K, \daleth_{\mathcal{U}_{\mathcal{P}2}})$, then $\phi$ is called a PNHSCM if $\phi^{-1}(\mathcal{A})$ is a PNHSOS of $(H, \daleth_{\mathcal{U}_{\mathcal{P}1}})$, for each $\mathcal{A} \in \daleth_{\mathcal{U}_{\mathcal{P}2}}$.*

**Definition 5.0.19**

*Let $\phi : (P, \daleth_{\mathcal{U}_{\mathcal{P}1}}) \to (K, \daleth_{\mathcal{U}_{\mathcal{P}2}})$ be a mapping from a PNHTS $(P, \daleth_{\mathcal{U}_{\mathcal{P}1}})$ to another PNHTS $(K, \daleth_{\mathcal{U}_{\mathcal{P}2}})$, then $\phi$ is called a PNHSOM if $\phi(\mathcal{A})$ is a PNHSOS for each $\mathcal{A} \in \daleth_{\mathcal{U}_{\mathcal{P}1}}$.*

**Definition 5.0.20**

*Let $\phi : (P, \daleth_{\mathcal{U}_{\mathcal{P}1}}) \to (K, \daleth_{\mathcal{U}_{\mathcal{P}2}})$ be a mapping from a PNHTS $(P, \daleth_{\mathcal{U}_{\mathcal{P}1}})$ to another PNHTS $(K, \daleth_{\mathcal{U}_{\mathcal{P}2}})$, then $\phi$ is called a PNHSCoM if $\phi(\mathcal{B})$ is a PNHSCoS for each PNHCoS $\mathcal{B}$ of $(P, \daleth_{\mathcal{U}_{\mathcal{P}1}})$.*

**Definition 5.0.21**

*A mapping $\phi : (M, \daleth_{\mathcal{U}_{\mathcal{P}1}}) \to (K, \daleth_{\mathcal{U}_{\mathcal{P}2}})$ is said to be a PNHACM if $\phi^{-1}(\mathcal{A}) \in (M, \daleth_{\mathcal{U}_{\mathcal{P}1}})$ for each PNHROS $\mathcal{A}$ of $(K, \daleth_{\mathcal{U}_{\mathcal{P}2}})$.*

**Definition 5.0.22**

*Let the pair $(F, E_\alpha) = M$ be a PNHS of a crisp group $\mathcal{U}$. Let $\urcorner_{\mathcal{U}_G}$ [from definition 5.0.15] be the PNHT on $M$ then $(M, \urcorner_{\mathcal{U}_G})$ is said to be PNHATG if the following conditions are satisfied:*

*(i) The mapping $\psi : (M, \urcorner_{\mathcal{U}_G}) \times (M, \urcorner_{\mathcal{U}_G}) \to (M, \urcorner_{\mathcal{U}_G})$ such that $\psi(x, y) = xy$, for all $x, y \in M = (F, E_\alpha)$, is relatively plithogenic neutrosophic hypersoft almost continuous.*

*(ii) The mapping $\mu : (M, \urcorner_{\mathcal{U}_G}) \to (M, \urcorner_{\mathcal{U}_G})$ such that $\mu(x) = x^{-1}$, for all $x \in M = (F, E_\alpha)$, is relatively plithogenic neutrosophic hypersoft almost continuous.*

*where $x = (b_1, r_1)$ and $y = (b_2, r_2)$. Then the pair $(M, \urcorner_{\mathcal{U}_G})$ is known as PNHATG.*

**Theorem 5.0.2**

*Let $(M, \urcorner_{\mathcal{P}_G})$ be a PNHATG and let $\sigma = (a_1, a_2) \in M$ be any element. Then*

*(i) A mapping $g_\sigma : (M, \urcorner_{\mathcal{U}_G}) \to (H, \urcorner_{\mathcal{U}_G})$ such that $g_\sigma(x) = \sigma x$, for all $x \in M$, is PNHACM;*

*(ii) A mapping $h_\sigma : (M, \urcorner_{\mathcal{U}_G}) \to (M, \urcorner_{\mathcal{U}_G})$ such that $h_\sigma(x) = x\sigma$, for all $x \in M$, is PNHACM.*

**Proof**

(i) Let $\delta = (a_3, a_4) \in M$ and let $W$ be a PNHROS containing $\sigma\delta$ in $M$. From Definition 5.0.22, $\exists$ plithogenic neutrosophic hypersoft open nbds $\mathcal{U}, \mathcal{V}$ of $\sigma, \delta$ in $M$so that $\mathcal{U}\mathcal{V} \subseteq W$. Especially, $\sigma\mathcal{V} \subseteq W$ that is $g_\sigma(\mathcal{V}) \subseteq W$. This shows that $g_\sigma$ is PNHACM at $\delta$ and therefore $g_\sigma$ is PNHACM.

(ii) Suppose $\delta = (a_3, a_4) \in M$ and $W \in$ PNHROS(M) containing $\delta\sigma$. Then $\exists$ PNHOSs $\delta \in \mathcal{U}$ and $\sigma \in \mathcal{V}$ in $M$ so that $\mathcal{U}\mathcal{V} \subseteq W$. This shows $\mathcal{U}_\sigma \subseteq W$, i.e., $h_\sigma(\mathcal{U}) \subseteq W$. This implies $h_\sigma$ is PNHACM at $\delta$. As arbitrary element $\delta$ is in $M$, therefore $h_\sigma$ is PNHACM.

**Theorem 5.0.3**

*Let $U$ be PNHROS in a PNHATG $(M, \urcorner_{\mathcal{U}_G})$. Then the following conditions are hold good, where $\sigma = (a_1, a_2)$*

*(i) $\sigma U \in$ PNHROS$(M)$, for all $\sigma \in M$.*

*(ii) $U\sigma \in$ PNHROS$(M)$, for all $\sigma \in M$.*

*(iii) $U^{-1} \in$ PNHROS$(M)$.*

**Proof**

(i) First, we have to prove that $\sigma U \in \urcorner_{\mathcal{U}_G}$. Let $\delta = (a_3, a_4) \in \sigma U$. Then from Definition 5.0.22 of PNHATGs, $\exists$ PNHOSs $\sigma^{-1} \in W_1$ and $\delta \in W_2$ in $M$ so that $W_1 W_2 \subseteq U$. Especially, $\sigma^{-1} W_2 \subseteq U$. i.e., equivalently $W_2 \subseteq \sigma U$. This shows that $\delta \in \text{PNH} - int(\sigma U)$ and thus, $\text{PNH} - int\,(\sigma U) = \sigma U$. i.e., $\sigma U \in \urcorner_{\mathcal{U}_G}$. Consequently, $\sigma U \subseteq \text{PNH} - int\big(\text{PNH} - cl(\sigma U)\big)$.
Now, we have to prove that $\text{PNH} - int\big(\text{PNH} - cl(\sigma U)\text{PNH}\big) \subseteq \sigma U$. Since $U$ is PNHOS, $\text{PNH} - cl(U) \in \text{PNHRCoS}(M)$. From Theorem 5.0.2, $g_{\sigma^{-1}} : (M, \urcorner_{\mathcal{U}_G}) \to (M, \urcorner_{\mathcal{U}_G})$ is PNHACM and therefore, $\sigma\text{PNH} - cl(U)$ is PNHCoS. Thus, $\text{PNH} - int\big(\text{PNH} - cl(\sigma U)\big) \subseteq \text{PNH} - cl(\sigma U) \subseteq \sigma\text{PNH} - cl(U)$. i.e., $\sigma^{-1}\text{PNH} - int\big(\text{PNH} - cl(\sigma U)\big) \subseteq \text{PNH} - cl(U)$. Since $\text{PNH} - int\big(\text{PNH} - cl(\sigma U)\big)$ is PNHROS, it follows that $\sigma^{-1}\text{PNH}{-}int\big(\text{PNH}{-}cl(\sigma U)\big) \subseteq \text{PNH} - int\big(PNH - cl(U)\big) = U$, i.e., $\text{PNH} - int\big(PNH - cl(\sigma U)\big) \subseteq \sigma U$. Thus, $\sigma U = \text{PNH} - int\big(\text{PNH} - cl(\sigma U)\big)$. This shows that $\sigma U \in \text{PNHROS}(M)$.

(ii) Following Theorem 5.0.3 (i), the proof is straightforward.

(iii) Let $x \in U^{-1}$, then $\exists$ PNHOS $\delta \in W$ in $H$ so that $W^{-1} \subseteq U \Rightarrow W \subseteq U^{-1}$. Therefore $U^{-1}$ has interior point $\delta$. Thus, $U^{-1}$ is PNHOS. i.e., $U^{-1} \subseteq \text{PNH} - int\big(\text{PNH} - cl(U^{-1})\big)$. Now we have to prove that $\text{PNH} - int\big(\text{PNH} - cl(U^{-1})\big) \subseteq U^{-1}$. Since $U$ is PNHOS, $\text{PNH} - cl(U)$ is PNHRCoS and hence $\text{PNH} - cl(U)^{-1}$ is PNHCoS in $M$. Therefore, $\text{PNH} - int\big(\text{PNH} - cl(U^{-1})\big) \subseteq \text{PNH} - cl(U^{-1}) \subseteq \text{PNH} - cl(U)^{-1} \Rightarrow \text{PNH} - int\big(\text{PNH} - cl(U^{-1})\big) \subseteq \big(\text{PNH} - cl(U)\big)^{-1} \subseteq U^{-1}$. Thus, $U^{-1} = \text{PNH} - int\big(\text{PNH} - cl(U^{-1})\big)$. This shows that $U^{-1} \in \text{PNHROS}(H)$.

**Corollary 5.0.1**

*Let $\mathcal{Q}$ be any PNHRCoS in a PNHATG in $M$. Then*

*(i) $\sigma\mathcal{Q} \in$ PNHRCoS$(M)$, for each $\sigma \in M$.*

*(ii) $\mathcal{Q}^{-1} \in$ PNHRCoS$(M)$.*

**Theorem 5.0.4**

*Let $U$ be any PNHROS in a PNHATG $M$. Then*

*(a) PNH $- cl(U\sigma) =$ PNH $- cl(U)\sigma$, for each $\sigma \in M$, where $\sigma = (a_1, a_2)$.*

*(b) PNH $- cl(\sigma U) = \sigma$PNH $- cl(U)$, for each $\sigma \in M$.*

*(c) PNH $- cl(U^{-1}) =$ PNH $- cl(U)^{-1}$.*

**Proof**

(a) Taking $\delta = (a_3, a_4) \in \text{PNH} - cl(U\sigma)$ and consider $q = \delta\sigma^{-1}$. Let $q \in W$ be PNHOS in $M$. Then $\exists$ PNHOSs $\sigma^{-1} \in V_1$ and $\delta \in V_2$ in $M$, so that $V_1V_2 \subseteq \text{PNH} - int\big(\text{PNH} - cl(W)\big)$. By assumption, there is $g \in U\sigma \bigcap V_2 \Rightarrow g\sigma^{-1} \in U \bigcap V_1V_2 \subseteq U \bigcap \text{PNH} -$

$int\big(\text{PNH} - cl(W)\big) \Rightarrow U \bigcap \text{PNH} - int\big(\text{PNH} - cl(W)\big) \neq \phi_{\mathcal{U}_\mathcal{P}}$ $\Rightarrow U \bigcap \big(\text{PNH} - cl(W)\big) \neq \phi_{\mathcal{U}_\mathcal{P}}$. Since $U$ is PNHOS, $U \bigcap W \neq \phi_{\mathcal{U}_\mathcal{P}}$. i.e., $x \in \text{PNH} - cl(U)\sigma$.

Conversely, let $q \in \text{PNH} - cl(U)\sigma$. Then $q = \delta g$ for some $\delta \in \text{PNH} - cl(U)$. To prove $\text{PNH} - cl(U)a \subseteq \text{PNH} - cl(Ua)$. Let $\delta g \in W$ be an PNHOS in $M$. Then $\exists$ PNHOSs $\sigma \in V_1$ in $M$ and $\delta \in V_2$ in $M$ so that $V_1V_2 \subseteq \text{PNH} - int\big(\text{PNH} - cl\,(W)\big)$. Since $\delta \in \text{PNH} - cl(U), U \bigcap V_2 \neq \phi_{\mathcal{U}_\mathcal{P}}$. There is $g \in U \bigcap V_2$. This gives $g\sigma \in (U\sigma) \bigcap \text{PNH} - int\big(\text{PNH} - cl(W)\big)$ $\Rightarrow (U\sigma) \bigcap \big(\text{PNH} - cl(W)\big) \neq \phi_{\mathcal{U}_\mathcal{P}}$. From Theorem 5.0.2, $U\sigma$ is PNHOS and thus $(U\sigma) \bigcap W \neq \phi_{\mathcal{U}_\mathcal{P}}$, therefore $q \in \text{PNH}-cl(U\sigma)$. Therefore $\text{PNH} - cl(U\sigma) = \text{PNH} - cl(U)\sigma$.

(b) Following Theorem 5.0.4 $(a)$, proof is straightforward.

(c) Since $\text{PNH} - cl(U)$ is PNHRCoS, $\text{PNH} - cl(U)^{-1}$ is PNHCoS in $M$. So, $U^{-1} \subseteq \text{PNH} - cl(U)^{-1}$ this implies $\text{PNH} - cl(U^{-1}) \subseteq \text{PNH} - cl(U)^{-1}$. Next, let $q \in \text{PNH} - cl(U)^{-1}$. Then $q = \delta^{-1}$, for some $\delta \in \text{PNH} - cl(U)$. Let $q \in V$ be any PNHOS in $M$. Then $\exists$ PNHOS $U$ in $M$ so that $\delta \in U$ with $U^{-1} \subseteq \text{PNH} - int\big(\text{PNH} - cl(V)\big)$. Also, there is $\sigma \in \mathcal{A} \bigcap U$ which implies $\sigma^{-1} \in \mathcal{A}^{-1} \bigcap \text{PNH} - int\big(\text{PNH} - cl(V)\big)$. That is, $\mathcal{A}^{-1} \bigcap \text{PNH} - int\big(\text{PNH} - cl(V)\big) \neq \phi_{\mathcal{U}_\mathcal{P}} \Rightarrow U^{-1} \bigcap \text{PNH} - cl(V) \neq \phi_{\mathcal{U}_\mathcal{P}} \Rightarrow \mathcal{A}^{-1} \bigcap V \neq \phi_{\mathcal{U}_\mathcal{P}}$, since $U^{-1}$ is PNHOS. Therefore, $q \in \text{PNH} - cl(U)^{-1}$. Hence $\text{PNH} - cl(U^{-1}) \subseteq \text{PNH} - cl(U)^{-1}$.

**Theorem 5.0.5**

*Let $\mathcal{Q}$ be plithogenic neutrosophic hypersoft regularly closed subset in a PNHATG $M$. Then the following statements are satisfied:*

*(a) PNH $- int(\sigma\mathcal{Q}) = \sigma$PNH $- int(\mathcal{Q})$, for all $\sigma \in M$, where $\sigma = (a_1, a_2)$.*

*(b) PNH* $-$ $int(\mathcal{Q}\sigma) =$ *PNH* $-$ $int(\mathcal{Q})\sigma$*, for all* $\sigma \in M$.

*(c) PNH* $-$ $int(\mathcal{Q}^{-1}) =$ *PNH* $-$ $int(\mathcal{Q})^{-1}$.

**Proof**

(a) Since $\mathcal{Q}$ is PNHRCoS, PNH $-$ $int(\mathcal{Q})$ is PNHROS in $M$. Consequently, $\sigma$PNH $-$ $int(\mathcal{Q}) \subseteq$ PNH $-$ $int(\sigma\mathcal{Q})$. Conversely, let $q$ be an arbitrary element of PNH $-$ $int(\sigma\mathcal{Q})$. Assume that $q = \sigma\delta$, for some $\delta = (a_3, a_4) \in \mathcal{Q}$. By assumption, this shows $\sigma\mathcal{Q}$ is PNHCoS and that is PNH$-int(\sigma\mathcal{Q})$ is PNHROS in $M$. Suppose $\sigma \in U$ and $\delta \in V$ be PNHOSs in $M$, so that $UV \subseteq$ PNH $-$ $int(\sigma\mathcal{Q})$. Then $\sigma V \subseteq \sigma\mathcal{Q}$, which follows that $\sigma V \subseteq \sigma$PNH $-$ $int(\mathcal{Q})$. Thus, PNH $-$ $int\,(\sigma\mathcal{Q}) \subseteq \sigma$PNH $-$ $int(\mathcal{Q})$. Hence the statement follows.

(b) Following Theorem 5.0.5 $(a)$, the proof is straightforward.

(c) Since PNH $-$ $int(\mathcal{Q})$ is PNHROS, so PNH $-$ $int(\mathcal{Q})^{-1}$ is PNHOS in $M$. Therefore, $\mathcal{Q}^{-1} \subseteq$ PNH $-$ $int(\mathcal{Q})^{-1}$ implies that PNH $-$ $int\left(\mathcal{Q}^{-1}\right) \subseteq$ PNH $-$ $int(\mathcal{Q})^{-1}$. Next, let $q$ be an arbitrary element of PNH $-$ $int(\mathcal{Q})^{-1}$. Then $q = \delta^{-1}$, for some $\delta \in$ PNH $-$ $int(\mathcal{Q})$. Let $q \in V$ be PNHOS in $M$. Then $\exists$ PNHOS $U$ is in $M$ so that $\delta \in U$ with $U^{-1} \subseteq$ PNH $-$ $cl\big($PNH $-$ $int(V)\big)$. Also, there is $g \in \mathcal{Q} \bigcap U$ which implies $g^{-1} \in \mathcal{Q}^{-1} \bigcap$ PNH $-$ $cl\big(PNH - int(V)\big)$. That is $\mathcal{Q}^{-1} \bigcap$ PNH$-cl\big($PNH$-int(V)\big) \neq \phi_{\mathcal{U}_{\mathcal{P}}} \Rightarrow \mathcal{Q}^{-1} \bigcap$ PNH$-int(V) \neq \phi_{\mathcal{U}_{\mathcal{P}}} \Rightarrow \mathcal{Q}^{-1} \bigcap V \neq \phi_{\mathcal{U}_{\mathcal{P}}}$, since $\mathcal{Q}^{-1}$ is PNHCoS. Hence PNH $-$ $int(\mathcal{Q}^{-1}) =$ PNH $-$ $int(\mathcal{Q})^{-1}$.

**Theorem 5.0.6**

*Let* $\mathcal{A}$ *be any PNHSOS in a PNHATG* $M$*. Then*

*(a) PNH* $-$ $cl(\sigma\mathcal{A}) \subseteq \sigma$*PNH* $-$ $cl(\mathcal{A})$*, for all* $\sigma \in M$*, where* $\sigma = (a_1, a_2)$.

*(b) PNH* $-\ cl(\mathcal{A}\sigma) \subseteq$ *PNH* $-\ cl(\mathcal{A})\sigma$, *for all* $\sigma \in M$.

*(c) PNH* $-\ cl(\mathcal{A}^{-1}) \subseteq$ *PNH* $-\ cl(\mathcal{A})^{-1}$.

**Proof**

(a) As $\mathcal{A}$ is PNHSOS, PNH $-\ cl(\mathcal{A})$ is PNHRCoS. From Theorem 5.0.2, $g_{\sigma^{-1}} : (M, \tau_{\mathcal{U}_G}) \to (M, \urcorner_{\mathcal{U}_G})$ is PNHACM. So, $\sigma$PNH $-$ $cl(\mathcal{A})$ is PNHCoS. Hence PNH $-\ cl(\sigma\mathcal{A}) \subseteq \sigma$PNH $-\ cl(\mathcal{A})$.

(b) As $\mathcal{A}$ is PNHSOS, PNH $-\ cl\,(\mathcal{A})$ is PNHRCoS. From Theorem 5.0.2, $h_{\sigma^{-1}} : (M, \urcorner_{\mathcal{U}_G}) \to (M, \urcorner_{\mathcal{U}_G})$ is PNHACM. So, PNH $-$ $cl(\mathcal{A})\sigma$ is PNHCoS. Thus, PNH $-\ cl(\mathcal{A}\sigma) \subseteq$ PNH $-\ cl(\mathcal{A})\sigma$.

(c) Since $\mathcal{A}$ is PNHSOS, so, PNH $-\ cl(\mathcal{A})$ is PNHRCoS and hence PNH$-cl(\mathcal{A})^{-1}$ is PNHCoS. Consequently, PNH$-cl(\mathcal{A}) \subseteq$ PNH$-$ $cl(\mathcal{A})^{-1}$.