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List of Publications

1. **Daimary, J.** and Roy Baruah, R.(2022). ***FRW Cosmological Models in presence of a Perfect Fluid within the framework of Sáez-Ballester Theory in five Dimensional Space Time***. Journal of Mathematical and Computational Science, 12:131:1-12.
2. **Daimary, J.** and Roy Baruah, R. (2022). ***Five Dimensional Bianchi Type-I Anisotropic Cloud String Cosmological Model With Electromagnetic Field in Sáez-Ballester Theory***. Frontiers in Astronomy and Space Sciences, 9:1-9.
3. **Daimary, J.** and Roy Baruah, R. (2022). ***Five Dimensional FRW Radiating Cosmological Model in Presence of Bulk Viscous Fluid in Scalar Tensor Theory of Gravitation***. Trends in Sciences, 19(20):1-10.
4. **Daimary, J.** and Roy Baruah, R. (2023). ***Anisotropic LRS Bianchi type-V Cosmological Model with Bulk Viscous String within the Framework of Sáez-Ballester Theory in Five Dimensional Space Time***. Journal of Scientific Research,1-11.(Accepted)

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FRW COSMOLOGICAL MODELS IN PRESENCE OF A PERFECT FLUID WITHIN THE FRAMEWORK OF SAEZ-BALLESTER THEORY IN FIVE DIMENSIONAL SPACE TIME

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Abstract: In the framework of Saez-Ballester scalar-tensor theory of gravity, five-dimensional FRW space-time is considered in the presence of perfect fluid. We employed a power law between the scalar field and the universe's scale factor to get a definite solution to the field equations. Models that radiate flat, closed and open universe are shown. The model's physical features are also discussed.

Keywords: five dimension; FRW cosmological models; perfect fluid; Saez-Ballester theory; radiating models.

2010 AMS Subject Classification: 83C10, 83C15.

1. INTRODUCTION

The study of five-dimensional space-time is crucial because the cosmos may have had a higher-dimensional phase earlier in its existence. The experimental observation of time change of fundamental constants, according to Marciano [1], provides significant evidence for the presence of extra dimension. The extra dimension in space-time contracted or remained invariant to a very

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small size of Planck length. Furthermore, extra dimensions produce a substantial quantity of entropy during the contraction process, providing a different solution to the flatness and horizon problem [2]. Higher-dimensional cosmology drew the attention of Witten [3], Appelquist et al. [4], and Chodos and Detweiler [5] because it has physical significance to the early cosmos before compactification changes. A number of authors have recently investigated higher-dimensional space time using various cosmological models [6-10].

Perfect fluid and viscous fluid are the two most commonly encountered fluids in simulating our cosmos. Perfect fluid scientists refer to fluids whose rest frame pressure and density can be completely described. The pressure of a perfect fluid is isotropic, meaning that it has the same pressure in all directions. There is no shear stress, no conduction, and no viscosity in a perfect fluid. A number of authors [11-15] have investigated the perfect fluid using different cosmological models and different scalar-tensor.

Different cosmological models for the universe have been discovered using Einstein's general theory of relativity. However, in recent years, various modifications to Einstein's theory of gravitation have been made to accommodate certain desirable aspects that the original theory lacked. The field equations, for example, do not properly include Mach's principle into Einstein's theory. Furthermore, the current hypothesis of early inflation and late-time accelerated expansions of the cosmos is unaffected by general theory of relativity. As a result, various modifications to general relativity have been made to integrate the above desirable qualities. Scalar-tensor theories of gravitation presented by Brans and Dicke [16] and Saez and Ballester [17] are noteworthy, as are modified theories of gravity such as Nojiri and Odinstov’s [18] f(R) theory of gravity and Harko et al’s [19] f(R, T) theory of gravity. The construction of cosmological models of the universe to understand the universe's origin, mechanics, and ultimate fate has sparked a lot of interest in recent years. Scientists are increasingly interested in cosmological models based on the Brans-Dicke and Saez-Ballester scalar-tensor theories of gravitation.

The Brans-Dicke theory is a well-known challenger to Einstein's gravitational theory. It is the most basic scalar-tensor theory, in which the gravitational interaction is mediated by a scalar field ϕ as well as Einstein's tensor field g_{ij}. The scalar field ϕ has the same dimension as the global gravitational constant in this theory. Saez-Ballester proposed a scalar-tensor theory in which the metric is simply coupled with a dimensionless scalar field ϕ. Despite the scalar field's dimensionlessness, an anti-gravity regime develops in this theory. In addition, this theory adequately describes weak fields and proposes a solution to the 'missing matter' problem in non-flat FRW cosmologies. Saez-Ballester's field equations for combined scalar and tensor fields are as follows:

$$R_{ij}-\frac{1}{2}g_{ij}R-\omega\phi^{n}\left(\phi_{,i}\phi_{,j}-\frac{1}{2}g_{ij}\phi_{,k}\phi^{,k}\right)=-8\pi T_{ij} \tag{1}$$

where the equation is satisfied by the scalar field ϕ.

$$2\phi^{n}\phi_{;i}^{,i}+n\phi^{n-1}\phi_{,k}\phi^{,k}=0 \tag{2}$$

We also have

$$T_{;j}^{ij}=0 \tag{3}$$

as a result of field equations (1) and (2). The conservation of matter source is physically represented by this equation. In this theory, we can also find equations of motion. The gravitational theory is strengthened by this equation. The constants are $\boldsymbol{\omega}$ and n, and partial and covariant differentiation are denoted by a comma and a semicolon, respectively. In general relativity, the other symbols have their usual meaning.

Some of the authors who have explored various parts of the Saez-Ballester theory in four and five dimensional space include Singh and Agrawal [20], Reddy and Rao [21], Reddy et al. [22], Mohanty and Sahoo [23], Adhav et al. [24], and Tripathy et al. [25]. Recently many researchers [26-32] have investigated Saez-Balester scalar-tensor theory using different cosmological models. We examine five-dimensional FRW models in Saez-Ballester theory of gravity as a result of the preceding investigations. The second section contains metric and field equations. We consider cosmological models in Saez-Ballester theory with an equation of state akin to disordered radiation in general relativity in Sect. 3. The physical explanation of the models is covered in Section 4, and the final section offers some conclusions.

2. METRIC AND FIELD EQUATIONS

In this paper, we look at the FRW five-dimensional space-time metric in the form

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1-mr^2} + r^2(d\Theta^2 + \sin^2\Theta\, d\phi^2) + (1-mr^2)d\psi^2\right] \tag{4}$$

The Einstein tensor's non-vanishing components for the metric (4) are

$$G_1^1 = G_2^2 = G_3^3 = G_4^4 = 3\frac{\ddot{a}}{a} + 3\frac{\dot{a}^2}{a^2} + 3\frac{m}{a^2} \tag{5}$$

$$G_5^5 = 6\frac{\dot{a}^2}{a^2} + 6\frac{m}{a^2} \tag{6}$$

where an overhead dot denotes ordinary differentiation with respect to t and $m = 1, -1, 0$ for closed, open, and flat models, respectively.

For a perfect fluid distribution, the energy momentum tensor is given by

$$T_j^i = (\rho + p)u^i u_j - g_{ij}p \tag{7}$$

where,

$$u^i u_i = 1, \qquad u^i u_j = 0 \tag{8}$$

The field equations (1)–(3) can be expressed as with the help of equations (5) – (8) for the metric (4) using co moving coordinates

$$6\frac{\dot{a}^2}{a^2} + 6\frac{m}{a^2} - \frac{\omega}{2}\frac{\dot{\phi}^2}{\phi^2} = -8\pi\rho \tag{9}$$

$$3\frac{\ddot{a}}{a} + 3\frac{\dot{a}^2}{a^2} + 3\frac{m}{a^2} + \frac{\omega}{2}\frac{\dot{\phi}^2}{\phi^2} = 8\pi p \tag{10}$$

$$\frac{\ddot{\phi}}{\phi} + 4\frac{\dot{a}}{a}\frac{\dot{\phi}}{\phi} + \frac{n}{2}\frac{\dot{\phi}^2}{\phi^2} = 0 \tag{11}$$

$$\dot{\rho} + 4\frac{\dot{a}}{a}(\rho + p) = 0 \tag{12}$$

Hubble parameter H and deceleration parameter q are the physical parameters of importance for the observation

$$H = \frac{\dot{a}}{a} \tag{13}$$

$$q = \frac{-\left(\dot{H} + H^2\right)}{H^2} \tag{14}$$

3. SOLUTION OF THE FIELD EQUATIONS

The field equations (9)–(11) are three independent equations with four unknowns: a, p, ρ and ϕ [Eq. (12) is the result of Eqs. (9) - (11)]. As a result, an additional condition is required to obtain a definite answer. So, we employ the well-known relationship between a scalar field and the universe's scale factor *a(t)* given by [33]

$$\phi = \phi_0 a^n \tag{15}$$

Here $n > 0$ and ϕ_0 are constants.

In this particular case, we obtain the following physically significant models

3.1. CASE (I): For m=1 (i.e., closed model)

In this situation, using Eq. (15) and the field equations (9)–(11), the scale factor solutions are as follows

$$a(t) = \left[\left(\frac{\frac{n^2}{2} + n + 4}{n\phi_0^{\frac{n+2}{2}}}\right)(a_0 t + t_0)\right]^{\frac{1}{\left(\frac{n^2}{2}+n+4\right)}} \tag{16}$$

We can now write the metric (4) with the help of (16) as (i.e. we choose $a_0 = 1, t_0 = 0$) with the right choice of coordinates and constants.

$$ds^2 = dt^2 - \left[\left(\frac{\frac{n^2}{2} + n + 4}{n\phi_0^{\frac{n+2}{2}}}\right)t\right]^{\frac{2}{\left(\frac{n^2}{2}+n+4\right)}} \left[\frac{dr^2}{1 - r^2} + r^2(d\Theta^2 + sin^2\,\Theta\, d\phi^2) + (1 - r^2)d\psi^2\right] \tag{17}$$

In the model, together with the scalar field, given by

$$\phi = \phi_0 \left[\left(\frac{\frac{n^2}{2} + n + 4}{n\phi_0^{\frac{n+2}{2}}} \right) t \right]^{\frac{n}{\left(\frac{n^2}{2}+n+4\right)}} \tag{18}$$

The model (17) is a five-dimensional FRW radiating model with the physical parameters Volume *V*, Hubble parameter *H*, Energy Density ρ, and Isotropic Pressure *p*, all of which are relevant in cosmology discussions.

$$V = a^4 = \left[\left(\frac{\frac{n^2}{2} + n + 4}{n\phi_0^{\frac{n+2}{2}}} \right) t \right]^{\frac{4}{\left(\frac{n^2}{2}+n+4\right)}} \tag{19}$$

$$H = \left(\frac{1}{\frac{n^2}{2} + n + 4} \right) \frac{1}{t} \tag{20}$$

$$\rho = \frac{1}{8\pi} \left[\frac{(\omega n^2 - 12)}{2t^2 \left(\frac{n^2}{2} + n + 4 \right)^2} \right] - 6 \left[\left(\frac{\frac{n^2}{2} + n + 4}{n\phi_0^{\frac{n+2}{2}}} t \right) \right]^{-\frac{2}{\left(\frac{n^2}{2}+n+4\right)}} \tag{21}$$

$$p = \frac{1}{8\pi} \left[\frac{(\omega n^2 + 6)}{2t^2 \left(\frac{n^2}{2} + n + 4 \right)^2} + 3 \left(\frac{\frac{n^2}{2} + n + 4}{n\phi_0^{\frac{n+2}{2}}} t \right)^{-\frac{2}{\left(\frac{n^2}{2}+n+4\right)}} - 3 \left(\frac{\frac{n^2}{2} + n + 3}{\frac{n^2}{2} + n + 4} \right) \right] \tag{22}$$

3.2. CASE (II): For m=-1 (i.e., open model)

In this situation, the model is

$$ds^2 = dt^2 - \left[\left(\frac{\frac{n^2}{2}+n+4}{n\phi_0^{\frac{n+2}{2}}}\right)t\right]^{\frac{2}{\left(\frac{n^2}{2}+n+4\right)}} \left[\frac{dr^2}{1+r^2} + r^2(d\Theta^2 + sin^2\Theta\, d\phi^2) + (1+r^2)d\psi^2\right] \tag{23}$$

Here the energy density is given by

$$\rho = \frac{1}{8\pi}\left[\frac{(\omega n^2 - 12)}{2t^2\left(\frac{n^2}{2}+n+4\right)^2}\right] + 6\left[\left(\frac{\frac{n^2}{2}+n+4}{n\phi_0^{\frac{n+2}{2}}}t\right)\right]^{-\frac{2}{\left(\frac{n^2}{2}+n+4\right)}} \tag{24}$$

and the pressure is given by

$$p = \frac{1}{8\pi}\left[\frac{(\omega n^2+6)}{2t^2\left(\frac{n^2}{2}+n+4\right)^2} - 3\left(\frac{\frac{n^2}{2}+n+4}{n\phi_0^{\frac{n+2}{2}}}t\right)^{-\frac{2}{\left(\frac{n^2}{2}+n+4\right)}} - 3\left(\frac{\frac{n^2}{2}+n+3}{\frac{n^2}{2}+n+4}\right)\right] \tag{25}$$

3.3. CASE (III): For m=0 (i.e., flat model)

In this situation, the model is transformed

$$ds^2 = dt^2 - \left[\left(\frac{\frac{n^2}{2}+n+4}{n\phi_0^{\frac{n+2}{2}}}\right)t\right]^{\frac{2}{\left(\frac{n^2}{2}+n+4\right)}} [dr^2 + r^2(d\Theta^2 + sin^2\Theta\, d\phi^2) + d\psi^2] \tag{26}$$

In this model, the energy density is

$$\rho = \frac{1}{8\pi}\left[\frac{(\omega n^2 - 12)}{2t^2\left(\frac{n^2}{2}+n+4\right)^2}\right] \tag{27}$$

The model's pressure is

$$p = \frac{1}{8\pi}\left[\frac{(\omega n^2 + 6)}{2t^2\left(\frac{n^2}{2} + n + 4\right)^2} - 3\left(\frac{\frac{n^2}{2} + n + 3}{\frac{n^2}{2} + n + 4}\right)\right] \tag{28}$$

For all the Cases (i.e. closed, open, flat), the scalar field ϕ, volume V and the Hubble parameter H are same which are given by equation (18), (19) and (20) respectively.

Also the deceleration parameter q is same for all the cases, which is define by as follows

$$q = \left(\frac{n^2}{2} + n + 3\right) \tag{29}$$

When $q > 0$, the cosmos decelerates in the usual fashion, and when $q < 0$, the universe accelerates. The models decelerate in the usual way here.

Here, $\pi = 3.14,\ n = 1,\ \omega = 500,\ \phi_0 = .001$ are used to draw all graphs.

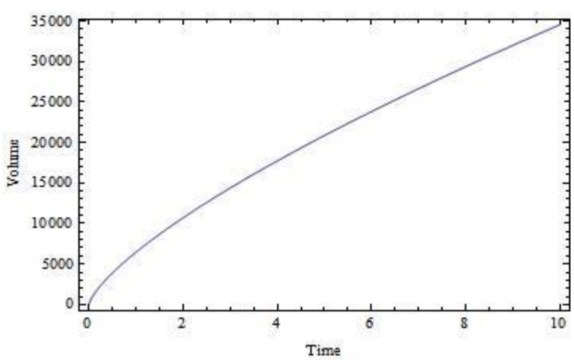

Fig-1. Volume V vs. time

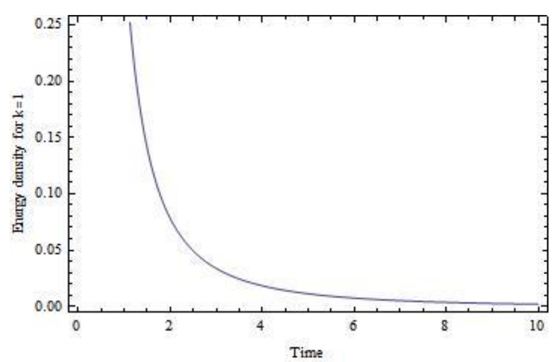

Fig-2. Density ρ vs. time for k=1

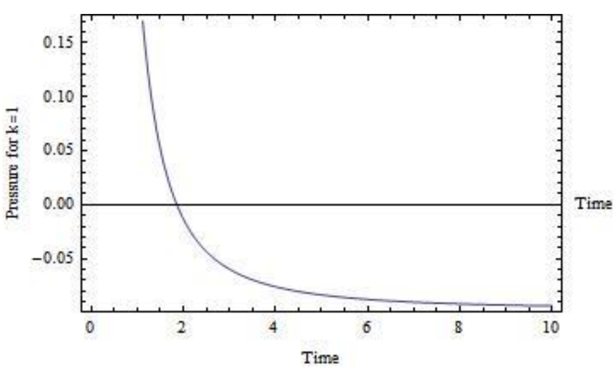

Fig-3. Pressure p vs. time for k=1

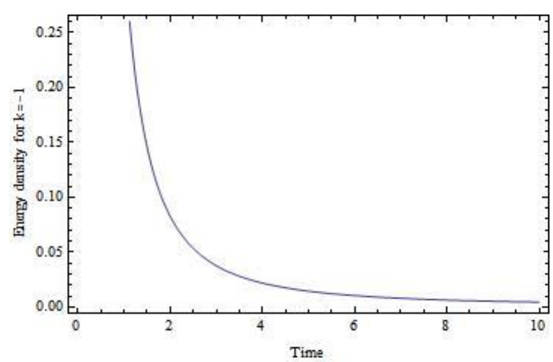

Fig-4. Density ρ vs. time for k= -1

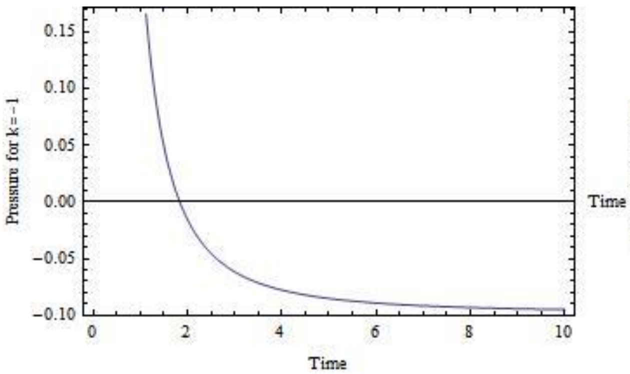

Fig-5. Pressure p vs. time for k= -1

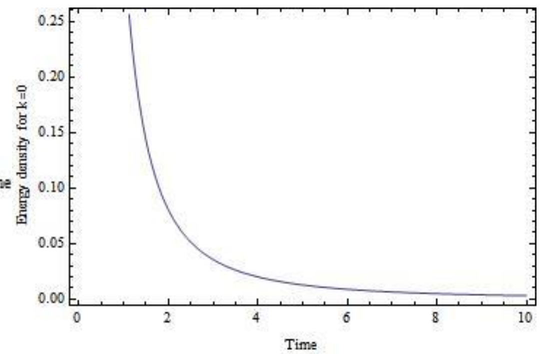

Fig-6. Density ρ vs. time for k= 0

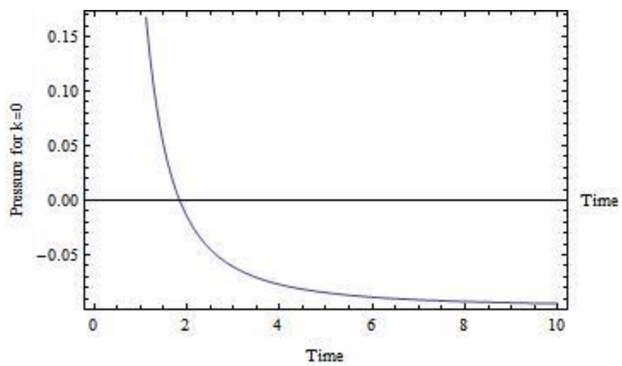

Fig-7, Pressure p vs. time for k= 0

4. PHYSICAL AND GEOMETRICAL INTERPRETATION

In Saez-Ballester theory, Equations (17), (23) and (26) represent FRW five-dimensional radiating closed, open, and flat models. It is worth noting that none of the models have an initial singularity. The pressure and energy density of closed and open models diverge at t = 0 and decrease over time [As shown in fig-2, fig-3, fig-4 and fig-5]. The energy density and pressure in the flat model diminish with time and eventually vanish for infinitely large t [As shown in fig-6 and fig-7]. They also diverge in the first epoch. All of the models have the same spatial volume, which grows with time and goes to infinity after an infinitely long period of time [As shown in fig-1]. For all models, the average Hubble's parameter, provided by Eq. (20), will diverge at the start epoch, i.e. at t = 0, and will approach infinity as time t gets indefinitely big. The models will aid in our understanding of spatially homogeneous isotropic universes in five dimensions immediately prior to compactification. In all of the models, the scalar field grows with time. In each Case, the deceleration parameter is $q = \left(\frac{n^2}{2} + n + 3\right)$, indicating that the models in five dimensions decelerate in a typical manner. The FRW five-dimensional models obtained here are significantly distinct from the Kaluza–Klein five-dimensional models obtained by various authors previously stated and are quite interesting.

5. CONCLUSION

In this research, cosmological models corresponding to perfect fluid dispersion with trace free matter source were constructed using the Saez-Ballester theory of gravitation. In five dimensions, the models obtained depict closed, open, and flat FRW radiating perfect fluid types. All physical quantities diverge at the beginning of the epoch and vanish for infinitely large values of cosmic time. The models will aid in our understanding of spatially homogeneous and isotropic universes in five dimensions just prior to compactification.

CONFLICT OF INTERESTS

The authors (s) declare that there is no conflict of interests.

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ORIGINAL RESEARCH
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Five Dimensional Bianchi Type-I Anisotropic Cloud String Cosmological Model With Electromagnetic Field in Saez-Ballester Theory

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Within the context of Saez-Ballester theory, we explored the interaction of a five-dimensional Bianchi type-I anisotropic cloud string cosmological model Universe with an electromagnetic field. With an electromagnetic field, the energy momentum tensor is assumed to be the sum of the rest energy density and string tension density in this paper. We use the average scale factor as an integrating function of time to get exact answers to Saez-Ballester equations. The dynamics and importance of the model's many physical parameters are also examined.

Keywords: five dimensions, Saez-Ballester theory, perfect fluid, electromagnetic field, Bianchi type-I, cloud string

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1 INTRODUCTION

Many authors have been attracted by the possibility of space-time having more than four dimensions. This has piqued the curiosity of cosmologists and theoretical physicists to the point where, in recent decades, there has been a trend among authors to investigate cosmology in higher-dimensional space-time. In an attempt to combine gravity and electromagnetism, the higher-dimensional model was introduced in (Kaluza, 1921; Klein, 1926). The late time expedited expanding paradigm can be illustrated using a higher dimensional model (Banik and Bhuyan, 2017). Investigation of higher dimensional space-time is a critical undertaking, as the cosmos may have passed through a higher dimensional epoch during its early history (Singh et al., 2004). Extra dimensions generate a large quantity of entropy, according to (Guth, 1981; Alvax and Gavela, 1983), which may provide a solution to the flatness and horizon problems. The discovery of time-varying fundamental constants can perhaps provide us the proof for extra dimensions, according to (Marciano, 1984). (Shinkai and Torii, 2015; Singh and Singh, 2019; Montefalcone et al., 2020; Daimary and Baruah, 2021; Singh and Baro, 2021) are a few of the noteworthy studies on higher dimensional space-time published in the previous few years.

Our Universe is growing at an accelerated rate, according to various types of literature. The Saez-Ballester theory of gravitation is regarded a perfect explanation to describe our expanding Universe in FRW space-time. Bull et al. (Bull et al., 2016) researched alternative cosmology and summarised the current position of ΛCDM as a physical theory in addition to the standard model ΛCDM in extending cosmology. Only an isotropic and homogeneous Universe is described by FRW space-time on a huge scale. However, recent discoveries and reasoning show that throughout the cosmic expansion of the Universe, an anisotropic phase exists before it transitions to an isotropic one. The homogeneous and anisotropic universes are represented by Bianchi type cosmological models, and their isotropy nature can be examined across time. In addition, anisotropic worlds have more generality than isotropic model universes from a theoretical standpoint. Several publications (Singh et al., 2021; Amirhashchi et al., 2009; Akarsu and Kilinc, 2010b,a; Sahoo and Mishra, 2015) looked at the anisotropic Bianchi type cosmological model from various angles. Kibble (Kibble, 1976) identified the stable topological flaws that occurred throughout the phase shift as strings in his study "Topology of Cosmic Domains and Strings." He also shown that the homogeneity group of the manifold of degenerate vacua affects the topological defects of domain structure. Letelier (Letelier, 1983) used the Bianchi type I and Kantowski-Sachs space-time cosmological models to research String Cosmology. The Bianchi type VII models, according to Collins and Hawking (Collins and Hawking, 1972), are the most generic uniform cosmological models that are infinite in all three spatial directions. These models include wide class initial conditions, allowing for the largest number of arbitrary constants imaginable. He also demonstrated that while all initial anisotropic universes do not attain isotropy, a subclass of universes with escape velocity does.

Because of the importance of string in characterising the early stages of our Universe's evolution, several authors have recently focused on string cosmological models. The string can describe the nature and fundamental configuration of the early cosmos at the same time. The most actively explored approach to quantum gravity is string theory, and it can be used to discuss the mechanics of the early cosmos. String theory unifies all matter and forces into a single theoretical structure and describes the early stages of our cosmos in terms of vibrating strings rather than particles. According to Kibble (Kibble, 1976), cosmic strings are stable line-like topological objects/defects that arise at some point during the phase transition in our Universe's early history. According to GUT (grand unified theories) [Zel'dovich et al. (Zel'dovich et al., 1974), Kibble (Kibble, 1976; Kibble, 1980), Everett (Everett, 1981), Vilenkin (Vilenkin, 1981a; Vilenkin, 1981b)], symmetry is broken during the phase transition in the early stages of the Universe after the big bang, and these strings appear when the cosmic temperature drops below a critical temperature. Strings can thus play an important part in studying the early stages of the Universe. Massive closed loops of strings also produce huge scale structures like galaxies and clusters of galaxies. The gravitational field couples with the cosmic strings, which may include stress-energy. As a result, one of the most exciting projects is the investigation of gravitational effects coming from cosmic strings.

Letelier (Letelier, 1983) succeeded in obtaining enormous string cosmological models in Bianchi type-I and Kantowski-Sachs space-times in 1983. Following Letelier, a slew of authors investigated string cosmological models in a variety of settings.

Krori (Krori et al., 1990) and Wang (Wang, 2003) explored the Letelier string cosmological model and obtained their exact solutions using Bianchi type-II, -VI0, -VIII, and -IX space-times. By taking the coefficient of bulk viscosity as a power function of energy density, Xing (Xing-Xiang, 2004) developed an exact solution string cosmological model with bulk viscosity using the LRS Bianchi type-I metric. Yavuz (Yavuz et al., 2005) investigated charged weird quark matter linked to the string cloud in spherical symmetric space-time and demonstrated the existence of a one-parameter group of conformal motions. In the setting of general relativity, Yilmaz (Yilmaz, 2006) established the Kaluza-Klein cosmological solutions for quark matter connected to the string cloud. Rao (Rao et al., 2008) established an exact perfect fluid cosmological model based on Lyra manifold where the displacement vector is a constant while investigating a Bianchi type-V space-time in a scalar-tensor theory. However, if β is a function of cosmic time t, then this model occurs solely in the case of radiation. In Saez-Ballester Scalar-Tensor theory, Tripathy (Tripathy et al., 2009) investigated an anisotropic and spatially homogeneous Bianchi type-VI0 space-time and developed string cloud cosmological models. In the Brans-Dicke theory of gravitation, Adhav et al. (Adhav et al., 2007) produced string cosmological models by solving the field equations with the condition that the sum of the tension density and energy density is zero. Pawar (Pawar et al., 2018) investigated the Kaluza–Klein string cosmological model in the context of the f (R, T) theory of gravity and solved the field equations using a power law relationship between scale factor and a time-varying deceleration parameter.

In the presence of an electromagnetic field, the cosmological model plays a critical role in the evolution of the cosmos and the construction of large scale structures such as galaxies and other celestial bodies. The presence of a cosmic electromagnetic field formed during inflation is responsible for the current phase of accelerated expansion of the cosmos. On cosmological scales, Jimenez and Maroto (Jimenez and Maroto, 2009) shown that the presence of a electromagnetic field provides an effective cosmological constant, which accounts for the Universe's accelerated expansion. In general relativity, Tripathy et al. (Tripathi et al., 2017) investigated an inhomogeneous string cosmological model with electromagnetic field. Parikh (Parikh et al., 2018) recently investigated a Bianchi type II string dust cosmological model in Lyra's Geometry with an electromagnetic field. Pradhan (Pradhan and Jaiswal, 2018) investigated a class of anisotropic and homogeneous Bianchi type-V cosmological models with heavy strings in the presence of a magnetic field in the f (R, T) theory of gravity. Grasso (Grasso and Rubinstein, 2000) looked into a wide range of characteristics of magnetic fields in the early Universe, as well as modern temporal fields and their evolution in galaxies and clusters. The magnetic field also contributes to the breakdown of statistical homogeneity and isotropy. Magnetic fields can be found in large scale galaxies and galaxy clusters. In his study, Subramanian (Subramanian, 2016) demonstrated that primordial magnetic fields have a significant impact on the creation of star structures, particularly on dwarf galaxy scales.

Alternative theories of gravity, such as the Brans-Dicke gravity theory (Brans and Dicke, 1961), the Saez-Ballester theory of gravity (Saez and Ballester, 1986), f (R) gravity (Capozziello et al., 2003; Nojiri and Odintsov, 2003), f (R,T) gravity (Harko et al., 2011) are all important. Each of these gravity theories has its own significance. We built a cosmological model in the Saez-Ballester theory of gravity to examine several features of the cosmos in this research. As it is known, the metric is easily coupled with a dimensionless scalar field in Sáez and Ballester theory. Despite the scalar field's dimensionless behaviour, this coupling affords a fair description of weak fields in which an accelerated growth regime arises. Many authors, including our peers, have built cosmological models based on the Saez-Ballester theory of gravity to investigate various features of the Universe (Singh and Agrawal, 1991; Aditya and Reddy, 2018; Mishra and Dua, 2019; Mishra and Chand, 2020; Rasouli et al., 2020). Santhi and Sobhanbabu (Santhi and Sobhanbabu, 2020) looked at the dynamics of the Bianchi type-III Tsallis holographic dark energy model in the Saez-Ballester theory of gravity and came up with some interesting findings. In the Saez-Ballester theory of gravity, Naidu et al. (Naidu et al., 2021) explored the behaviour of Kaluza-Klein FRW dark energy models, finding the solution of field equations utilising I hybrid expansion law and (ii) variable deceleration parameter. Saez-Ballester gives the Einstein field equations for the combined scalar and tensor fields

$$R_{ij} - \frac{1}{2}Rg_{ij} - \omega\phi^m\left(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,k}\phi^{,k}\right) = -T_{ij} \quad (1)$$

and the scalar field also fulfils the equation

$$2\phi^m\phi^{,i}_{;i} + m\phi^{m-1}\phi_{,k}\phi^{,k} = 0 \quad (2)$$

We solved the Saez-Ballester equations for energy momentum tensor for cloud string interacting with electromagnetic field and addressed in depth all elements of physical and kinematic properties in this article, which was motivated by the study of the above literatures. The following is how the paper is structured: The metric and field equations are presented in **Section 2**; the solution of the field equations is offered in **Section 3**; physical features of our model Universe are presented in **Section 4**; and illustrations and figures are presented in **Section 5**. Finally, in **Section 6**, we review the findings and provide some last observations.

2 METRIC AND FIELD EQUATIONS

We consider the Bianchi type-I metric, which is spatially homogenous and anisotropic

$$ds^2 = -dt^2 + A^2\left(dx^2 + dy^2\right) + B^2dz^2 + C^2d\psi^2 \quad (3)$$

A, B, and C are all functions of cosmic time t.

For the metric (3) we assume

$x^1 = x, x^2 = y, x^3 = z, x^4 = \psi, and\ x^5 = t$

A cloud string's energy-momentum tensor (Reddy, 2003) is of the form

$$T_{ij} = \rho u_i u_j - \lambda x_i x_j + E_{ij} \quad (4)$$

where ρ_p is the particle density and λ is the string tension density, and ρ is the rest energy density of the cloud of strings with particles attached to them $\rho = \rho_p + \lambda$. x_i is a unit space-like vector that represents the direction of strings, with $x^2 = 0 = x^3 = x^4 = x^5$ and $x^1 \neq 0$, and u_i is the five velocity vector that meets the following conditions

$$u^i u_i = -x^i x_i = -1 \quad (5)$$

and

$$u^i x_i = 0 \quad (6)$$

The five velocity vectors u^i as well as the string's direction x^i, are given by

$$u^i = (0,0,0,0,1) \quad (7)$$

and

$$x^i = \left(\frac{1}{A}, 0,0,0,0\right) \quad (8)$$

where the strings direction are parallel to the x-axis.

The electromagnetic field E_{ij}, which is a component of the energy momentum tensor, is referred to as

$$E_{ij} = \frac{1}{4\pi}\left(g^{\alpha\beta}F_{i\alpha}F_{i\beta} - \frac{1}{4}g_{ij}F_{\alpha\beta}F^{\alpha\beta}\right) \quad (9)$$

Here $F_{\alpha\beta}$ is denoted the electromagnetic field tensor.

F_{15} is the only non-vanishing component of the electromagnetic field tensor F_{ij} if we quantize the magnetic field along the x axis. Assuming infinite electromagnetic conductivity, it can be obtain that $F_{12} = F_{13} = F_{14} = F_{23} = F_{24} = F_{25} = F_{34} = F_{35} = 0$ (Singh et al., 2020).

As a result, the nontrivial components of the electromagnetic field E_{ij} can be calculated using **Eq. 9** as given below

$$E^1_1 = -E^2_2 = -E^3_3 = -E^4 = E^1_1 = -\frac{1}{2}g^{11}g^{55}F^2_{15} = \frac{1}{2A^2}F^2_{15} \quad (10)$$

Or, If we take the magnetic field along the x-axis in co-moving coordinates, F_{34} is the only non-vanishing component of the electromagnetic field tensor F_{ij}. Assuming infinite electromagnetic conductivity, it can be define that $F_{15} = F_{25} = F_{35} = F_{12} = F_{13} = F_{14} = 0$ (Singh et al., 2020).

As a result, the nontrivial components of the electromagnetic field E_{ij} can be calculated using **Eq. 9**

$$E^1_1 = -E^2_2 = -E^3_3 = -E^4 = E^1_1 = -\frac{1}{2A^2}F^2_{34} \quad (11)$$

Important physical quantities for the metric (3) include the spatial volume V, the average scale factor R, the expansion scalar θ, the Hubble parameter H, the deceleration parameter q, the shear scalar σ^2 and the mean anisotropy parameter $\triangle$:

$$V = R^4 = A^2BC \quad (12)$$

$$\theta = u^i_{;i} = 2\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \quad (13)$$

$$H = \frac{\dot{R}}{R} = \frac{1}{4}\left(2\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) \quad (14)$$

$$q = -\frac{R\ddot{R}}{\dot{R}^2} = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 \tag{15}$$

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{1}{2}\left(2\frac{\dot{A}^2}{A^2} + \frac{\dot{B}^2}{B^2} + \frac{\dot{C}^2}{C^2}\right) - \frac{\theta^2}{8} \tag{16}$$

$$\triangle = \frac{1}{4}\sum_{i=1}^{4}\left(\frac{H_i - H}{H}\right)^2 \tag{17}$$

With regard to cosmic time t, an over head dot represents the first derivative, while a double over head dot represents the second. Also, $H_i(i = 1,2,3,4)$ denotes the directional Hubble Parameters in the direction of x, y, z and ψ axes and are obtained for metric (1) as $H_1 = H_2 = \frac{\dot{A}}{A}$, $H_3 = \frac{\dot{B}}{B}$, and $H_4 = \frac{\dot{C}}{C}$.

The Saez-Ballester field **Eqs 1**, **2** are reduced to the following system of equations when combined with **Eq. 4** for the line-element **Eq. 3**:

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} = \lambda - \frac{1}{2A^2}F_{15}^2 + \omega\phi^m\frac{\dot{\phi}^2}{2} \tag{18}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} = \frac{1}{2A^2}F_{15}^2 + \omega\phi^m\frac{\dot{\phi}^2}{2} \tag{19}$$

$$2\frac{\ddot{A}}{A} + \frac{\dot{A}^2}{A^2} + \frac{\ddot{C}}{C} + 2\frac{\dot{A}}{A}\frac{\dot{C}}{C} = \frac{1}{2A^2}F_{15}^2 + \omega\phi^m\frac{\dot{\phi}^2}{2} \tag{20}$$

$$2\frac{\ddot{A}}{A} + \frac{\dot{A}^2}{A^2} + \frac{\ddot{B}}{B} + 2\frac{\dot{A}}{A}\frac{\dot{B}}{B} = \frac{1}{2A^2}F_{15}^2 + \omega\phi^m\frac{\dot{\phi}^2}{2} \tag{21}$$

$$\frac{\dot{A}^2}{A^2} + 2\frac{\dot{A}}{A}\frac{\dot{B}}{B} + 2\frac{\dot{A}}{A}\frac{\dot{C}}{C} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} = \rho - \frac{1}{2A^2}F_{15}^2 - \omega\phi^m\frac{\dot{\phi}^2}{2} \tag{22}$$

$$\ddot{\phi} + \dot{\phi}\left(2\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) + \frac{m}{2}\frac{\dot{\phi}^2}{\phi} = 0 \tag{23}$$

3 SOLUTION OF THE FIELD EQUATIONS

The field **Eqs 18–22** are a set of five equations with seven unknown parameters (A, B, C, ρ, λ, ϕ and F_{15}). As a result, two additional constraints linking these parameters are necessary in order to achieve explicit system solutions. These two relationships are considered to be

1) We can take advantage of the fact that the shear scalar σ is proportional to the scalar expansion θ (Collins et al., 1983; Rao et al., 2015).

$$B = C^n \tag{24}$$

here n is constant.

2) Berman (Berman, 1983) and Ram et al. (Ram et al., 2010) derived a relationship between the Hubble parameter H and the average scale factor R. Berman, and Gomide (Berman and Gomide, 1988), and Ram et al. (Ram et al., 2010) solved FRW models using this type of relation, whereas Ram et al. (Ram et al., 2010) solved Bianchi Type V cosmological models in Lyra's Geometry.

$$H = aR^{-k_1} \tag{25}$$

$a \geq 0$, $k_1 \geq 0$ are constants.

Equation 25 provides us with

$$R = (ak_1 t + k_2)^{\frac{1}{k_1}} \qquad if \qquad k_1 \neq 0 \tag{26}$$

and

$$R = k_3 e^{at} \qquad if \qquad k_1 = 0 \tag{27}$$

k_2 and k_3 are constants.

As a result of **Equation 15**, the deceleration parameter q is determined

$$q = (k_1 - 1) \qquad if \qquad k_1 \neq 0 \tag{28}$$

$$q = -1 \qquad if \qquad k_1 = 0 \tag{29}$$

Case I: When $k_1 \neq 0$, we have

The **Eqs 19**, **20**, **24**, **26** will provide us with the following result

$$C = k_5 e^{\frac{k_4 (ak_1 t + k_2)^{\frac{k_1 - 4}{k_1}}}{a(k_1 - 4)}} \tag{30}$$

k_4, k_5 are constants in this equation. We can assume $k_4 = k_5 = 1$ without losing generality, giving us

$$C = e^{\frac{(ak_1 t + k_2)^{\frac{k_1 - 4}{k_1}}}{a(k_1 - 4)}} \tag{31}$$

As a result, the **Eqs 12**, **24**, **26** will provide us with

$$A = (ak_1 t + k_2)^{\frac{2}{k_1}} e^{-\frac{(ak_1 t + k_2)^{\frac{k_1 - 4}{k_1}}(n+1)}{2a(k_1 - 4)}} \tag{32}$$

and

$$B = e^{\frac{n(ak_1 t + k_2)^{\frac{k_1 - 4}{k_1}}}{a(k_1 - 4)}} \tag{33}$$

As a result, the line element **Eq. 3** is transformed into

$$ds^2 = -dt^2 + (ak_1 t + k_2)^{\frac{4}{k_1}} e^{-\frac{(ak_1 t + k_2)^{\frac{k_1 - 4}{k_1}}(n+1)}{a(k_1 - 4)}} (dx^2 + dy^2) + e^{\frac{2n(ak_1 t + k_2)^{\frac{k_1 - 4}{k_1}}}{a(k_1 - 4)}} dz^2 + e^{\frac{2(ak_1 t + k_2)^{\frac{k_1 - 4}{k_1}}}{a(k_1 - 4)}} d\psi^2 \tag{34}$$

Equation 34 is a Bianchi Type I cosmological model Universe with electromagnetic field and Hubble parameter special law which is becomes singular for $k_1 = 4$.

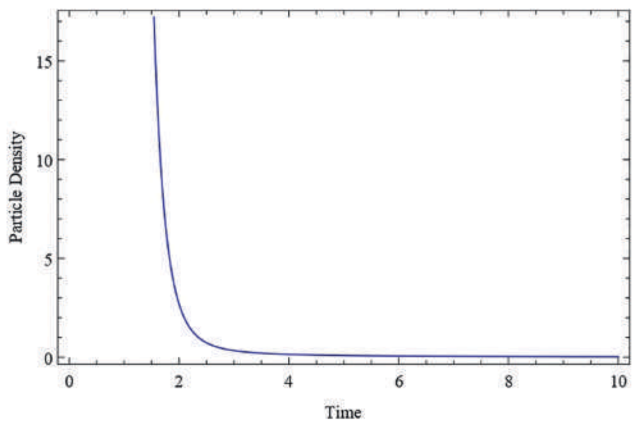

FIGURE 1 | Particle Density ρ_p vs. time.

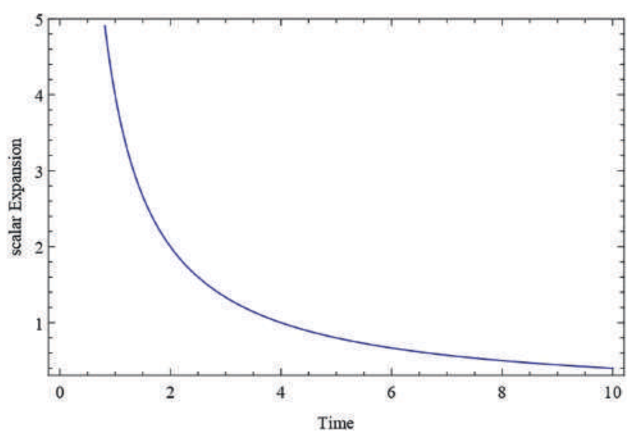

FIGURE 2 | Scalar Expansion θ vs. time.

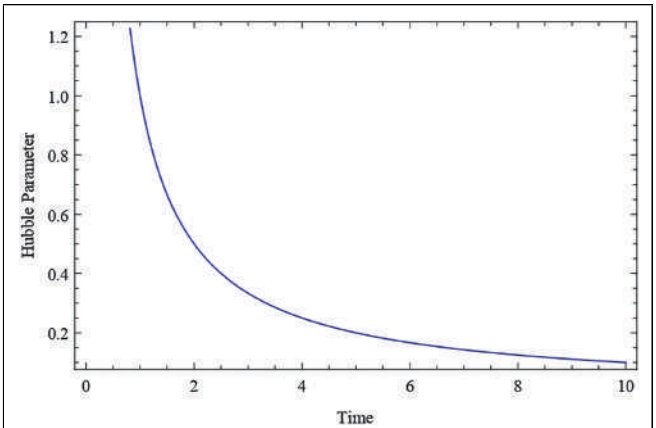

FIGURE 3 | Hubble Parameter H vs. time.

4 SOME PHYSICAL PROPERTIES OF THE MODEL

Now, using **Equations 18–22**, we can calculate the values of energy density ρ, String tension density λ, and electromagnetic field tensor F_{15} for our model Universe provided by **Equation 34**.

$$\rho = -4a^2(k_1-4)(ak_1t+k_2)^{-2} \tag{35}$$

$$\lambda = \frac{1}{2}(3n^2+2n+3)(ak_1t+k_2)^{\frac{-8}{k_1}} - 2a^2(3k_1-8)(ak_1t+k_2)^{-2} - 4a(n+1)(ak_1t+k_2)^{-\left(\frac{k_1+4}{k^1}\right)} - \omega k_6^2(ak_1t+k_2)^{-\frac{8}{k_1}} \tag{36}$$

$$F_{15} = \sqrt{2}\left[\frac{1}{4}(3n^2+2n+3)(ak_1t+k_2)^{-\frac{4}{k_1}} - 4a^2(k_1-3)(ak_1t+k_2)^{-2\left(\frac{k_1-2}{k_1}\right)} - 2a(n+1)(ak_1t+k_2)^{-1} - \omega\frac{k_6^2}{2}(ak_1t+k_2)^{-\frac{8}{k_1}}\right]^{\frac{1}{2}} \times e^{\frac{(ak_1t+k_2)^{\frac{k_1-4}{k_1}}(n+1)}{a(k_1-4)}} \tag{37}$$

Using the relationship $\rho = \rho_p + \lambda$ we now obtain

$$\rho_p = 2a^2k_1(ak_1t+k_2)^{-2} - \frac{1}{2}(3n^2+2n+3)(ak_1t+k_2)^{-\frac{8}{k_1}} + 4a(n+1)(ak_1t+k_2)^{-\left(\frac{k_1+4}{k_1}\right)} + \omega k_6^2(ak_1t+k_2)^{-\frac{8}{k_1}} \tag{38}$$

The physical quantities proper volume V, Hubble parameter H, expansion scalar θ, scalar field ϕ shear scalar σ^2, and mean anisotropy parameter $\triangle$ are calculated as follows:

$$V = R = (ak_1t+k_2)^{\frac{4}{k_1}} \tag{39}$$

$$H = \frac{a}{(ak_1t+k_2)} \tag{40}$$

$$\theta = \frac{4a}{(ak_1t+k_2)} \tag{41}$$

$$\phi = \left[\frac{k_6(m+2)}{2a(k_1-4)}\right]^{\frac{2}{m+2}}\left[(ak_1t+k_2)^{\left(\frac{k_1-4}{k_1}\right)\left(\frac{2}{m+2}\right)}\right] \tag{42}$$

$$\sigma^2 = \frac{(3n^2+2n+3)}{4}(ak_1t+k_2)^{-\frac{8}{k_1}} - 2a(n+1)(ak_1t+k_2)^{-\left(\frac{k_1+4}{k_1}\right)} + 2a^2(ak_1t+k_2)^{-2} \tag{43}$$

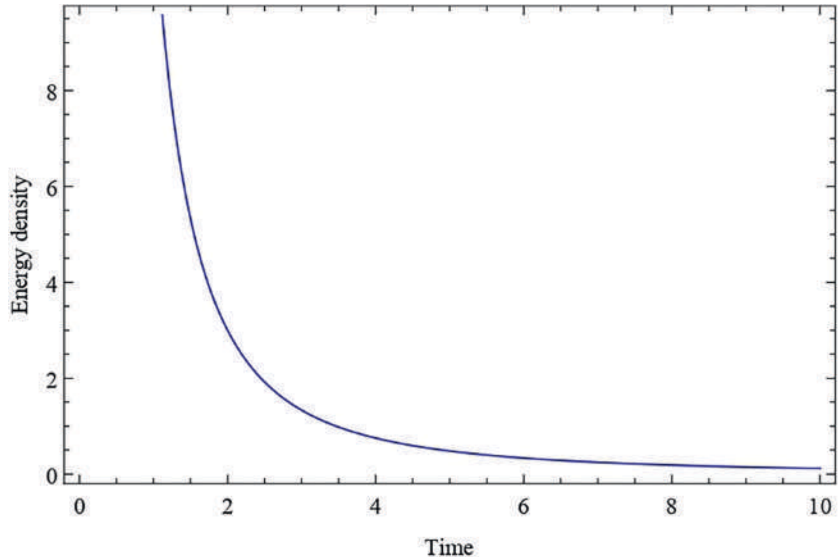

FIGURE 4 | Energy density ρ vs. time.

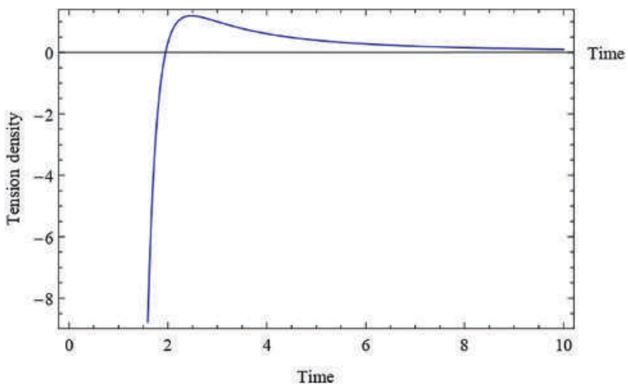

FIGURE 5 | Tension Density λ vs. time.

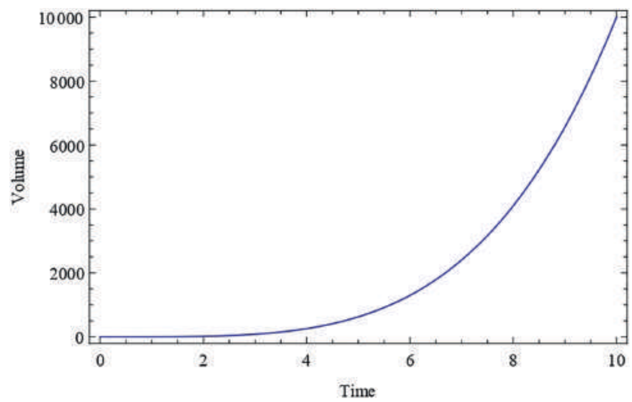

FIGURE 6 | Volume V vs. time.

$$\triangle = 1 - \frac{(n+1)}{a}(ak_1 t + k_2)^{\left(\frac{k_1-4}{k_1}\right)} + \frac{(3n^2+2n+3)}{2a^2}(ak_1 t + k_2)^{2\left(\frac{k_1-4}{k_1}\right)} \quad (44)$$

From **Equations 41**, **43**, we get

$$\lim_{t\to\infty}\frac{\sigma^2}{\theta^2} \neq 0 \quad (45)$$

The fluctuation of some of the parameters is depicted using the values $a = k_1 = k_6 = n = 1$ and $k_2 = 0$.

5 PHYSICAL INTERPRATATION

The deceleration parameter q is constant and negative, as shown in the expression **Eq. 28**. This indicates that since cosmic time

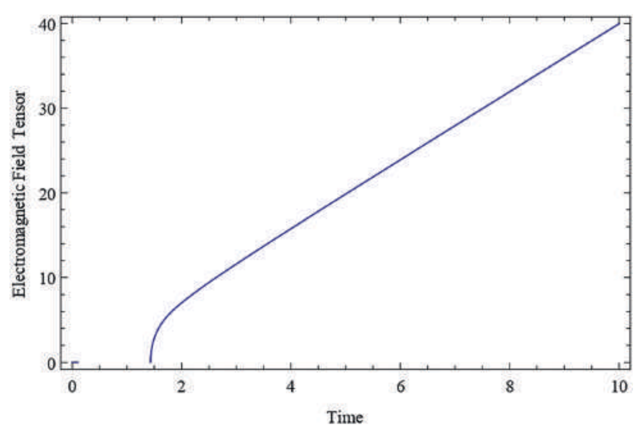

FIGURE 7 | Electromagnetic Field tensor F_{15} vs. time.

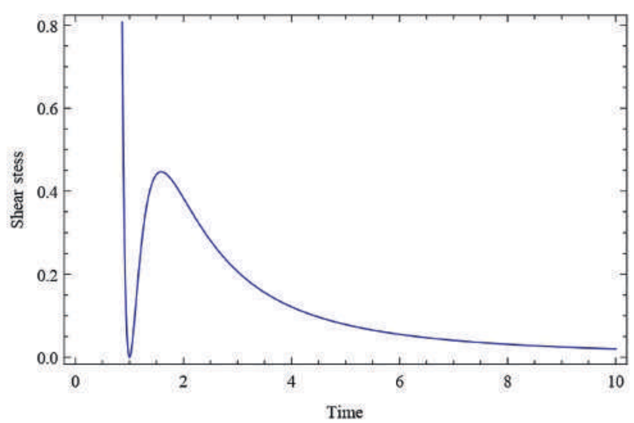

FIGURE 8 | Shear Stress σ^2 vs. time.

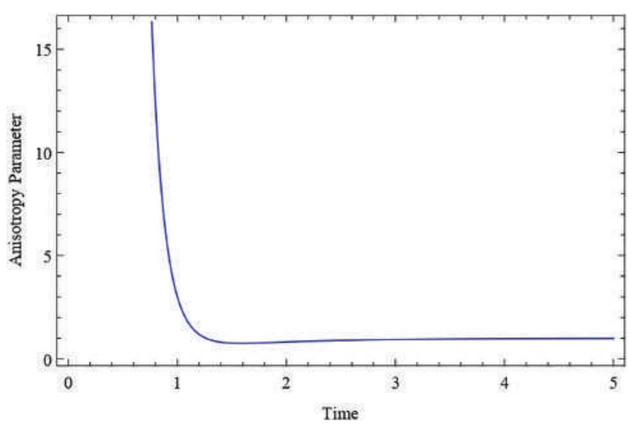

FIGURE 9 | Anisotropy Parameter $\triangle$ vs. time.

t = 0, our proposed model Universe has been inflating and expanding at a constant rate. Throughout the course of evolution, the Universe's expansion rate is constant. q = constant (negative value) for $k_1 < 1$. Also, at $t = 0$ at the time of the big bang, the particle density has a huge positive value. It approaches a positive value as time passes, eventually reaching a finite constant value at $t \to \infty$ (As shown in **Figure 1**). This demonstrates that the particle continues to dominate the cosmos as time passes, and that the total number of particles in the Universe remains constant. The expansion scalar, a Hubble parameter, exhibits similar fluctuations throughout time. After evolving from infinity at t = 0 to a finite value at $t \to \infty$, both the Hubble's parameter H and the expansion scalar diminish with cosmic time t (shown in **Figures 2**, **3**). The expansion of the cosmos appears to be accelerating, and the deceleration parameter is negative in this scenario, $\ddot{R} > 0$. Our model satisfies the energy conditions $\rho \geq 0$ and $\rho_p \geq 0$. The electromagnetic field changes the behaviour of the model, as well as the physical parameter expressions. In the presence of an electromagnetic field, the rest energy density and string tension tensity decrease (As shown in **Figures 4**, **5**). Particle density and string tension density are also comparable, although string tension density vanishes faster than particle density. This shows that our model displays a matter-dominated Universe at a late time scale, which is in line with current observational data. We can see from the expression **Eq. 39**, that the proper volume $V = k_2^{\frac{4}{k_1}}$ at $t = 0$ increases with cosmic time t (Shown in **Figure 6**). This implies that the Universe begins with a finite volume at $t = 0$ and then grows as cosmic time passes. The appropriate volume V becomes infinite at time t. **Equation 37** illustrates that the non-vanishing electromagnetic field tensor F_{15} grows exponentially as a function of cosmic time t (Shown in **Figure 7**). If $a \neq 0$, we can see that the electromagnetic field tensor F_{15} does not vanish. It has a significant impact on the formation of strings during the early stages of the Universe's evolution. In this scenario, the string and electromagnetic field are found to coexist.

At the first singularity, the parameters shear scalar σ^2 and mean anisotropy parameter $\triangle$ diverge (As shown in **Figures 8**, **9**). The model depicts a shearing, non-rotating cosmos with the potential for a large crunch at some $t = 0$ beginning epoch. Also, we can observe from the mathematical statement **Eq. 45**, the isotropic condition $\lim_{t\to\infty} \frac{\sigma^2}{\theta 2} \neq 0$(constant), remains constant throughout the Universe's evolution (from early to late-time), indicating the model does not attain isotropy (Sahoo et al., 2017).

6 CONCLUSION

The interaction of an anisotropic Bianchi Type-I string cosmological model Universe with an electromagnetic field is investigated in the general theory of relativity. In the above cosmological model $H > 0$ and $q < 0$, which demonstrates that the Universe expands according to a power law after the big bang, starting with a finite volume at cosmic time $t = 0$ and expanding with an acceleration. For the above cosmological model Universe, the hypothesised relationship between Hubble's parameter and average scale factor results in a constant negative value of the deceleration parameter. A point type (MacCallum, 1971) singularity exists in the derived model at time $t = 0$. The density of particles and the tension density of the string are comparable, but tension density falls off more quickly than particle density, showing that particles dominate the cosmos as time passes. In our hypothesis, the Universe has a chance of being anisotropic at any point during its existence, from the beginning to the end. According to recent research, there is a discrepancy in estimating microwave intensities emitted from various directions of the sky. This prompted us to investigate the world using the anisotropic Bianchi Type-I metric to better describe our Universe. Several CMB (de Bernardis et al., 2000; Hanany et al., 2000) anomalies, such as temperature anisotropies in the CMB that are inconsistent with the exact homogeneous and isotropic FRW model recorded by COBE/WMAP (Bennet et al., 2003) satellites, foregrounds/systematics, and novel topologies, are also evidence that we live in a globally anisotropic Universe. Shear reduces during inflation, finally resulting in an isotropic phase with no shear. In order to produce any significant amount of shear in recent years, one must first induce anisotropy in space-time.

DATA AVAILABILITY STATEMENT

The original contributions presented in the study are included in the article, further inquiries can be directed to the corresponding author.

AUTHOR CONTRIBUTIONS

All of the authors listed have contributed a significant, direct, and intellectual contribution to the work and have given their permission for it to be published.

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Five Dimensional FRW Radiating Cosmological Model in Presence of Bulk Viscous Fluid in Scalar Tensor Theory of Gravitation

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Abstract

In this paper using the Saez-Ballester scalar-tensor theory of gravitation, we examined a 5-dimensional FRW cosmic space-time in a source of bulk viscous fluid in the article. To examine determinate solutions of the field equations, we used a power law between a scalar field and the universe's scale factor. Our research considers radiating flat, closed, and open models. The physical and kinematical properties of the models were explored in each scenario. In this study, we show that our model expands and is free of initial singularities, as well as that our models decelerate in a conventional manner.

Keywords: Bulk viscous model, Radiating models, Scalar-Tensor theory, FRW models

Introduction

We know that the matter distribution in our universe is appropriately illustrated by a perfect fluid due to the vast scale distribution of galaxies. For a practical study of the problem, a state of material dissemination other than perfect fluid is required. When neutrino decoupling occurs, Misner [1] characterized the matter as a viscous fluid in the early phases of the universe. Riess *et al*. [2] and Perlmutter *et al*. [3] explored the function of bulk viscosity in the recent scenario of accelerated expansion of the universe, which is known as the inflationary phase in cosmology. As a result, there has been a lot of interest in studying cosmological models with bulk viscosity in recent years. Pavon *et al*. [4], Mohanty and Pradhan [5], Pimentel [6], Rao *et al*. [7], and Naidu *et al*. [8] have all looked into general relativity and modified theories of gravity. Banerjee *et al*. [9] have also defined Bianchi type-I cosmological models with viscous fluid in higher dimensional space time. Mohanty *et al*. [10] have investigated higher dimensional string cosmological model with bulk viscous fluid in Lyra manifold. Naidu *et al*. [11] have defined 5 dimensional Kaluza-Klein bulk viscous models in modified theories of gravitation. In a simple method, Saez and Ballester [12] investigated a theory in which the metric is paired with a dimensionless scalar field. This combination provides a sufficient explanation for the weak fields. Because of the scalar field's dimensionlessness, an antigravity system emerges. In the non-flat FRW model, this theory suggests a possible solution to define the missing matter problem. In this idea, Saez examined the original singularity and extension universe, as well as the fact that an antigravity system exists at either the beginning of the extension era or before it. Many authors (Reddy and Rao [13], Mohanty and Sahu [14], Reddy and Naidu [15], Singh *et al*. [16], Pradhan *et al*. [17], Reddy *et al*. [18], Katore and Shaikh [19], Rao *et al*. [20], Ram *et al*. [21], Yadav *et al*. [22], Santhi and Sobhanbabub [23]) have studied Saez-Ballester scalar-tensor in different cosmological models. Recently Mishra and Dua [24], Naidu *et al*. [25] had investigated Saez-Ballester Scalar tensor using different cosmological models.

The field equations for combined scalar and tensor fields, proposed by Saez-Ballester are;

$$R_{ij} - \frac{1}{2} g_{ij} R - \omega \phi^n \left(\phi_{,i} \phi_{,j} - \frac{1}{2} g_{ij} \phi_{,k} \phi^{,k} \right) = -8\pi T_{ij} \tag{1}$$

where the scalar field ϕ satisfies the equation

$$2\phi^n \phi^{,i}_{;i} + n\phi^{n-1} \phi_{,k} \phi^{,k} = 0 \tag{2}$$

and

$$T^{ij}_{;j} = 0. \tag{3}$$

Eq. (3) is a consequence of the field Eqs. (1) and (2), $\boldsymbol{\omega}$ and n are constants. Comma and semicolon represent partial and covariant differentiation respectively.

There has been huge attract in the investigation of higher dimensional space-time in current years due to the evidence that the cosmos at its beginning period of expansion of the universe that might have had a higher dimensional epoch. This evidence had attracted several authors (Venkateswarlu and Kumar [26], Khadekar and Avachar [27], Bahrehbakhsh *et al*. [28], Biswal *et al*. [29], Venkateswarlu *et al*. [30] Oli [31], Ramprasad *et al*. [32], Rao *et at*. [33], Aygun *et al*. [34], Caglar *et al*. [35], Caglar and Aygun [36], Singh and Singh [37]) to study to the field of higher dimensions. We know that at beginning period of times before the universe has undergone compactification transitions the results of the field equations in general relativity and in scalar-tensor theories in higher dimensional space-time are of substantial purpose probably. Marciano [38] has advised that the investigational conclusion of the theoretical constants with varying time might develop the information of additional dimensions. Currently, Gomez *et al*. [39], Trivedi and Bhabor [40], Das and Bharali [41] have studied 5 dimensional FRW models using different type of scalar tensor.

In this study, we define 5-dimensional FRW radiating models in the presence of bulk viscous cosmological models in the scalar-tensor theory of gravitation, as a result of the previous investigations and discussion. We discussed about bulk viscous fluid, Saez-Ballester, and FRW models in Section 1. The Section 2, contains metric and field equations. In Section 3, we look at Saez-Ballester cosmological models with an equation of state that is equivalent to disordered radiation in general relativity. The physical explanation of the models is covered in Section 4, and the conclusions are given in Section 5.

Metric and field equations

Here we assume the 5-dimensional FRW metric in the following form;

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{(1-kr^2)} + r^2(d\Theta^2 + \sin^2\Theta\, d\phi^2) + (1-kr^2)d\psi^2\right] \tag{4}$$

From Eq. (4), we get the non-vanishing components of Einstein tensor in the following way;

$$G^1_1 = G^2_2 = G^3_3 = G^4_4 = 3\frac{a''}{a} + 3\frac{a'^2}{a^2} + 3\frac{k}{a^2} \tag{5}$$

and

$$G^5_5 = 6\frac{a'^2}{a^2} + 6\frac{k}{a^2} \tag{6}$$

Here a prime symbol means differentiation with respect to time t. Also $k = +1, -1, 0$ denotes *closed*, *open* and *flat* models respectively.

For bulk viscous fluid the energy momentum tensor is given by in the following way;

$$T_{ij} = (\rho + \bar{p})u_i u_j - g_{ij}\bar{p} \quad i, j = 1,2,3,4,5 \tag{7}$$

Together with,

$$u_i u_i = 1 \quad and \quad u_i u_j = 0 \tag{8}$$

Therefore,

$$T = \rho - 4\bar{p} \tag{9}$$

The total pressure with the proper pressure containing bulk viscosity coefficient ζ and Hubble expansion parameter H are defined in the following way;

$$\bar{p} = p - 3\zeta H = \varepsilon\rho, \qquad where \qquad p = \varepsilon_0\rho, \qquad and \qquad \varepsilon = \varepsilon_0 - \beta \tag{10}$$

By applying co moving coordinates and with the help of Eqs. (3) and (5) - (9), the Saez-Ballester field Eqs. (1) and (2) for the matric (4), becomes in the following way;

$$6\frac{a'^2}{a^2}+6\frac{k}{a^2}-\frac{\omega}{2}\frac{\phi'^2}{\phi^2}=-8\pi\rho \tag{11}$$

$$3\frac{a''}{a}+3\frac{a'^2}{a^2}+3\frac{k}{a^2}+\frac{\omega}{2}\frac{\phi'^2}{\phi^2}=8\pi\bar{p} \tag{12}$$

$$\frac{\phi''}{\phi}+4\frac{a'}{a}\frac{\phi'}{\phi}+\frac{n}{2}\frac{\phi'^2}{\phi^2}=0 \tag{13}$$

$$\rho'+4\frac{a'}{a}(\rho+\bar{p})=0 \tag{14}$$

We know that,
Hubble parameter H is

$$H=\frac{a'}{a} \tag{15}$$

and the deceleration parameter q is

$$q=-\frac{(H'+H^2)}{H^2} \tag{16}$$

Solutions and the models

In this paper we apply the condition given by Eq. (10). We also apply the relation between scalar field ϕ and the scale factor of the universe a(t) which was defined by Pimental [42]; Johri and Kalyani [43].

$$\phi=\phi_0 a^n \tag{17}$$

where and n > 0 are constants.

Here we find the solutions of the field Eqs. (11) - (13) for $k=+1,-1,0$ (i.e closed, open and flat model) respectively.

Case (i): For k = 1, (i. e. Closed model)

Using Eq. (17) in the field Eqs. (11) - (13) we get the scale factor in the following form;

$$a(t)=\left[\left\{\frac{\left(\frac{n^2}{2}+n+4\right)}{n\phi_0\left(\frac{n+2}{2}\right)}\right\}(a_0t+t_0)\right]^{\frac{1}{\left(\frac{n^2}{2}+n+4\right)}} \tag{18}$$

Now by choosing a_0 = 1 and $t_0=0$ and using (18), the metric (4) becomes in the following form;

$$ds^2=dt^2-\left[\left\{\frac{\left(\frac{n^2}{2}+n+4\right)}{n\phi_0\left(\frac{n+2}{2}\right)}\right\}t\right]^{\frac{2}{\left(\frac{n^2}{2}+n+4\right)}}\left[\frac{dr^2}{(1-r^2)}+r^2(d\Theta^2+\sin^2\Theta\, d\phi^2)+(1-r^2)d\psi^2\right] \tag{19}$$

where the scalar field is

$$\phi=\phi_0\left[\left\{\frac{\left(\frac{n^2}{2}+n+4\right)}{n\phi_0\left(\frac{n+2}{2}\right)}\right\}t\right]^{\frac{n}{\left(\frac{n^2}{2}+n+4\right)}}. \tag{20}$$

The Eq. (19) denotes a 5-dimensional FRW bulk viscous radiating model.

The Spatial volume is

$$V = a^4 = \left[\left\{\frac{\left(\frac{n^2}{2}+n+4\right)}{n\phi_0\left(\frac{n+2}{2}\right)}\right\} t\right]^{\frac{4}{\left(\frac{n^2}{2}+n+4\right)}} \tag{21}$$

Hubble's parameter H is

$$H = \left(\frac{1}{\frac{n^2}{2}+n+4}\right)\frac{1}{t} \tag{22}$$

The energy density ρ is

$$8\pi\rho = \left\{\frac{(\omega n^2-12)}{2t^2\left(\frac{n^2}{2}+n+4\right)^2}\right\} - 6\left\{\left(\frac{\frac{n^2}{2}+n+4}{n\phi_0\left(\frac{n+2}{2}\right)}\right) t\right\}^{-\frac{2}{\left(\frac{n^2}{2}+n+4\right)}} \tag{23}$$

The isotropic pressure p is

$$8\pi p = \boldsymbol{\varepsilon_0}\left[\left\{\frac{(\omega n^2-12)}{2t^2\left(\frac{n^2}{2}+n+4\right)^2}\right\} - 6\left\{\left(\frac{\frac{n^2}{2}+n+4}{n\phi_0\left(\frac{n+2}{2}\right)}\right) t\right\}^{-\frac{2}{\left(\frac{n^2}{2}+n+4\right)}}\right] \tag{24}$$

The coefficient of bulk $\boldsymbol{\zeta}$ viscosity is

$$8\pi\zeta = \left\{\frac{\left(\frac{n^2}{2}+n+4\right)(\boldsymbol{\varepsilon}-\boldsymbol{\varepsilon_0})t}{3}\right\}\left[\left\{\frac{(\omega n^2-12)}{2t^2\left(\frac{n^2}{2}+n+4\right)^2}\right\} - 6\left\{\left(\frac{\frac{n^2}{2}+n+4}{n\phi_0\left(\frac{n+2}{2}\right)}\right) t\right\}^{-\frac{2}{\left(\frac{n^2}{2}+n+4\right)}}\right] \tag{25}$$

Case (ii): For k = -1, (i. e. Open model)
Here the model is open and is given by

$$ds^2 = dt^2 - \left[\left\{\frac{\left(\frac{n^2}{2}+n+4\right)}{n\phi_0\left(\frac{n+2}{2}\right)}\right\} t\right]^{\frac{2}{\left(\frac{n^2}{2}+n+4\right)}}\left[\frac{dr^2}{(1+r^2)} + r^2(d\Theta^2 + \sin^2\Theta\, d\phi^2) + (1+r^2)d\psi^2\right] \tag{26}$$

Along with the energy density ρ

$$8\pi\rho = \left\{\frac{(\omega n^2-12)}{2t^2\left(\frac{n^2}{2}+n+4\right)^2}\right\} + 6\left\{\left(\frac{\frac{n^2}{2}+n+4}{n\phi_0\left(\frac{n+2}{2}\right)}\right) t\right\}^{-\frac{2}{\left(\frac{n^2}{2}+n+4\right)}} \tag{27}$$

Isotropic pressure p is

$$8\pi p = \boldsymbol{\varepsilon_0}\left[\left\{\frac{(\omega n^2-12)}{2t^2\left(\frac{n^2}{2}+n+4\right)^2}\right\} + 6\left\{\left(\frac{\frac{n^2}{2}+n+4}{n\phi_0\left(\frac{n+2}{2}\right)}\right) t\right\}^{-\frac{2}{\left(\frac{n^2}{2}+n+4\right)}}\right] \tag{28}$$

The coefficient of bulk viscosity ζ is

$$8\pi\zeta = \left\{\frac{\left(\frac{n^2}{2}+n+4\right)(\boldsymbol{\varepsilon}-\boldsymbol{\varepsilon_0})t}{3}\right\}\left[\left\{\frac{(\omega n^2-12)}{2t^2\left(\frac{n^2}{2}+n+4\right)^2}\right\} + 6\left\{\left(\frac{\frac{n^2}{2}+n+4}{n\phi_0\left(\frac{n+2}{2}\right)}\right) t\right\}^{-\frac{2}{\left(\frac{n^2}{2}+n+4\right)}}\right] \tag{29}$$

In this case the scalar field, spatial volume and Hubble parameter are given by Eqs. (20) - (22), respectively.

Case (iii): For k = 0, (i. e. Flat model)

In this case the model is flat and define by as follows;

$$ds^2 = dt^2 - \left[\left\{\frac{\left(\frac{n^2}{2}+n+4\right)}{n\phi_0^{\left(\frac{n+2}{2}\right)}}\right\} t\right]^{\frac{2}{\left(\frac{n^2}{2}+n+4\right)}} [dr^2 + r^2(d\Theta^2 + \sin^2\Theta\, d\phi^2) + d\psi^2] \tag{30}$$

The energy density ρ is

$$8\pi\rho = \left\{\frac{(\omega n^2 - 12)}{2t^2\left(\frac{n^2}{2}+n+4\right)^2}\right\} \tag{31}$$

Pressure p is

$$8\pi p = \boldsymbol{\varepsilon_0}\left[\left\{\frac{(\omega n^2 - 12)}{2t^2\left(\frac{n^2}{2}+n+4\right)^2}\right\}\right] \tag{32}$$

The coefficient of bulk viscosity ζ is

$$8\pi\zeta = \left\{\frac{\left(\frac{n^2}{2}+n+4\right)(\varepsilon - \boldsymbol{\varepsilon_0})t}{3}\right\}\left[\left\{\frac{(\omega n^2 - 12)}{2t^2\left(\frac{n^2}{2}+n+4\right)^2}\right\}\right] \tag{33}$$

Here also the scalar field, spatial volume and Hubble parameter are given by Eqs. (20) - (22) respectively.

The deceleration parameter q for all the 3 cases (i. e. $k = +1, -1, 0$) is

$$q = \left(\frac{n^2}{2} + n + 3\right) \tag{34}$$

Here the models decelerate in the standard way for all the above 3 cases (since we know that if $q > 0$ the universe decelerates in the standard way and accelerates when $q < 0$).

We plot all the following graphs with $\pi = 3.14,\ n = 1,\ \ \phi_0 = .001,\ \omega = 500,\ \varepsilon_0 = -1, \varepsilon = \frac{1}{3}$.

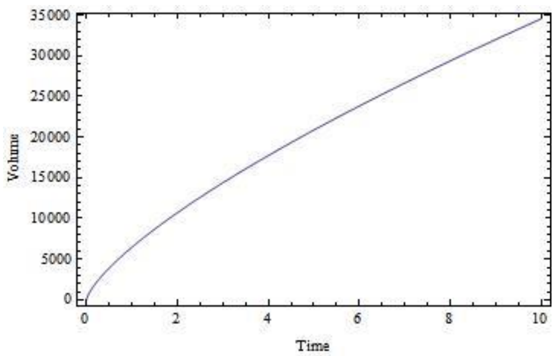

Figure 1 Volume V vs. time ($\pi = 3.14, n = 1, \phi_0 = 0.001$).

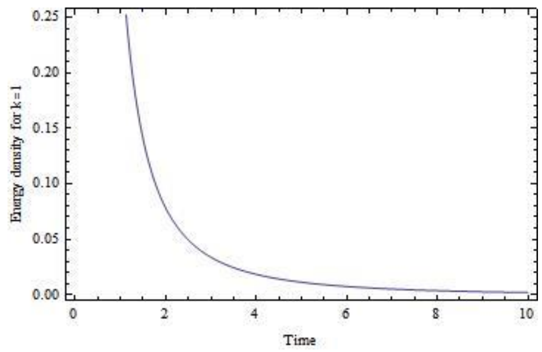

Figure 2 Energy density ρ vs. time for k = 1 ($\pi = 3.14, n = 1,\ \omega = 500,\ \phi_0 = 0.001$).

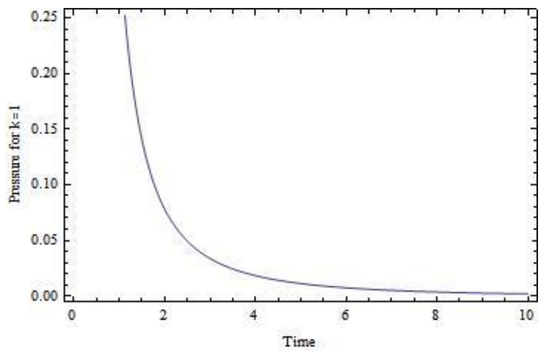

Figure 3 Pressure p vs. time for $k = 1$ ($\pi = 3.14, n = 1, \omega = 500,\ \phi_0 = 0.001$).

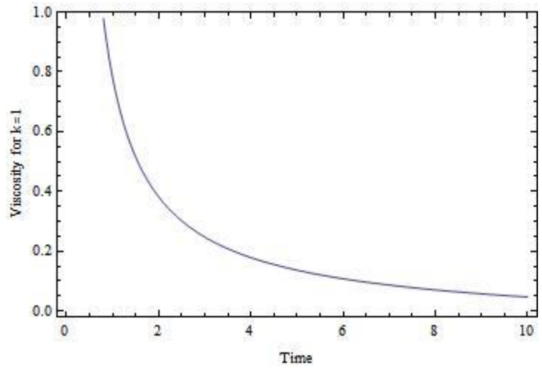

Figure 4 Viscosity ζ vs. time for $k = 1$ ($\pi = 3.14, n = 1, \omega = 500, \phi_0 = 0.001, \varepsilon_0 = -1, \varepsilon = \frac{1}{3}$).

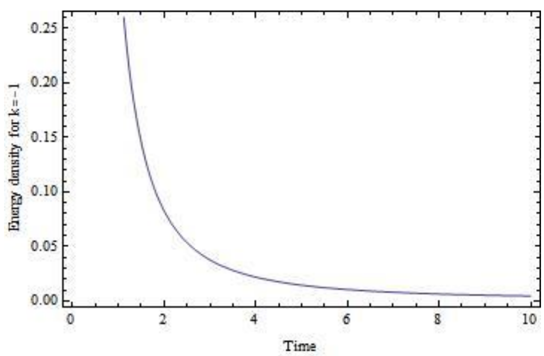

Figure 5 Energy density ρ vs. time for $k = -1$ ($\pi = 3.14, n = 1, \omega = 500,\ \phi_0 = 0.001$).

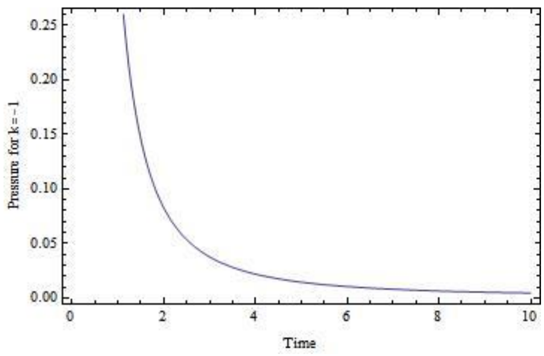

Figure 6 Pressure p vs. time for $k = -1$ ($\pi = 3.14, n = 1, \omega = 500, \ \phi_0 = 0.001$).

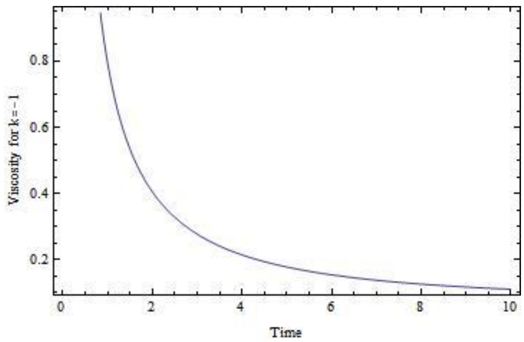

Figure 7 Viscosity ζ vs. time for $k = -1$ ($\pi = 3.14, n = 1, \omega = 500, \phi_0 = 0.001, \varepsilon_0 = -1, \varepsilon = \frac{1}{3}$).

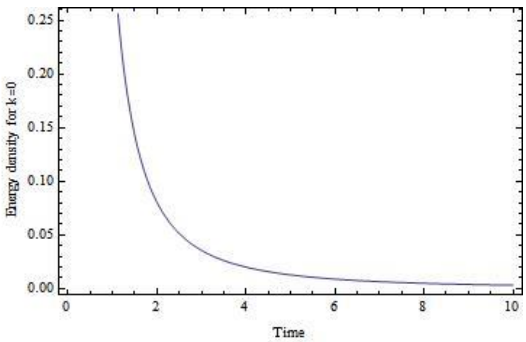

Figure 8 Energy density ρ vs. time for $k = 0$ ($\pi = 3.14, n = 1, \omega = 500, \ \phi_0 = 0.001$).

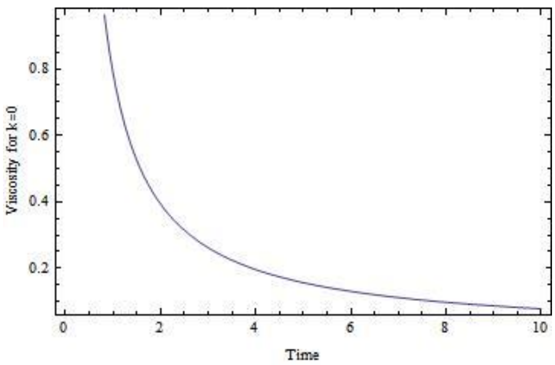

Figure 9 Pressure p vs. time for $= 0$ ($\pi = 3.14, n = 1, \omega = 500, \ \phi_0 = 0.001$).

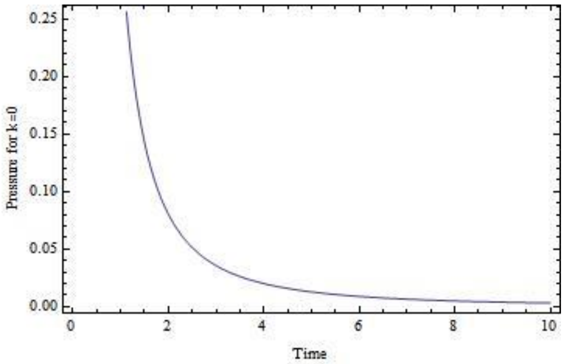

Figure 10 Viscosity ζ vs. time for $k = 0$ ($\pi = 3.14, n = 1, \omega = 500, \phi_0 = 0.001, \varepsilon_0 = -1, \varepsilon = \frac{1}{3}$).

Physical interpretation

In this paper the energy density, pressure and coefficient of bulk viscosity diverge at $t = 0$ and decrease with time for both in closed and open models (**Figures 2** - **7**). The energy density, the pressure and bulk viscosity decrease with time and will vanish for infinitely large time t in the flat model (**Figures 8** - **10**). All of them are diverge at the initial epoch. For all the models the spatial volume is same and increase with time but tends to infinity for infinitely large time (**Figure 1**). Eq. (22) is the average Hubble's parameter for all the models and will diverge at the initial epoch and will approach infinity as t becomes infinitely large. Also, Eqs. (19), (26) and (30) denotes FRW 5 dimensional radiating closed, open and flat models in Saez-Ballester theory respectively. For all the models the deceleration parameter is $q = (\frac{n^2}{2} + n + 3)$. Hence the models represented by Eqs. (19), (26) and (30) in 5 dimensions decelerates in the standard way. (Since we Know that if $n > 0$ the universe decelerates in the standard way and when $n < 0$ the universe accelerates). In this paper the models defined, in 5 dimensions FRW, are quite distinct from the Lyra geometry 5 dimensional models and the Kaluza-Klein 5 dimensional Models defined by many researchers.

Conclusions

Using the Saez-Ballester theory and a 5-dimensional FRW space-time as a source of bulk viscous fluid, we have obtained cosmological models that can be assumed to be equivalent radiating models in closed, open, and flat space-times. We have obtained models [i.e. Eqs. (19), (26) and (30)] that are expanding and free of initial singularity for all cases [i.e. Case I Case (ii), and Case (iii)]. The deceleration parameter found here decelerates in a conventional manner. The findings of this research aid our understanding of Saez-Ballester cosmology in 5 dimensions soon before compactification transition.

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