CHAPTER 6

CHAPTER 6

NEUTROSOPHIC MULTI TOPOLOGICAL GROUP

In this chapter, to study some properties of the NMTG, the definitions of NMSOS, NMSCoS, NMROS, NMRCoS, and NMCM are introduced and studied the definition of a NMTG and some of its properties. Also, since the concept of the almost topological group is very new, the definition of the NMATG has been introduced.

Definition 6.0.1

Let $(X, \daleth_X)$ *be NMTS. Then for a NMS* $\mathcal{A} = \big\{\langle x, \mu_{\mathcal{N}_i}, \sigma_{\mathcal{N}_i}, \delta_{\mathcal{N}_i}\rangle : x \in X\big\}$*, neutrosophic interior of* $\mathcal{A}$ *can be defined as*

$$NM \sim Int(\mathcal{A}) = \big\{\langle x, \cup\mu_{\mathcal{N}_i},\ \cap\,\sigma_{\mathcal{N}_i},\ \cap\,\delta_{\mathcal{N}_i}\rangle : x \in X\big\}.$$

Definition 6.0.2

Let $(X, \daleth_X)$ *be NMTS. Then for a NMS* $\mathcal{A} = \big\{\langle x, \mu_{\mathcal{N}_i}, \sigma_{\mathcal{N}_i}, \delta_{\mathcal{N}_i}\rangle : x \in X\big\}$*, neutrosophic closure of* $\mathcal{A}$ *can be defined as*

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$$NM \sim Cl(\mathcal{A}) = \big\{\langle x, \cap\mu_{\mathcal{N}_i},\ \cup\sigma_{\mathcal{N}_i},\ \cup\delta_{\mathcal{N}_i}\rangle : x \in X\big\}.$$

Definition 6.0.3

Let $\mathcal{G}$ be a NMG on a group X. Let $\urcorner_X$ be a NMT on $\mathcal{G}$ then $(\mathcal{G}, \urcorner_X)$ is known as a neutrosophic multi topological group (NMTG) if it satisfies the given conditions:

- *(i) The mapping $\alpha : (\mathcal{G}, \urcorner_X)\times(\mathcal{G}, \urcorner_X) \to (\mathcal{G}, \urcorner_X)$ defined by $\alpha(m, n) = mn, \forall\, m, n \in X$, is relatively neutrosophic multi continuous and*
- *(ii) The mapping $\beta : (\mathcal{G}, \urcorner_X) \to (\mathcal{G}, \urcorner_X)$ defined by $\beta(m) = m^{-1}, \forall\, m \in X$, is relatively neutrosophic multi continuous.*

Definition 6.0.4

Let $\mathcal{A}$ be an NMS of a NMTS $(X, \urcorner_X)$, then $\mathcal{A}$ is called a NMSOS of X if $\exists$ a $\mathcal{B} \in \urcorner_X$ such that $\mathcal{A} \subseteq MN \sim Int(MN \sim Cl(\mathcal{B}))$.

Example 6.0.1

Let $X = \{a, b\}$ and

$$\mathcal{A} = \begin{Bmatrix} \langle a, 0.8, 0.1, 0.2\rangle, \langle a, 0.7, 0.1, 0.3\rangle, \langle a, 0.6, 0.2, 0.4\rangle, \\ \langle b, 0.7, 0.2, 0.3\rangle, \langle b, 0.6, 0.3, 0.4\rangle, \langle b, 0.4, 0.2, 0.5\rangle \end{Bmatrix};$$

$$\mathcal{B} = \begin{Bmatrix} \langle a, 0.9, 0.1, 0.1\rangle, \langle a, 0.8, 0.1, 0.2\rangle, \langle a, 0.7, 0.2, 0.3\rangle, \\ \langle b, 0.8, 0.2, 0.2\rangle, \langle b, 0.7, 0.2, 0.3\rangle, \langle b, 0.5, 0.2, 0.4\rangle \end{Bmatrix}.$$

Then $\urcorner = \{0_X, 1_X, \mathcal{B}\}$ is neutrosophic multi topological space.
Then $Cl(\mathcal{B}) = 1_X$, $Int(Cl(\mathcal{B})) = 1_X$.
Hence, $\mathcal{B}$ is NMSOS.

Definition 6.0.5

Let $\mathcal{A}$ be an NMS of a NMTS $(X, \urcorner_X)$, then $\mathcal{A}$ is called a NMSCoS of X if $\exists$ a $\mathcal{B}^c \in \urcorner_X$ such that $MN \sim Cl(MN \sim Int(\mathcal{B})) \subseteq \mathcal{A}$.

Lemma 6.0.1

Let $\phi : X \to Y$ be a mapping and $\mathcal{A}_\alpha$ be a family of NMSs of Y, then

(i) $\phi^{-1}(\cup \mathcal{A}_\alpha) = \bigcup \phi^{-1}(\mathcal{A}_\alpha)$ *and*

(ii) $\phi^{-1}(\cap \mathcal{A}_\alpha) = \cap \phi^{-1}(\mathcal{A}_\alpha)$.

The proof is straightforward.

Lemma 6.0.2

Let $\mathcal{A}$ and $\mathcal{B}$ be NMSs of X and Y respectively, then

$$1_X - \mathcal{A} \times \mathcal{B} = (\mathcal{A}^c \times 1_X) \cup (1_X \times \mathcal{B}^c).$$

Proof

Let (p, q) be any element of $X \times Y$, then

$$\begin{aligned}(1_X - \mathcal{A} \times \mathcal{B})(p, q) &= max\Big\{\big(1_X - \mathcal{A}(p), 1_X - \mathcal{B}(q)\big)\Big\} \\ &= max\Big\{(\mathcal{A}^c \times 1_X)(p, q), (\mathcal{B}^c \times 1_X)(p, q)\Big\} \\ &= (\mathcal{A}^c \times 1_X) \cup (1_X \times \mathcal{B}^c)(p, q), \text{ for each } (p, q) \in X \times Y.\end{aligned}$$

Lemma 6.0.3

Let $\phi : X_i \to Y_i$ and $\mathcal{A}_i$ be NMSs of $Y_i, i = 1, 2$; then $(\phi_1 \times \phi_2)^{-1}(\mathcal{A}_1 \times \mathcal{A}_2) = {\phi_1}^{-1}(\mathcal{A}_1) \times {\phi_2}^{-1}(\mathcal{A}_2)$.

Proof

For each $(p_1, p_2) \in X_1 \times X_2$, we have

$$\begin{aligned}(\phi_1 \times \phi_2)^{-1}(\mathcal{A}_1 \times \mathcal{A}_2)(p_1, p_2) &= (\mathcal{A}_1 \times \mathcal{A}_2)\Big((\phi_1(p_1), \phi_2(p_2)\Big) \\ &= min\big\{\mathcal{A}_1\phi_1(p_1), \mathcal{A}_2\phi_2(p_2)\big\} \\ &= min\Big\{\phi_1^{-1}(\mathcal{A}_1)(p_1), \phi_2^{-1}(\mathcal{A}_2)(p_2)\Big\} \\ &= \Big(\phi_1^{-1}(\mathcal{A}_1) \times \phi_2^{-1}(\mathcal{A}_2)\Big)(p_1, p_2).\end{aligned}$$

Lemma 6.0.4

Let $\psi : X \to X \times Y$ be the graph of a mapping $\psi : X \to Y$. Then, if $\mathcal{A}$ and $\mathcal{B}$ be NMSs of X and Y respectively, then $\psi^{-1}(\mathcal{A} \times \mathcal{B}) = \mathcal{A} \cap \phi^{-1}(\mathcal{B})$.

Proof

For each $p \in X$, we have

$$\begin{aligned}\psi^{-1}(\mathcal{A} \times \mathcal{B})(p) &= (\mathcal{A} \times \mathcal{B})\psi(p)\\ &= (\mathcal{A} \times \mathcal{B})\big(p, \phi(p)\big)\\ &= min\Big\{\mathcal{A}(p), \mathcal{B}\big(\phi(p)\big)\Big\}\\ &= \Big(\mathcal{A} \cap \phi^{-1}(\mathcal{B})\Big)(p).\end{aligned}$$

Lemma 6.0.5

For a family $\{\mathcal{A}\}_\alpha$ of NMSs of NMTS $(X, \daleth_X), \cup NM \sim Cl(\mathcal{A}_\alpha) \subseteq NM \sim Cl\big(\cup(\mathcal{A}_\alpha)\big)$. In case $\mathcal{B}$ is a finite set, $\cup NM \sim Cl(\mathcal{A}_\alpha) \subseteq NM \sim Cl\big(\cup(\mathcal{A}_\alpha)\big)$. Also, $\cup NM \sim Int(\mathcal{A}_\alpha) \subseteq NM \sim Int\big(\cup(\mathcal{A}_\alpha)\big)$, where a subfamily $\mathcal{B}$ of $(X, \daleth_X)$ is said to be subbase for $(X, \daleth_X)$ if the collection of all intersections of members of $\mathcal{B}$ forms a base for $(X, \daleth_X)$.

Lemma 6.0.6

For an NMS $\mathcal{A}$ of a NMTS $(X, \daleth_X)$,

(a) $1_{NM} - NM \sim Int(\mathcal{A}) = NM \sim Cl(1_{NM} - \mathcal{A})$, *and*

(b) $1_{NM} - NM \sim Cl(\mathcal{A}) = NM \sim Int(1_{NM} - \mathcal{A})$.

The proof is straightforward.

Theorem 6.0.1

The following statements are equivalent:

(a) $\mathcal{A}$ *is a NMCoS,*

(b) $\mathcal{A}^c$ *is a NMOS,*

(c) $NM \sim Int\Big(NM \sim Cl(\mathcal{A})\Big) \subseteq \mathcal{A}$*, and*

(d) $NM \sim Cl\Big(NM \sim Int(\mathcal{A}^c)\Big) \supseteq \mathcal{A}^c$*.*

Theorem 6.0.2

(i) Arbitrary union of NMSOSs is a NMSOS, and

(ii) Arbitrary intersection of NMSCoSs is a NMSCoS.

Remark 6.0.1
It is clear that every NMOS (NMCoS) is a NMSOS (NMSCoS). The converse is not true.

Example 6.0.2
From Example 6.0.1, it is clear that $\mathcal{B}$ *is a NMSOS, but* $\mathcal{B}$ *is not NMOS.*

Theorem 6.0.3
If $(X, \daleth_X)$ *and* $(Y, \daleth_Y)$ *are NMTSs and* X *is product related to* Y*. Then the product* $\mathcal{A} \times \mathcal{B}$ *of a NMSOS* $\mathcal{A}$ *of* X *and a NMSOS* $\mathcal{B}$ *of* Y *is a NMSOS of the neutrosophic multi product space* $X \times Y$*.*

Definition 6.0.6
An NMS $\mathcal{A}$ *of a NMTS* $(X, \daleth_X)$ *is called a NMROS of* $(X, \daleth_X)$ *if* $NM \sim Int\Big(NM \sim Cl(\mathcal{A})\Big) = \mathcal{A}$*.*

Example 6.0.3
Let $X = \{a, b\}$ *and*

$$\mathcal{A} = \left\{ \begin{array}{c} \langle a, 0.4, 0.5, 0.5\rangle, \langle a, 0.3, 0.5, 0.6\rangle, \langle a, 0.2, 0.6, 0.7\rangle, \\ \langle b, 0.5, 0.7, 0.6\rangle, \langle b, 0.4, 0.5, 0.7\rangle, \langle b, 0.3, 0.5, 0.8\rangle \end{array} \right\}.$$

Then $\daleth = \big\{0_X, 1_X, \mathcal{A}\big\}$ *is neutrosophic multi topological space.*
Clearly, $NM \sim Cl(\mathcal{A}) = \mathcal{A}^c, NM \sim Int\big(NM \sim Cl(\mathcal{A})\big) = \mathcal{A}$*.*
Hence, $\mathcal{A}$ *is NMROS.*

Definition 6.0.7

An NMS $\mathcal{A}$ *of a NMTS* $(X, \daleth_X)$ *is called a NMRCoS of* $(X, \daleth_X)$ *if* $NM \sim Cl\Big(NM \sim Int(\mathcal{A})\Big) = \mathcal{A}$*.*

Theorem 6.0.4

An NMS $\mathcal{A}$ *of NMTS* $(X, \daleth_X)$ *is a NMROS iff* $\mathcal{A}^c$ *is NMRCoS.*

Remark 6.0.2

It is obvious that every NMROS (NMRCoS) is a NMOS (NMCoS). The converse need not be true.

Example 6.0.4

Let $X = \{a, b\}$ *and* $\daleth = \big\{0_X, 1_X, \mathcal{A}, \mathcal{B}, \mathcal{A}\bigcup\mathcal{B}\big\}$ *is neutrosophic multi topological space, where*

$$\mathcal{A} = \left\{\begin{matrix}\langle a, 0.4, 0.5, 0.6\rangle, \langle a, 0.3, 0.5, 0.7\rangle, \langle a, 0.2, 0.6, 0.8\rangle, \\ \langle b, 0.7, 0.5, 0.3\rangle, \langle b, 0.6, 0.5, 0.4\rangle, \langle b, 0.4, 0.5, 0.6\rangle\end{matrix}\right\};$$

$$\mathcal{B} = \left\{\begin{matrix}\langle a, 0.6, 0.5, 0.4\rangle, \langle a, 0.7, 0.5, 0.3\rangle, \langle a, 0.8, 0.4, 0.2\rangle, \\ \langle b, 0.3, 0.5, 0.7\rangle, \langle b, 0.4, 0.5, 0.6\rangle, \langle b, 0.6, 0.5, 0.4\rangle\end{matrix}\right\};$$

$$\mathcal{A}\bigcup\mathcal{B} = \left\{\begin{matrix}\langle a, 0.6, 0.5, 0.4\rangle, \langle a, 0.7, 0.5, 0.3\rangle, \langle a, 0.8, 0.4, 0.2\rangle, \\ \langle b, 0.7, 0.5, 0.3\rangle, \langle b, 0.6, 0.5, 0.4\rangle, \langle b, 0.4, 0.5, 0.6\rangle\end{matrix}\right\}.$$

Here, $NM \sim Cl(\mathcal{A}) = \mathcal{B}^c, NM \sim Int\big(NM \sim Cl(\mathcal{A})\big) = \mathcal{A}$*, and* $NM \sim Cl(\mathcal{B}) = \mathcal{A}^c, NM \sim Int\big(NM \sim Cl(\mathcal{B})\big) = \mathcal{B}$*.*
Then $NM \sim Cl(\mathcal{A}\bigcup\mathcal{B}) = 1_X$*.*
Thus, $NM \sim Int\Big(NM \sim Cl(\mathcal{A}\bigcup\mathcal{B})\Big) = 1_X$*.*
Hence, $\mathcal{A}$ *and* $\mathcal{B}$ *is NMROS, but* $\mathcal{A}\bigcup\mathcal{B}$ *is not NMROS.*

Remark 6.0.3

The union (intersection) of any two NMROSs (NMRCoS) need not be a NMROS (NMRCoS).

Theorem 6.0.5

(i) *The intersection of any two NMROSs is a NMROS, and*

(ii) *The union of any two NMRCoSs is a NMRCoS.*

Theorem 6.0.6

(i) *The closure of a NMOS is a NMRCoS, and*

(ii) *The interior of a NMCoS is a NMROS.*

Definition 6.0.8

Let $\phi : (X, \daleth_X) \to (Y, \daleth_Y)$ *be a mapping from a NMTS* $(X, \daleth_X)$ *to another NMTS* $(Y, \daleth_Y)$*, then* ϕ *is known as a NMCM, if* $\phi^{-1}(\mathcal{A}) \in \daleth_X$ *for each* $\mathcal{A} \in \daleth_Y$*; or equivalently* $\phi^{-1}(\mathcal{B})$ *is a NMCoS of* X *for each NMCoS* $\mathcal{B}$ *of* Y.

Example 6.0.5

Let $X = Y = \{a, b, c\}$ *and*

$$\mathcal{A} = \left\{ \begin{matrix} \langle a, 0.4, 0.5, 0.6 \rangle, \langle a, 0.3, 0.5, 0.7 \rangle, \langle a, 0.2, 0.6, 0.8 \rangle, \\ \langle b, 0.3, 0.5, 0.4 \rangle, \langle b, 0.2, 0.5, 0.6 \rangle, \langle b, 0.1, 0.5, 0.7 \rangle, \\ \langle c, 0.4, 0.5, 0.6 \rangle, \langle c, 0.3, 0.5, 0.7 \rangle, \langle c, 0.2, 0.6, 0.8 \rangle \end{matrix} \right\};$$

$$\mathcal{B} = \left\{ \begin{matrix} \langle a, 0.6, 0.1, 0.2 \rangle, \langle a, 0.5, 0.1, 0.3 \rangle, \langle a, 0.4, 0.2, 0.4 \rangle, \\ \langle b, 0.3, 0.5, 0.4 \rangle, \langle b, 0.2, 0.5, 0.6 \rangle, \langle b, 0.1, 0.5, 0.7 \rangle, \\ \langle c, 0.4, 0.5, 0.6 \rangle, \langle c, 0.3, 0.5, 0.7 \rangle, \langle c, 0.2, 0.6, 0.8 \rangle \end{matrix} \right\}.$$

Then $\daleth_X = \{0_X, 1_X, \mathcal{A}\}$ *and* $\daleth_Y = \{0_Y, 1_Y, \mathcal{B}\}$ *are neutrosophic multi topological spaces.*

Now, define a mapping $f : (X, \daleth_X) \to (Y, \daleth_Y)$ *by* $f(a) = f(c) = c$ *and* $f(b) = b$.
Thus, f *is NMCM.*

Definition 6.0.9
Let $\phi : (X, \daleth_X) \to (Y, \daleth_Y)$ *be a mapping from a NMTS* $(X, \daleth_X)$ *to another NMTS* $(Y, \daleth_Y)$*, then* ϕ *is called a NMOM, if* $\phi(\mathcal{A}) \in \daleth_Y$ *for each* $\mathcal{A} \in \daleth_X$.

Definition 6.0.10 *Let* $\phi : (X, \daleth_X) \to (Y, \daleth_Y)$ *be a mapping from a NMTS* $(X, \daleth_X)$ *to another NMTS* $(Y, \daleth_Y)$*, then* ϕ *is said to be a NM-CoM if* $\phi(\mathcal{B})$ *is a NMCoS of* Y *for each NMCoS* $\mathcal{B}$ *of* X.

Definition 6.0.11
Let $\phi : (X, \daleth_X) \to (Y, \daleth_Y)$ *be a mapping from a NMTS* $(X, \daleth_X)$ *to another NMTS* $(Y, \daleth_Y)$*, then* ϕ *is called a NMSCM, if* $\phi^{-1}(\mathcal{A})$ *is the NMSOS of* X*, for each* $\mathcal{A} \in \daleth_Y$.

Definition 6.0.12
Let $\phi : (X, \daleth_X) \to (Y, \daleth_Y)$ *be a mapping from a NMTS* $(X, \daleth_X)$ *to another NMTS* $(Y, \daleth_Y)$*, then* ϕ *is called a NMSOM, if* $\phi(\mathcal{A})$ *is a NMSOS for each* $\mathcal{A} \in \daleth_X$.

Example 6.0.6
Let $X = Y = \{a, b, c\}$ *and*

$$\mathcal{A} = \left\{ \begin{array}{c} \langle a, 0.6, 0.1, 0.2\rangle, \langle a, 0.5, 0.1, 0.3\rangle, \langle a, 0.4, 0.2, 0.4\rangle, \\ \langle b, 0.3, 0.5, 0.4\rangle, \langle b, 0.2, 0.5, 0.6\rangle, \langle b, 0.1, 0.5, 0.7\rangle, \\ \langle c, 0.4, 0.5, 0.6\rangle, \langle c, 0.3, 0.5, 0.7\rangle, \langle c, 0.2, 0.6, 0.8\rangle \end{array} \right\};$$

$$\mathcal{B} = \left\{ \begin{array}{c} \langle a, 0.3, 0.5, 0.4\rangle, \langle a, 0.2, 0.5, 0.6\rangle, \langle a, 0.1, 0.5, 0.7\rangle, \\ \langle b, 0.6, 0.1, 0.2\rangle, \langle b, 0.5, 0.1, 0.3\rangle, \langle b, 0.4, 0.2, 0.4\rangle, \\ \langle c, 0.4, 0.5, 0.6\rangle, \langle c, 0.3, 0.5, 0.7\rangle, \langle c, 0.2, 0.6, 0.8\rangle \end{array} \right\}.$$

Then $\daleth_X = \{0_X, 1_X, \mathcal{A}\}$ *and* $\daleth_Y = \{0_Y, 1_Y, \mathcal{B}\}$ *are neutrosophic multi topological spaces.*
Clearly, $\mathcal{A}$ *is neutrosophic multi semi-open set.*
Then a mapping $f : (X, \daleth_X) \to (Y, \daleth_Y)$ *by* $f(a) = b, f(b) = a$ *and* $f(c) = c$*.*
Hence, f *is NMSOM.*

Definition 6.0.13
Let $\phi : (X, \daleth_X) \to (Y, \daleth_Y)$ *be a mapping from a NMTS* $(X, \daleth_X)$ *to another NMTS* $(Y, \daleth_Y)$*, then* ϕ *is called a NMSCoM, if* $\phi(\mathcal{B})$ *is a NMSCoS for each NMCoS* $\mathcal{B}$ *of* X*.*

Remark 6.0.4
From Remark 6.0.1, a NMCM (NMOM, NMCoM) is also a NMSCM (NMSOM, NMSCoM).

Example 6.0.7
Let $X = Y = \{a, b, c\}$ *and*

$$\mathcal{A} = \begin{Bmatrix} \langle a, 0.4, 0.5, 0.6\rangle, \langle a, 0.3, 0.5, 0.7\rangle, \langle a, 0.2, 0.6, 0.8\rangle, \\ \langle b, 0.3, 0.5, 0.4\rangle, \langle b, 0.2, 0.5, 0.6\rangle, \langle b, 0.1, 0.5, 0.7\rangle, \\ \langle c, 0.4, 0.5, 0.6\rangle, \langle c, 0.3, 0.5, 0.7\rangle, \langle c, 0.2, 0.6, 0.8\rangle \end{Bmatrix};$$

$$\mathcal{B} = \begin{Bmatrix} \langle a, 0.4, 0.5, 0.6\rangle, \langle a, 0.3, 0.5, 0.7\rangle, \langle a, 0.2, 0.6, 0.8\rangle, \\ \langle b, 0.4, 0.6, 0.4\rangle, \langle b, 0.3, 0.5, 0.5\rangle, \langle b, 0.2, 0.5, 0.6\rangle, \\ \langle c, 0.6, 0.5, 0.5\rangle, \langle c, 0.4, 0.5, 0.6\rangle, \langle c, 0.2, 0.6, 0.9\rangle \end{Bmatrix}.$$

Then $\daleth_X = \{0_X, 1_X, \mathcal{A}\}$ *and* $\daleth_Y = \{0_Y, 1_Y, \mathcal{B}\}$ *are neutrosophic multi topological spaces.*
Let us define a mapping $f : (X, \daleth_X) \to (Y, \daleth_Y)$ *by* $f(a) = f(c) = c$ *and* $f(b) = b$*.*
Thus, f *is NMSCM, which is not NMCM.*

Theorem 6.0.7

Let X_1, X_2, Y_1 and Y_2 be NMTSs such that X_1 is product related to X_2. Then, the product $\phi_1 \times \phi_2 : X_1 \times X_2 \to Y_1 \times Y_2$ of NMSCMs $\phi_1 : X_1 \to Y_1$ and $\phi_2 : X_2 \to Y_2$ is NMSCM.

Theorem 6.0.8

Let X, X_1 and X_2 be a NMTSs and $p_i : X_1 \times X_2 \to X_i$ $(i = 1, 2)$ be the projection of $X_1 \times X_2$ onto X_i. Then, if $\phi : X \to X_1 \times X_2$ is a NMSCM, $p_i\phi$ is also NMSCM.

Theorem 6.0.9

Let $\phi : X \to Y$ be a mapping from a NMTS X to another NMTS Y. Then if the graph $\psi : X \to X \times Y$ of ϕ is NMSCM, ϕ is also NMSCM.

Remark 6.0.5

The converse of Theorem 6.0.9 is not true.

Definition 6.0.14

A mapping $\phi : (X, \daleth_X) \to (Y, \daleth_Y)$ from a NMTS X to another NMTS Y is known as NMACM, if $\phi^{-1}(\mathcal{A}) \in \daleth_X$ for each NMROS $\mathcal{A}$ of Y.

Example 6.0.8

Let $X = Y = \{a, b\}$ and

$$\mathcal{A} = \left\{ \begin{array}{c} \langle a, 0.4, 0.5, 0.5\rangle, \langle a, 0.3, 0.5, 0.6\rangle, \langle a, 0.2, 0.6, 0.7\rangle, \\ \langle b, 0.5, 0.7, 0.6\rangle, \langle b, 0.4, 0.5, 0.7\rangle, \langle b, 0.3, 0.5, 0.8\rangle, \end{array} \right\};$$

$$\mathcal{B} = \left\{ \begin{array}{c} \langle a, 0.5, 0.7, 0.6\rangle, \langle a, 0.4, 0.5, 0.7\rangle, \langle a, 0.3, 0.5, 0.8\rangle, \\ \langle b, 0.4, 0.5, 0.5\rangle, \langle b, 0.3, 0.5, 0.6\rangle, \langle b, 0.2, 0.6, 0.7\rangle \end{array} \right\}.$$

Then $\daleth_X = \{0_X, 1_X, \mathcal{A}\}$ and $\daleth_Y = \{0_Y, 1_Y, \mathcal{B}\}$ are neutrosophic multi topological spaces.
Clearly, $NM \sim Cl(\mathcal{B}) = \mathcal{B}^c, NM \sim Int\big(NM \sim Cl(\mathcal{B})\big) = \mathcal{B}$.

Hence, $\mathcal{B}$ is NMROS.
Now, let us define a mapping $f : (X, \daleth_X) \to (Y, \daleth_Y)$ by $f(a) = b$ and $f(b) = a$.
Thus, f is NMACM.

Theorem 6.0.10
Let $\phi : (X, \daleth_X) \to (Y, \daleth_Y)$ be a mapping. Then the following statements are equivalent:

(a) ϕ *is a NMACM,*

(b) $\phi^{-1}(\mathcal{F})$ *is a NMCoS, for each NMRCoS $\mathcal{F}$ of Y,*

(c) $\phi^{-1}(\mathcal{A}) \subseteq NM \sim Int\Big(\phi^{-1}\Big(NM \sim Int\Big(NM \sim Cl(\mathcal{A})\Big)\Big)\Big)$, *for each NMOS $\mathcal{A}$ of Y,*

(d) $NM \sim Cl\Big(\phi^{-1}\Big(NM \sim Cl\Big(NM \sim Int(\mathcal{F})\Big)\Big)\Big) \subseteq \phi^{-1}(\mathcal{F})$, *for each NMCoS $\mathcal{F}$ of Y.*

Remark 6.0.6
Clearly, a NMCM is a NMACM. That the converse need not be true.

Example 6.0.9 *Let $X = \{a, b\}$ and*

$$\mathcal{A} = \left\{ \begin{array}{c} \langle a, 0.4, 0.5, 0.5\rangle, \langle a, 0.3, 0.5, 0.6\rangle, \langle a, 0.2, 0.6, 0.7\rangle, \\ \langle b, 0.5, 0.7, 0.6\rangle, \langle b, 0.4, 0.5, 0.7\rangle, \langle b, 0.3, 0.5, 0.8\rangle, \end{array} \right\};$$
$$\mathcal{B} = \left\{ \begin{array}{c} \langle a, 0.5, 0.5, 0.6\rangle, \langle a, 0.6, 0.5, 0.7\rangle, \langle a, 0.2, 0.6, 0.9\rangle, \\ \langle b, 0.4, 0.4, 0.7\rangle, \langle b, 0.3, 0.5, 0.5\rangle, \langle b, 0.4, 0.5, 0.6\rangle \end{array} \right\}.$$

Then $\daleth_X = \big\{0_X, 1_X, \mathcal{A}\big\}$ and $\daleth_Y = \big\{0_Y, 1_Y, \mathcal{B}\big\}$ are neutrosophic multi topological spaces.
Clearly, $NM \sim Cl(\mathcal{B}) = \mathcal{B}^c, NM \sim Int\big(NM \sim Cl(\mathcal{B})\big) = \mathcal{B}$.
Hence, $\mathcal{B}$ is NMROS in $\daleth_Y$.
Now, a mapping $f : (X, \daleth_X) \to (Y, \daleth_Y)$ defined by $f(a) = a$ and

$f(b) = b$.

Then clearly, f is NMACM but not NMCM.

Theorem 6.0.11

Neutrosophic multi semi-continuity and neutrosophic multi almost continuity are independent notions.

Definition 6.0.15

A NMTS $(X, \daleth_X)$ is called a neutrosophic multi semi-regularly space (NMSRS) if and only if the collection of all NMROSs of X forms a base for NMT $\daleth_X$.

Theorem 6.0.12

Let $\phi : (X, \daleth_X) \to (Y, \daleth_Y)$ be a mapping from a NMTS $(X, \daleth_X)$ to a NMSRS $(Y, \daleth_Y)$. Then ϕ is NMACM iff ϕ is NMCM.

Theorem 6.0.13

Let X_1, X_2, Y_1 and Y_2 be the NMTSs such that Y_1 is product related to Y_2. Then the product $\phi_1 \times \phi_2 : X_1 \times X_2 \to Y_1 \times Y_2$ of NMACMs $\phi_1 : X_1 \to Y_1$ and $\phi_2 : X_2 \to Y_2$ is NMACM.

Theorem 6.0.14

Let X, X_1 and X_2 be a NMTSs and $p_i : X_1 \times X_2 \to X_i$ $(i = 1, 2)$ be the projection of $X_1 \times X_2$ onto X_i. Then if $\phi : X \to X_1 \times X_2$ is a NMACM, $p_i\phi$ is also a NMACM.

Theorem 6.0.15

Let X and Y be NMTSs such that X is product related to Y and let $\phi : X \to Y$ be a mapping. Then, the graph $\psi : X \to X \times Y$ of ϕ is NMACM iff ϕ is NMACM.

NOTE: *The proof of the theorems from 6.0.1 to 6.0.15 is straightforward following* **Chapter-2**.

Definition 6.0.16

Let $\mathcal{G}$ be a NMG on a group X. Now, if $\urcorner_X$ be a NMT on $\mathcal{G}$, then $(\mathcal{G}, \urcorner_X)$ is said to be a NMATG if the given conditions are satisfied:

(i) $\alpha : (\mathcal{G}, \urcorner_X) \times (\mathcal{G}, \urcorner_X) \to (\mathcal{G}, \urcorner_X) : \alpha(m, n) = mn$ *is NMACM and*

(ii) $\beta : (\mathcal{G}, \urcorner_X) \to (\mathcal{G}, \urcorner_X) : \beta(m) = m^{-1}$ *is NMACM.*

Then $(\mathcal{G}, \urcorner_X)$ is known as a NMATG.

Remark 6.0.7

$(\mathcal{G}, \urcorner_X)$ is a NMATG if the following conditions hold good:

(i) for $g_1, g_2 \in \mathcal{G}$ and every NMROS $\mathcal{P}$ containing $g_1 g_2$ in $\mathcal{G}, \exists$ open neighborhoods $\mathcal{R}$ and $\mathcal{S}$ of g_1 and g_2 in $\mathcal{G}$ such that $\mathcal{R} * \mathcal{S} \subseteq \mathcal{P}$ and

(ii) for $g \in \mathcal{G}$ and every NMROS $\mathcal{Q}$ in $\mathcal{G}$ containing $g^{-1}, \exists$ open neighborhood $\mathcal{R}$ of g in $\mathcal{G}$ so that $\mathcal{R}^{-1} \subseteq \mathcal{S}$.

Remark 6.0.8

For any $\mathcal{P}, \mathcal{Q} \subseteq \mathcal{G}$, we denote $\mathcal{P} * \mathcal{Q}$ by $\mathcal{P}\mathcal{Q}$ and defined as $\mathcal{P}\mathcal{Q} = \{gh : g \in \mathcal{P}, h \in \mathcal{Q}\}$ and $\mathcal{P}^{-1} = \{g^{-1} : g \in \mathcal{P}\}$. If $\mathcal{P} = a$ for each $a \in \mathcal{G}$, we denote $\mathcal{P} * \mathcal{Q}$ by $a\mathcal{Q}$ and $\mathcal{Q} * \mathcal{P}$ by $\mathcal{P}a$.

Example 6.0.10

Let $\mathcal{G} = (\mathbb{Z}_3, +)$ be a classical group and

$$\mathcal{A} = \left\{ \begin{matrix} \langle 0, 0.4, 0.5, 0.6\rangle, \langle 0, 0.3, 0.5, 0.7\rangle, \langle 0, 0.2, 0.6, 0.8\rangle, \\ \langle 1, 0.3, 0.5, 0.4\rangle, \langle 1, 0.2, 0.5, 0.6\rangle, \langle 1, 0.1, 0.5, 0.7\rangle, \\ \langle 2, 0.4, 0.5, 0.6\rangle, \langle 2, 0.3, 0.5, 0.7\rangle, \langle 2, 0.2, 0.6, 0.8\rangle \end{matrix} \right\}.$$

Then $\urcorner_X = \{0_G, 1_G, \mathcal{A}\}$ is NTS and $\alpha : (\mathcal{G}, \urcorner_{\mathcal{G}}) \times (\mathcal{G}, \urcorner_{\mathcal{G}}) \to (\mathcal{G}, \urcorner_{\mathcal{G}}) : \alpha(m, n) = mn$, $\beta : (\mathcal{G}, \urcorner_{\mathcal{G}}) \to (\mathcal{G}, \urcorner_{\mathcal{G}}) : \beta(m) = m^{-1}$ are NMACM. Hence, $(\mathcal{G}, \urcorner_{\mathcal{G}})$ is NMATG.

Theorem 6.0.16

Let $(\mathcal{G}, \urcorner_X)$ be a NMATG and let a be any element of $\mathcal{G}$. Then

(i) $\mu_a : (\mathcal{G}, \urcorner_X) \to (\mathcal{G}, \urcorner_X) : \mu_a(x) = ax, \forall x \in \mathcal{G}$, *is NMACM;*

(ii) $\lambda_a : (\mathcal{G}, \urcorner_X) \to (\mathcal{G}, \urcorner_X) : \lambda_a(x) = xa, \forall x \in \mathcal{G}$, *is NMACM.*

Proof

(i) Let $p \in \mathcal{G}$ and let $\mathcal{R}$ be a NMROS containing ap in $\mathcal{G}$. By definition 6.0.16, $\exists$ open neighborhoods $\mathcal{P}, \mathcal{Q}$ of a, p in $\mathcal{G}$ such that $\mathcal{P}\mathcal{Q} \subseteq \mathcal{R}$. Especially, $a\mathcal{Q} \subseteq \mathcal{R}$ i.e., $\mu_a(\mathcal{Q}) \subseteq \mathcal{R}$. This proves that μ_a is NMACM at p and hence μ_a is NMACM.

(ii) Suppose $p \in \mathcal{G}$ and $\mathcal{R} \in$ NMRO $(\mathcal{G})$ containing pa. Then $\exists$ open sets $p \in \mathcal{P}$ and $a \in \mathcal{Q}$ in $\mathcal{G}$ such that $\mathcal{P}\mathcal{Q} \subseteq \mathcal{R}$. This proves $\mathcal{P}a \subseteq \mathcal{R}$. This gives λ_a is NMACM at p. Since arbitrary element p is in $\mathcal{G}$, hence λ_a is NMACM.

Theorem 6.0.17

Let $\mathcal{U}$ be NMROS in a NMATG $(\mathcal{G}, \urcorner_X)$. The following conditions hold good:

(a) $m\mathcal{U} \in$ *NMROS*$(\mathcal{G}), \forall\, m \in \mathcal{G}$.

(b) $\mathcal{U}m \in$ *NMROS*$(\mathcal{G}), \forall\, m \in \mathcal{G}$.

(c) $\mathcal{U}^{-1} \in$ *NMROS*$(\mathcal{G})$.

Proof

(a) We first show that $m\mathcal{U} \in \urcorner_X$. Let $p \in m\mathcal{U}$. Then by definition 6.0.16 of NMATGs, $\exists$ NMOSs $m^{-1} \in W_1$ and $p \in W_2$ in $\mathcal{G}$ such that $W_1 W_2 \subseteq \mathcal{U}$. Especially, $m^{-1} W_2 \subseteq \mathcal{U}$. That is, equivalently $W_2 \subseteq m\mathcal{U}$. This indicates that $p \in NM \sim Int(m\mathcal{U})$ and thus,

$NM \sim Int(m\mathcal{U}) = m\mathcal{U}$. That is $m\mathcal{U} \in \daleth_X$. Consequently, $m\mathcal{U} \subseteq NM \sim Int\Big(NM \sim Cl(m\mathcal{U})\Big)$.

Now, we have to prove that $NM \sim Int\Big(NM \sim Cl(m\mathcal{U})\Big) \subseteq m\mathcal{U}$. As $\mathcal{U}$ is NMOS, $NM \sim Cl(\mathcal{U}) \in$ NMRCS($\mathcal{G}$). By Theorem 6.0.16, $\mu_{m^{-1}} : (\mathcal{G}, \daleth_X) \to (\mathcal{G}, \daleth_X)$ is NMACM and therefore, $mNM \sim Cl(\mathcal{U})$ is NMCoS. Thus, $NM \sim Int\Big(NM \sim Cl(m\mathcal{U})\Big) \subseteq NM \sim Cl(m\mathcal{U}) \subseteq mNM \sim Cl(\mathcal{U})$. i.e., $m^{-1}NM \sim Int\Big(NM \sim Cl(m\mathcal{U})\Big) \subseteq NM \sim Cl(\mathcal{U})$. Since $NM \sim Int\Big(NM \sim Cl(m\mathcal{U})\Big)$ is NMROS, it follows that $m^{-1}NM \sim Int\Big(NM \sim Cl(m\mathcal{U})\Big) \subseteq NM \sim Int\Big(NM \sim Cl(\mathcal{U})\Big) = \mathcal{U}$, i.e., $NM \sim Int(NM \sim Cl\Big(m\mathcal{U})\Big) \subseteq m\mathcal{U}$. Thus, $m\mathcal{U} = NM \sim Int\Big(NM \sim Cl(m\mathcal{U})\Big)$. This proves that $m\mathcal{U} \in$ NMROS($\mathcal{G}$).

(b) Following the same steps as in part (a) above, we can prove that $\mathcal{U}m \in$ NMROS$(\mathcal{G}), \forall\, m \in \mathcal{G}$.

(c)) Let $p \in \mathcal{U}^{-1}$, then $\exists$ open set $p \in W$ in $\mathcal{G}$ such that $W^{-1} \subseteq \mathcal{U} \Rightarrow W \subseteq \mathcal{U}^{-1}$. So, $\mathcal{U}^{-1}$ has interior point p. Thus, $\mathcal{U}^{-1}$ is NMOS. That is $\mathcal{U}^{-1} \subseteq NM \sim Int\Big(NM \sim Cl(\mathcal{U}^{-1})\Big)$. Now we have to prove that $NM \sim Int\Big(NM \sim Cl(\mathcal{U}^{-1})\Big) \subseteq \mathcal{U}^{-1}$. Since $\mathcal{U}$ is NMOS, $NM \sim Cl(\mathcal{U})$ is NMRCoS and thus $NM \sim Cl(\mathcal{U})^{-1}$ is NMCoS in $\mathcal{G}$. So, $NM \sim Int\Big(NM \sim Cl(\mathcal{U}^{-1})\Big) \subseteq NM \sim Cl(\mathcal{U}^{-1}) \subseteq NM \sim Cl(\mathcal{U})^{-1} \Rightarrow NM \sim Int\Big(NM \sim Cl(\mathcal{U}^{-1})\Big) \subseteq \Big(NM \sim Cl(\mathcal{U})\Big)^{-1} \subseteq \mathcal{U}^{-1}$. Thus, $\mathcal{U}^{-1} = NM \sim Int\Big(NM \sim Cl(\mathcal{U}^{-1}t)\Big)$. This proves that $\mathcal{U}^{-1} \in$ NMROS($\mathcal{G}$).

Corollary 6.0.1

Let $\mathcal{Q}$ be any NMRCoS in a NMATG in $\mathcal{G}$. Then *(i)* $m\mathcal{Q} \in$ *NMRCS*$(\mathcal{G})$*, for each* $m \in \mathcal{G}$*.*

(ii) $\mathcal{Q}^{-1} \in$ *NMRCS*$(\mathcal{G})$*.*

The proof is straightforward.

Theorem 6.0.18

Let $\mathcal{U}$ *be any NMROS in a NMATG* $\mathcal{G}$*. Then*

(a) $NM \sim Cl(\mathcal{U}m) = NM \sim Cl(\mathcal{U})m$*, for each* $m \in \mathcal{G}$*.*

(b) $NM \sim Cl(m\mathcal{U}) = mNM \sim Cl(\mathcal{U})$*, for each* $m \in \mathcal{G}$*.*

(c) $NM \sim Cl(\mathcal{U}^{-1}) = NM \sim Cl(\mathcal{U})^{-1}$*.*

Proof

(a) Assume $p \in NM \sim Cl(\mathcal{U}m)$ and consider $q = pm^{-1}$. Let $q \in W$ be NMOS in $\mathcal{G}$. Then $\exists$ NMOSs $m^{-1} \in V_1$ and $p \in V_2$ in $\mathcal{G}$, such that $V_1V_2 \subseteq NM \sim Int\Big(NM \sim Cl(W)\Big)$. By hypothesis, there is $g \in \mathcal{U}m \bigcap V_2 \Rightarrow gm^{-1} \in \mathcal{U} \bigcap V_1V_2 \subseteq \mathcal{U} \bigcap NM \sim Int\Big(NM \sim Cl(W)\Big) \Rightarrow \mathcal{U} \bigcap NM \sim Int\Big(NM \sim Cl(W)\Big) \neq 0_{NM} \Rightarrow \mathcal{U} \bigcap \Big(NM \sim Cl(W)\Big) \neq 0_{NM}$. Since $\mathcal{U}$ is NMOS, $\mathcal{U} \bigcap W \neq 0_{NM}$. That is, $m \in NM \sim Cl(\mathcal{U})m$.

Conversely, let $q \in NM \sim Cl(\mathcal{U})m$. Then $q = pg$ for some $p \in NM \sim Cl(\mathcal{U})$. To prove $NM \sim Cl(\mathcal{U})m \subseteq NM \sim Cl(\mathcal{U}m)$. Let $pg \in W$ be an NMOS in $\mathcal{G}$. Then $\exists$ NMOSs $m \in V_1$ in $\mathcal{G}$ and $p \in V_2$ in $\mathcal{G}$ so that $V_1V_2 \subseteq NM \sim Int\big(NM \sim Cl(W)\big)$. Since $p \in NM \sim Cl(\mathcal{U})$,$\mathcal{U} \bigcap V_2 \neq 0_{NM}$. There is $g \in \mathcal{U} \bigcap V_2$. This implies $gm \in (\mathcal{U}m) \bigcap NM \sim Int\Big(NM \sim Cl(W)\Big) \Rightarrow (\mathcal{U}m) \bigcap \Big(NM \sim Cl(W)\Big) \neq 0_{NM}$. From Theorem 6.0.17, $\mathcal{U}m$ is NMOS and thus $(\mathcal{U}m) \bigcap W \neq 0_{NM}$, therefore $q \in NM \sim Cl(\mathcal{U}m)$. Therefore $NM \sim Cl(\mathcal{U}m) = NM \sim Cl(\mathcal{U})m$.

(b) Following the same steps as in part (a) above, we can prove that $NM \sim Cl(m\mathcal{U}) = mNM \sim Cl(\mathcal{U})$.

(c) Since $NM \sim Cl(\mathcal{U})$ is NMRCoS, $NM \sim Cl(\mathcal{U})^{-1}$ is NMCoS in $\mathcal{G}$. Therefore, $\mathcal{U}^{-1} \subseteq NM \sim Cl(\mathcal{U})^{-1}$ this gives $NM \sim Cl(\mathcal{U}^{-1}) \subseteq NM \sim Cl(\mathcal{U})^{-1}$. Next, let $q \in NM \sim Cl(\mathcal{U})^{-1}$. Then $q = p^{-1}$, for some $p \in NM \sim Cl(\mathcal{U})$. Let $q \in V$ be any NMOS in $\mathcal{G}$. Then $\exists$ open set $\mathcal{U}$ in $\mathcal{G}$ such that $p \in \mathcal{U}$ with $\mathcal{U}^{-1} \subseteq NM \sim Int\Big(NM \sim Cl(V)\Big)$. Also, there is $m \in \mathcal{A}\bigcap\mathcal{U}$ which implies $m^{-1} \in \mathcal{U}^{-1}\bigcap NM \sim Int\Big(NM \sim Cl(V)\Big)$. That is, $\mathcal{U}^{-1}\bigcap NM \sim Int\Big(NM \sim Cl(V)\Big) \neq 0_{NM} \Rightarrow \mathcal{U}^{-1}\bigcap NM \sim Cl(V) \neq 0_{NM} \Rightarrow \mathcal{U}^{-1}\bigcap V \neq 0_{NM}$, since $\mathcal{U}^{-1}$ is NMOS. Therefore, $q \in NM \sim Cl(\mathcal{U})^{-1}$. Hence $NM \sim Cl(\mathcal{U}^{-1}) \subseteq NM \sim Cl(\mathcal{U})^{-1}$.

Theorem 6.0.19

Let $\mathcal{Q}$ be NMRCoS subset in a NMATG $\mathcal{G}$. Then the following assertions are true:

(a) $NM \sim Int(m\mathcal{Q}) = aNM \sim Int(\mathcal{Q}), \forall\, m \in \mathcal{G}$.

(b) $NM \sim Int(\mathcal{Q}m) = NM \sim Int(\mathcal{Q})a, \forall\, m \in \mathcal{G}$.

(c) $NM \sim Int(\mathcal{Q}^{-1}) = NM \sim Int(\mathcal{Q})^{-1}$.

Proof

(a) Since $\mathcal{Q}$ is NMRCoS, $NM \sim Int(\mathcal{Q})$ is NMROS in $\mathcal{G}$. Consequently, $mNM \sim Int(\mathcal{Q}) \subseteq NM \sim Int(m\mathcal{Q})$. Conversely, let $q \in NM \sim Int(m\mathcal{Q})$ be an arbitrary element. Suppose $q = mp$, for some $p \in \mathcal{Q}$. By hypothesis, this proves $m\mathcal{Q}$ is NMCoS and that is $NM \sim Int(m\mathcal{Q})$ is NMROS in $\mathcal{G}$. Assume that $m \in U$ and $p \in V$ be NMOSs in $\mathcal{G}$, such that $UV \subseteq NM \sim Int(m\mathcal{Q})$.

Then $mV \subseteq m\mathcal{Q}$, which follows that $mV \subseteq mNM \sim Int(\mathcal{Q})$. Thus, $NM \sim Int(m\mathcal{Q}) \subseteq mNM \sim Int(\mathcal{Q})$.

(b) Following the same steps as in part (a) above, we can prove that $NM \sim Int(\mathcal{Q}m) \subseteq NM \sim Int(\mathcal{Q})m$.

(c) Since $NM \sim Int(\mathcal{Q})$ is NMROS, so $NM \sim Int(\mathcal{Q})^{-1}$ is NMOS in $\mathcal{G}$. Therefore, $\mathcal{Q}^{-1} \subseteq NM \sim Int(\mathcal{Q})^{-1}$ implies that $NM \sim Int(\mathcal{Q}^{-1}) \subseteq NM \sim Int(\mathcal{Q})^{-1}$. Next, let q be an arbitrary element of $NM \sim Int(\mathcal{Q})^{-1}$. Then $q = p^{-1}$, for some $p \in NM \sim Int(\mathcal{Q})$. Let $q \in V$ be NMOS in $\mathcal{G}$. Then $\exists$ NMOS U is in $\mathcal{G}$ such that $p \in U$ with $U^{-1} \subseteq NM \sim Cl\Big(NM \sim Int(V)\Big)$. Also, there is $g \in \mathcal{Q} \bigcap U$ which implies $g^{-1} \in \mathcal{Q}^{-1} \bigcap NM \sim Cl\Big(NM \sim Int(V)\Big)$. That is $\mathcal{Q}^{-1} \bigcap NM \sim Cl\Big(NM \sim Int(V)\Big) \neq 0_{NM} \Rightarrow \mathcal{Q}^{-1} \bigcap NM \sim Int(V) \neq 0_{NM} \Rightarrow \mathcal{Q}^{-1} \bigcap V \neq 0_{NM}$, since $\mathcal{Q}^{-1}$ is NMCoS. Hence $NM \sim Int(\mathcal{Q}^{-1}) = NM \sim Int(\mathcal{Q})^{-1}$.

Theorem 6.0.20

Let $\mathcal{U}$ be any NMSOS in a NMATG $\mathcal{G}$. Then

(a) $NM \sim Cl(m\mathcal{U}) \subseteq mNM \sim Cl(\mathcal{U}), \forall\, m \in \mathcal{G}$.

(b) $NM \sim Cl(\mathcal{U}m) \subseteq NM \sim Cl(\mathcal{U})m, \forall\, m \in \mathcal{G}$.

(c) $NM \sim Cl(\mathcal{U}^{-1}) \subseteq NM \sim Cl(\mathcal{U})^{-1}$.

Proof

(a) As $\mathcal{U}$ is NMSOS, $NM \sim Cl(\mathcal{U})$ is NMRCoS. From Theorem 6.0.16, $\mu_{m^{-1}} : (\mathcal{G}, \urcorner_X) \to (\mathcal{G}, \urcorner_X)$ is NMACM. So, $mNM \sim Cl(\mathcal{U})$ is NMCoS. Hence $NM \sim Cl(m\mathcal{U}) \subseteq mNM \sim Cl(\mathcal{U})$.

(b) As $\mathcal{U}$ is NMSOS, $NM \sim Cl(\mathcal{U})$ is NMRCoS. From Theorem 6.0.16, $\lambda_{m^{-1}} : (\mathcal{G}, \urcorner_X) \to (\mathcal{G}, \urcorner_X)$ is NMACM. So, $NM \sim Cl(\mathcal{U})m$ is NMCoS. Thus, $NM \sim Cl(\mathcal{U}m) \subseteq NM \sim Cl(\mathcal{U})m$.

(c) Since $\mathcal{U}$ is NMSOS, so, $NM \sim Cl(\mathcal{U})$ is NMRCoS and hence $NM \sim Cl(\mathcal{U})^{-1}$ is NMCoS. Consequently, $NM \sim Cl(\mathcal{U}) \subseteq NM \sim Cl(\mathcal{U})^{-1}$.

Theorem 6.0.21

Let $\mathcal{U}$ be both NMSOS and NMSCoS subset of a NMATG $\mathcal{G}$. Then the following statements hold:

(a) $NM \sim Cl(m\mathcal{U}) = mNM \sim Cl(\mathcal{U})$, for each $m \in \mathcal{G}$.

(b) $NM \sim Cl(\mathcal{U}m) = NM \sim Cl(\mathcal{U})m$, for each $m \in \mathcal{G}$.

(c) $NM \sim Cl(\mathcal{U}^{-1}) = NM \sim Cl(\mathcal{U})^{-1}$.

Proof

(a) Since $\mathcal{U}$ is NMSOS, $NM \sim Cl(\mathcal{U})$ is NMRCoS, from which it implies that $NM \sim Cl(m\mathcal{U}) \subseteq mNM \sim Cl(\mathcal{U})$. Further, neutrosophic multi semi-openness of $\mathcal{U}$ gives $NM \sim Cl(\mathcal{U}) = NM \sim Cl\Big(NM \sim Int(\mathcal{U})\Big) \Rightarrow mNM \sim Cl(\mathcal{U}) = mNM \sim Cl\Big(NM \sim Int(\mathcal{U})\Big)$. As $\mathcal{U}$ is NMSCoS, $NM \sim Int(\mathcal{U})$ is NMROS in $\mathcal{G}$. From Theorem 6.0.20, $mNM \sim Cl(\mathcal{U}) = mNM \sim Cl\Big(NM \sim Int(\mathcal{U})\Big) = NM \sim Cl\Big(mNM \sim Int(\mathcal{U})\Big) \subseteq NM \sim Cl(m\mathcal{U})$. Hence $NM \sim Cl(m\mathcal{U}) = mNM \sim Cl(\mathcal{U})$.

(b) Following the same steps as in part (a) above, we can prove that $NM \sim Cl(\mathcal{U}m) = NM \sim Cl(\mathcal{U})m$.

(c) By hypothesis, this proves $NM \sim Cl(\mathcal{U})$ is NMRCoS and therefore $NM \sim Cl(\mathcal{U})^{-1}$ is NMCoS. Consequently, $NM \sim Cl(\mathcal{U}^{-1}) \subseteq NM \sim Cl(\mathcal{U})^{-1}$. Next, since $\mathcal{U}$ is NMSOS, $NM \sim Cl(\mathcal{U}) = NM \sim Cl\Big(NM \sim Int(\mathcal{U})\Big) \Rightarrow NM \sim Cl(\mathcal{U})^{-1} = NM \sim Cl\Big(NM \sim Int(\mathcal{U})\Big)$. Also, as $\mathcal{U}$ is NMSCoS, $NM \sim Int(\mathcal{U})$

is NMROS. From Theorem 6.0.18, $NM \sim Cl(\mathcal{U})^{-1} = NM \sim Cl\Big(NM \sim Int(\mathcal{U})^{-1}\Big) \subseteq NM \sim Cl(\mathcal{U}^{-1})$. This shows that $NM \sim Cl(\mathcal{U}^{-1}) = NM \sim Cl(\mathcal{U})^{-1}$.

Corollary 6.0.2

From Theorem 6.0.21, the following statements are hold:

(a) $NM \sim Int(m\mathcal{U}) = mNM \sim Int(\mathcal{U})$*, for each* $m \in \mathcal{G}$.

(b) $NM \sim Int(\mathcal{U}m) = NM \sim Int(\mathcal{U})m$*, for each* $m \in \mathcal{G}$.

(c) $NM \sim Int(\mathcal{U}^{-1}) = NM \sim Int(\mathcal{U})^{-1}$.

Proof

(a) As $\mathcal{U}$ is NMSCoS, $NM \sim Int(\mathcal{U})$ is NMROS. From Theorem 6.0.16, $\mu_{m^{-1}} : (\mathcal{G}, \urcorner_X) \to (\mathcal{G}, \urcorner_X)$ is NMACM. So, ${\mu^{-1}}_{m^{-1}}\Big(NM \sim Int(\mathcal{U})\Big) = mNM \sim Int(\mathcal{U})$ is NMOS. Thus, $mNM \sim Int(\mathcal{U}) \subseteq NM \sim Int(m\mathcal{U})$. Next, by assumption, it implies that $NM \sim Int(\mathcal{U}) = R\ NM \sim Int\Big(NM \sim Cl(\mathcal{U})\Big) \Rightarrow mNM \sim Int(\mathcal{U}) = mNM \sim Int\Big(NM \sim Cl(\mathcal{U})\Big)$. As $\mathcal{U}$ is NMSOS, $NM \sim Cl(\mathcal{U})$ is NMRCoS. From Theorem 6.0.19, $mNM \sim Int\Big(NM \sim Cl(\mathcal{U})\Big) = NM \sim Int\Big(mNM \sim Cl(\mathcal{U})\Big) \supseteq NM \sim Int(m\mathcal{U})$. That is, $NM \sim Int(m\mathcal{U}) \subseteq mNM \sim Int(\mathcal{U})$. Therefore, we have, $NM \sim Int(m\mathcal{U}) = mNM \sim Int(\mathcal{U})$. Hence proved.

(b) As $\mathcal{U}$ is NMSCoS, $NM \sim Int(\mathcal{U})$ is NMROS. From Theorem 6.0.16, $\mu_{m^{-1}} : (\mathcal{G}, \urcorner_X) \to (\mathcal{G}, \urcorner_X)$ is NMACM. So, ${\lambda^{-1}}_{m^{-1}}\Big(NM \sim Int(\mathcal{U})\Big) = mNM \sim Int(\mathcal{U})$ is NMOS. Thus, $NM \sim Int(\mathcal{U})m \subseteq NM \sim Int(\mathcal{U}m)$. Next, by assumption, this proves that $NM \sim Int(\mathcal{U}) = NM \sim Int\Big(NM \sim Cl(\mathcal{U})\Big) \Rightarrow NM \sim$

$Int(\mathcal{U})m = NM \sim Int\Big(NM \sim Cl(\mathcal{U})\Big)m$. As $\mathcal{U}$ is NMSOS, $NM \sim Cl(\mathcal{U})$ is NMRCoS. From Theorem 6.0.19, $NM \sim Int\Big(NM \sim Cl(\mathcal{U})\Big)m = NM \sim Int\Big(NM \sim Cl(\mathcal{U})m\Big) \supseteq NM \sim Int(\mathcal{U}m)$. That is, $NM \sim Int(\mathcal{U}m) \subseteq NM \sim Int(\mathcal{U})m$. Therefore, $NM \sim Int(\mathcal{U}m) = NM \sim Int(\mathcal{U})m$. Hence proved.

(c) From assumption, this proves that $NM \sim Int(\mathcal{U})$ is NMROS and therefore $NM \sim Int(\mathcal{U})^{-1}$ is NMOS. Consequently, $NM \sim Int(\mathcal{U}^{-1}) \subseteq NM \sim Int(\mathcal{U})^{-1}$. Next, as $\mathcal{U}$ is NMSCoS, $NM \sim Int(\mathcal{U}) = NM \sim Int\Big(NM \sim Cl(\mathcal{U})\Big) \Rightarrow NM \sim Int(\mathcal{U})^{-1} = NM \sim Int\Big(NM \sim Cl(\mathcal{U})\Big)^{-1}$. Also, as $\mathcal{U}$ is NMSOS, $NM \sim Cl(\mathcal{U})$ is NMRCoS. From Theorem 6.0.19, $NM \sim Int(\mathcal{U})^{-1} = NM \sim Int\Big(NM \sim Cl(\mathcal{U})^{-1}\Big) \subseteq NM \sim Int(\mathcal{U}^{-1})$. This proves that $NM \sim Int(\mathcal{U}^{-1}) = NM \sim Int(\mathcal{U})^{-1}$.

Theorem 6.0.22

Let $\mathcal{A}$ be NMOS in a NMATG $\mathcal{G}$. Then $a\mathcal{A} \subseteq NM \sim Int\Big(aNM \sim Int\Big(NM \sim Cl(\mathcal{A})\Big)\Big)$ for $a \in \mathcal{G}$.

Proof

Since $\mathcal{A}$ is NMOS, so $\mathcal{A} \subseteq NM \sim Int\Big(NM \sim Cl(\mathcal{A})\Big) \Rightarrow a\mathcal{A} \subseteq aNM \sim Int\Big(NM \sim Cl(\mathcal{A})\Big)$. From Theorem 6.0.17, $aNM \sim Int\Big(NM \sim Cl(\mathcal{A})\Big)$ is NMOS $\Big($in fact, NMROS$\Big)$. Hence $a\mathcal{A} \subseteq NM \sim Int\Big(aNM \sim Int\Big(NM \sim Cl(\mathcal{A})\Big)\Big)$.

Theorem 6.0.23

Let $\mathcal{Q}$ be any neutrosophic multi closed subset in a NMATG $\mathcal{G}$. Then $NM \sim Cl\Big(aNM \sim Cl\Big(NM \sim Int(\mathcal{A})\Big)\Big) \subseteq a\mathcal{Q}$ for each $a \in \mathcal{G}$.

Proof

Since $\mathcal{Q}$ is NMCoS, so $\mathcal{Q} \supseteq NM \sim Cl\big(NM \sim Int(\mathcal{Q})\big) \Rightarrow a\mathcal{Q} \supseteq$

$aNM \sim Cl\big(NM \sim Int(\mathcal{Q})\big)$. From Theorem 6.0.17, $aNM \sim Cl\Big(NM \sim Int(\mathcal{Q})\Big)$ is NMCoS $\Big($in fact, NMRCoS$\Big)$. Therefore, $a\mathcal{Q} \supseteq NM \sim Cl\Big(aNM \sim Cl\Big(NM \sim Int(\mathcal{A}\Big)\Big)$. Hence $NM \sim Cl\Big(aNM \sim Cl\Big(NM \sim Int(\mathcal{A})\Big)\Big) \subseteq a\mathcal{Q}$.