*Chapter 6*

# Five Dimensional FRW Radiating Models in Presence of Bulk Viscous Cosmological Models in Scalar-Tensor Theory of Gravitation

## 6.1 Introduction

Due to the large scale dissemination of galaxies in our universe the matter dissemination is properly illustrated by a perfect fluid. The situation of material dissemination other than perfect fluid is needed for a practical and realistic treatment of the problem. Misner (Misner, 1968) defined that the matter behaves as a viscous fluid in a beginning stages of the universe when neutrino decoupling occurs. Riess et al. (Reiss and et al., 1998) and Perlmutter et al. (Perlmutter and et al., 1999) have investigated that in getting the recent scenario of accelerated extension of the universe function of bulk viscosity plays a vital role in cosmology which is popularly known as the inflationary phase.

The work presented in this chapter has been published in "Trends in Sciences", volume 19(20), pp.1-10, (2022)

## 6.2 Metric and field equations

The five-dimensional FRW metric is assumed in the following form

$$ds^2 = dt^2 - a^2(t)[\frac{dr^2}{(1-kr^2)} + r^2(d\theta^2 + \sin^2\theta d\phi^2) + (1-kr^2)d\psi^2] \tag{6.1}$$

Sáez-Ballester's field equations for coupled scalar and tensor fields are

$$R_{ij} - \frac{1}{2}g_{ij}R - \omega\phi^n(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,k}\phi^{,k}) = -8\pi T_{ij} \tag{6.2}$$

where the scalar field $\phi$ fulfils the equation

$$2\phi^n\phi^{;i}_{;i} + n\phi^{n-1}\phi_{,k}\phi^{,k} = 0 \tag{6.3}$$

and

$$T^{ij}_{;j} = 0 \tag{6.4}$$

Equation (6.4) follows from field equations (6.2) and (6.3), where $\omega$ and *n* are constants. The semicolon and comma express partial and covariant differentiation respectively.

We can derive the non-vanishing components of the Einstein tensor from equation (6.1) as

$$G^1_1 = G^2_2 = G^3_3 = G^4_4 = 3\frac{\ddot{a}^2}{a} + 3\frac{\dot{a}^2}{a^2} + 3\frac{k}{a^2} \tag{6.5}$$

and

$$G^5_5 = 6\frac{\dot{a}^2}{a^2} + 6\frac{k}{a^2} \tag{6.6}$$

A dot symbol here denotes differentiation with respect to time *t*. Also, $k = +1, -1, 0$ denotes closed, open and flat models.

The energy momentum tensor for bulk viscous fluid is given by in the following way

$$T_{ij} = (\rho + \bar{p})u_iu_j - g_{ij}\bar{p} \quad where \quad i,j = 1,2,3,4,5 \tag{6.7}$$

along with,

$$u_i u_i = 1 \quad and \quad u_i u_j = 0 \tag{6.8}$$

Therefore,

$$T = \rho - 4\bar{p} \tag{6.9}$$

The total pressure with the proper pressure, which includes the coefficient of bulk viscosity $\zeta$ and the Hubble's expansion parameter *H* is defined as follows

$$\bar{p} = p - 3\zeta H = \varepsilon\rho, \;\; where \;\; p = \varepsilon_0\rho \;\; and \;\; \varepsilon = \varepsilon_0 - \beta \tag{6.10}$$

Using co-moving coordinates and equations (6.4) and (6.5)-(6.9), the S$\acute{a}$ez-Ballester field equations (6.2) and (6.3) for the metric (6.1) becomes

$$6\frac{\dot{a}^2}{a^2} + 6\frac{k}{a^2} - \frac{\omega}{2}\frac{\dot{\phi}^2}{\phi^2} = -8\pi\rho \tag{6.11}$$

$$3\frac{\ddot{a}}{a} + 3\frac{\dot{a}2}{a^2} + 3\frac{k}{a^2} + \frac{\omega}{2}\frac{\dot{\phi}2}{\phi^2} = 8\pi\bar{p} \tag{6.12}$$

$$\frac{\ddot{\phi}}{\phi} + 4\frac{\dot{a}}{a}\frac{\dot{\phi}}{\phi} + \frac{n}{2}\frac{\dot{\phi}^2}{\phi^2} = 0 \tag{6.13}$$

$$\dot{\rho} + 4\frac{\dot{a}}{a}(\rho + \bar{p}) = 0 \tag{6.14}$$

The Hubble's parameter *H* is

$$H = \frac{\dot{a}}{a} \tag{6.15}$$

and the deceleration parameter *q* is

$$q = -\frac{(\dot{H} + H^2)}{H^2} \tag{6.16}$$

## 6.3 Solution of the field equations

We use the condition given by equation (6.10) in this chapter. We also use the relationship provided by Pimental (Pimentel, 1985); Johri and Kalyani (Johri and Kalyani, 1994) between the scalar field $\phi$ and the scale factor *a(t)* of the universe.

$$\phi = \phi_0 a^n \tag{6.17}$$

where $\phi_0$ and *n* $> 0$ are constants.

We solve the field equations (6.11)-(6.13) in the following physically significant cases.

### 6.3.1 For k=1, (i.e. Closed model)

Using equation (6.17) in the field equations (6.11)-(6.13), we obtain the scale factor as given below

$$a(t) = [(\frac{(\frac{n^2}{2} + n + 4)}{n\phi_0^{(\frac{n+2}{2})}})(a_0 t + t_0)]^{\frac{1}{(\frac{n^2}{2}+n+4)}} \tag{6.18}$$

Now using (6.18) and by choosing $a_0 = 1$ and $t_0 = 0$, the metric (6.1) becomes

$$ds^2 = dt^2 - [(\frac{(\frac{n^2}{2} + n + 4)}{n\phi_0^{(\frac{n+2}{2})}})t]^{\frac{2}{(\frac{n^2}{2}+n+4)}} [\frac{dr^2}{(1-r^2)} + r^2(d\theta^2 + \sin^2\theta d\phi^2) + (1-r^2)d\psi^2] \tag{6.19}$$

and the scalar field is

$$\phi = \phi_0[(\frac{(\frac{n^2}{2} + n + 4)}{n\phi_0^{(\frac{n+2}{2})}})t]^{\frac{n}{(\frac{n^2}{2}+n+4)}} \tag{6.20}$$

The equation (6.19) represents a five-dimensional FRW bulk viscous radiating model.

The spatial volume is defined as

$$V = a^4 == [(\frac{(\frac{n^2}{2} + n + 4)}{n\phi_0^{(\frac{n+2}{2})}})t]^{\frac{4}{(\frac{n^2}{2}+n+4)}} \tag{6.21}$$

Hubble's parameter *H* is

$$H = (\frac{1}{\frac{n^2}{2}+n+4})\frac{1}{t} \quad (6.22)$$

The energy density $\rho$ is

$$8\pi\rho = [\frac{(\omega n^2-12)}{2t^2(\frac{n^2}{2}+n+4)^2}] - 6[(\frac{\frac{n^2}{2}+n+4}{n\phi_0^{(\frac{n+2}{2})}})t]^{-\frac{2}{(\frac{n^2}{2}+n+4)}} \quad (6.23)$$

The isotropic pressure *p* is

$$8\pi p = \varepsilon_0[\frac{(\omega n^2-12)}{2t^2(\frac{n^2}{2}+n+4)^2}] - 6[(\frac{\frac{n^2}{2}+n+4}{n\phi_0^{(\frac{n+2}{2})}})t]^{-\frac{2}{(\frac{n^2}{2}+n+4)}} \quad (6.24)$$

The coefficient of bulk $\zeta$ viscosity is

$$8\pi\zeta = [\frac{(\frac{n^2}{2}+n+4)(\varepsilon-\varepsilon_0)t}{3}][(\frac{(\omega n^2-12)}{2t^2(\frac{n^2}{2}+n+4)^2}) - 6((\frac{\frac{n^2}{2}+n+4}{n\phi_0^{(\frac{n+2}{2})}})t)^{-\frac{2}{(\frac{n^2}{2}+n+4)}}] \quad (6.25)$$

### 6.3.2 For k=-1, (i.e. Open model)

The model is open in this case and is provided by

$$ds^2 = dt^2 - [(\frac{(\frac{n^2}{2}+n+4)}{n\phi_0^{(\frac{n+2}{2})}})t]^{\frac{2}{(\frac{n^2}{2}+n+4)}}[\frac{dr^2}{(1+r^2)} + r^2(d\theta^2+\sin^2\theta d\phi^2) + (1+r^2)d\psi^2] \quad (6.26)$$

along with the energy density $\rho$

$$8\pi\rho = [\frac{(\omega n^2-12)}{2t^2(\frac{n^2}{2}+n+4)^2}] + 6[(\frac{\frac{n^2}{2}+n+4}{n\phi_0^{(\frac{n+2}{2})}})t]^{-\frac{2}{(\frac{n^2}{2}+n+4)}} \quad (6.27)$$

The isotropic pressure *p*

$$8\pi p = \varepsilon_0[\frac{(\omega n^2-12)}{2t^2(\frac{n^2}{2}+n+4)^2}] + 6[(\frac{\frac{n^2}{2}+n+4}{n\phi_0^{(\frac{n+2}{2})}})t]^{-\frac{2}{(\frac{n^2}{2}+n+4)}} \quad (6.28)$$

and the coefficient of bulk viscosity $\zeta$ is

$$8\pi\zeta = [\frac{(\frac{n^2}{2}+n+4)(\varepsilon-\varepsilon_0)t}{3}][(\frac{(\omega n^2-12)}{2t^2(\frac{n^2}{2}+n+4)^2}) + 6((\frac{\frac{n^2}{2}+n+4}{n\phi_0^{(\frac{n+2}{2})}})t)^{-\frac{2}{(\frac{n^2}{2}+n+4)}}] \quad (6.29)$$

In this case the scalar field, spatial volume and the Hubble's parameter are given by equation (20), (21) and (22) respectively.

### 6.3.3 For k=0, (i.e. Flat model)

The model in this case is flat and is defined as follows

$$ds^2 = dt^2 - [(\frac{(\frac{n^2}{2}+n+4)}{n\phi_0^{(\frac{n+2}{2})}})t]^{\frac{2}{(\frac{n^2}{2}+n+4)}}[dr^2 + r^2(d\theta^2+\sin^2\theta d\phi^2)+d\psi^2] \quad (6.30)$$

and the energy density $\rho$ is

$$8\pi\rho = [\frac{(\omega n^2-12)}{2t^2(\frac{n^2}{2}+n+4)^2}] \quad (6.31)$$

The pressure $p$ is

$$8\pi p = \varepsilon_0[\frac{(\omega n^2-12)}{2t^2(\frac{n^2}{2}+n+4)^2}] \quad (6.32)$$

and the coefficient of bulk viscosity $\zeta$ is

$$8\pi\zeta = [\frac{(\frac{n^2}{2}+n+4)(\varepsilon-\varepsilon_0)t}{3}][\frac{(\omega n^2-12)}{2t^2(\frac{n^2}{2}+n+4)^2}] \quad (6.33)$$

The scalar field, spatial volume and the Hubble's parameter are given by equations (6.20), (6.21) and (6.22).

For all the three cases (i.e. $k=+1,-1,0$), the deceleration parameter $q$ is

$$q = (\frac{n^2}{2}+n+3) \quad (6.34)$$

All of the graphs below were drawn using $\pi = 3.14$, $n = 1$ , $\phi_0 = .001$, $\omega = 500$, $\varepsilon_0 = -1$, $\varepsilon = \frac{1}{3}$.

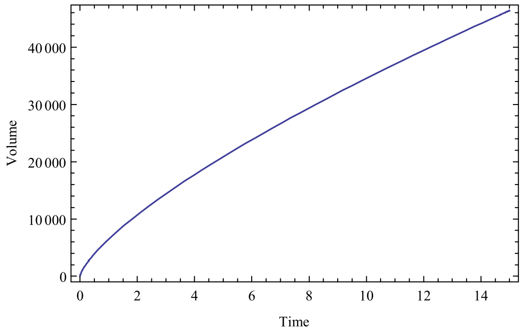


Figure 6.1: $V$ vs. $t$ (billion years)

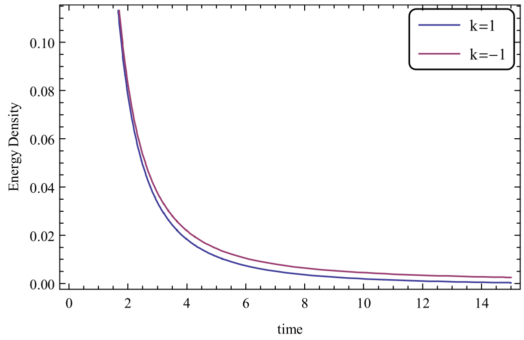


Figure 6.2: $\rho$ for $k = 1, -1$ vs. $t$ (billion years)

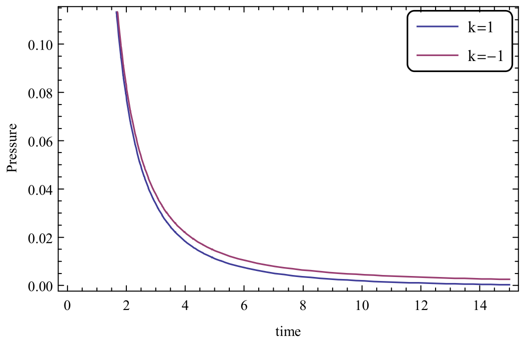


Figure 6.3: $p$ for $k = 1, -1$ vs. $t$ (billion years)

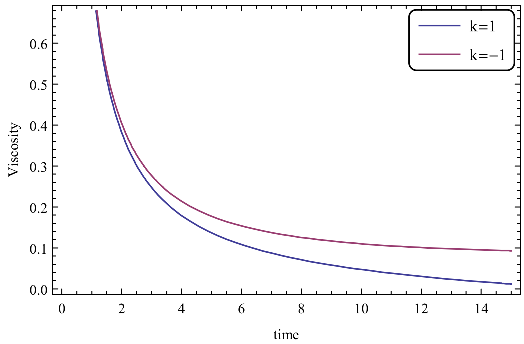


Figure 6.4: $\zeta$ for $k = 1, -1$ vs. $t$ (billion years)

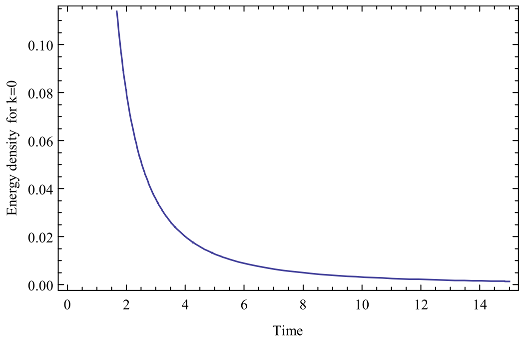


Figure 6.5: $\rho$ for $k = 0$ vs. $t$ (billion years)

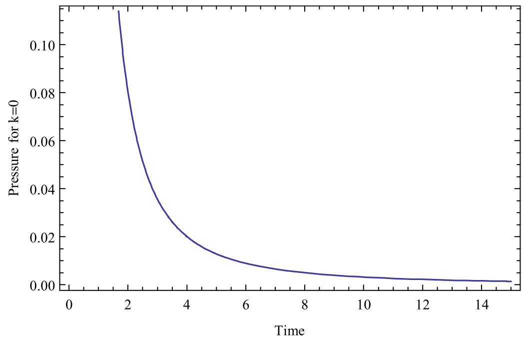


Figure 6.6: $p$ for $k = 0$ vs. $t$ (billion years)

.

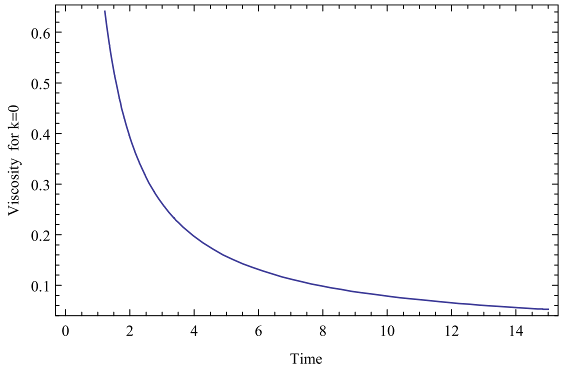


Figure 6.7: $\zeta$ for $k = 0$ vs. $t$ (billion years)

.

## 6.4 Physical and geometrical discussion

In this chapter, the energy density, pressure and the coefficient of bulk viscosity diverges at $t = 0$ and decreases with time in both closed and open models. As $t \to \infty$ they will vanishes (shown in fig-6.2, fig-6.3, fig-6.4). In the flat model, the energy density, pressure and the bulk viscosity decreases with time and eventually vanishes over an infinitely large time *t*(shown in fig-6.5, fig-6.6, and fig-6.7). At the initial epoch, they all diverges. For all the models, the spatial volume increases with time but approaches to infinity for indefinitely large time. Volume is vanishes when $t = 0$ (shown in fig-6.1). Equation (6.22) is the average Hubble's parameter for all the models and it will diverges at the initial epoch. As $t \to \infty$, Hubble's parameter will vanishes. In S$\acute{a}$ez-Ballester theory, equations (6.19), (6.26) and (6.30) denoted the FRW five-dimensional radiating closed, open and flat models respectively. The deceleration parameter for all the models is $q = (\frac{n^2}{2} + n + 3)$. Because the deceleration parameter $q > 0$ for all values of t, the model represents a standard decelerating universe. Despite the fact that the universe decelerates in this case, it will accelerate in finite time due to cosmic re-collapse, where the universe inflates "decelerates and then accelerates" (Nojiri and Odintsov, 2003; Rao et al., 2015a; Reddy and Naidu, 2007; Vishwakarma, 2003).