Chapter 3

FRW Cosmological Models in Presence of a Perfect Fluid within the Framework of Sáez-Ballester Theory in Five Dimensional Space Time

3.1 Introduction

The two most commonly encountered fluids in replicating our universe are perfect fluid and viscous fluid. Perfect fluids are defined by scientists as fluids whose rest frame pressure and density can be perfectly specified. A perfect fluid's pressure is isotropic, which means it has the same pressure in all directions. A perfect fluid has no shear stress, no conduction and no viscosity. Several authors (Aditya and Reddy, 2019; Chirde et al., 2020; Das and et al., 2018; Nath and Sahu, 2019; Vinutha and Vasavi, 2021) have researched the perfect fluid using various cosmological models.

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3.2 The metric and field equations

The five-dimensional FRW metric is considered in the form

$$ds^2 = dt^2 - a^2(t)[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) + (1-kr^2)d\psi^2] \tag{3.1}$$

The field equations of S$\acute{a}$ez-Ballester for coupled scalar and tensor fields are as follows:

$$R_{ij} - \frac{1}{2}g_{ij}R - \omega\phi^k(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,k}\phi^{,k}) = -8\pi T_{ij} \tag{3.2}$$

where the scalar field ϕ satisfies the equation

$$2\phi^k\phi^{;i}_{;i} + k\phi^{n-1}\phi_{,k}\phi^{,k} = 0 \tag{3.3}$$

Furthermore, we have

$$T^{ij}_{;i} = 0 \tag{3.4}$$

This is a result of the field equations (3.2) and (3.3). This equation physically expresses the matter source's conservation and also provides equations of motion. Here, ω and *n* are constants and the comma and semicolon denotes partial and covariant differentiation. As in general relativity, the other symbols have their usual meaning.

The non-vanishing components of the Einstein tensor for the metric (3.1) are define as follows

$$G^1_1 = G^2_2 = G^3_3 = G^4_4 = 3\frac{\ddot{a}}{a} + 3\frac{\dot{a}^2}{a^2} + 3\frac{k}{a^2} \tag{3.5}$$

$$G^5_5 = 6\frac{\dot{a}^2}{a^2} + 6\frac{k}{a^2} \tag{3.6}$$

where an overhead dot represents ordinary differentiation with respect to *t* and $k = 1, -1, 0$ for closed, open and flat models respectively

The perfect fluid distribution's energy momentum tensor is given by

$$T_{ij} = (\rho + p)u_iu_j - g_{ij}p \tag{3.7}$$

Here,

$$u_i u_i = 1, \qquad u_i u_j = 0 \tag{3.8}$$

The field equations (3.2)-(3.4), with the help of equations (3.5)-(3.8), for the metric (3.1) can be expressed using co-moving coordinates as

$$6\frac{\dot{a}^2}{a^2} + 6\frac{k}{a^2} - \frac{\omega}{2}\frac{\dot{\phi}^2}{\phi^2} = -8\pi\rho \tag{3.9}$$

$$3\frac{\ddot{a}}{a} + 3\frac{\dot{a}^2}{a^2} + 3\frac{k}{a^2} + \frac{\omega}{2}\frac{\dot{\phi}^2}{\phi^2} = 8\pi p \tag{3.10}$$

$$\frac{\ddot{\phi}}{\phi} + 4\frac{\dot{a}}{a}\frac{\dot{\phi}}{\phi} + \frac{n}{2}\frac{\dot{\phi}^2}{\phi^2} = 0 \tag{3.11}$$

$$\dot{\rho} + 4\frac{\dot{a}}{a}(\rho + p) = 0 \tag{3.12}$$

The Hubble's parameter *H* and the deceleration parameter *q* are defined by

$$H = \frac{\dot{a}}{a} \tag{3.13}$$

$$q = \frac{-(\dot{H} + H^2)}{H^2} \tag{3.14}$$

3.3 Solution of the field equations

The field equations (3.9)-(3.11) are composed of three independent equations with four unknowns: *a*, *p*, ρ and ϕ [equation (3.12) is the consequence of equations (3.9) - (3.11).] As a result, to achieve a definitive solution, an additional condition is required. So we apply well-known relationship between a scalar field ϕ and the universe's scale factor *a(t)*(Johri and Kalyani, 1994).

$$\phi = \phi_0 a^n \tag{3.15}$$

where $n > 0$ and ϕ_0 are constants.

In this particular case, we obtain the physically significant models as

3.3.1 Case (I): For $k = 1$ (i.e., closed model)

Using equation (3.15) and the field equations (3.9)-(3.11), the scale factor solutions in this case are given below,

$$a(t) = [(\frac{\frac{n^2}{2} + n + 4}{n{\phi_0}^{\frac{n+2}{2}}})(a_0 t + t_0)]^{\frac{1}{(\frac{n^2}{2}+n+4)}} \tag{3.16}$$

We can now write the metric (3.1) using (3.16) (i.e. we choose $a_0 = 1$, $t_0 = 0$) with the correct coordinates and constants as

$$ds^2 = dt^2 - [(\frac{\frac{n^2}{2} + n + 4}{n{\phi_0}^{\frac{n+2}{2}}})t]^{\frac{2}{(\frac{n^2}{2}+n+4)}}[\frac{dr^2}{1-r^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) + (1-r^2)d\psi^2] \tag{3.17}$$

along with the scalar field, given by

$$\phi = \phi_0[(\frac{\frac{n^2}{2} + n + 4}{n{\phi_0}^{\frac{n+2}{2}}})t]^{\frac{n}{(\frac{n^2}{2}+n+4)}} \tag{3.18}$$

The model (3.17) is a five-dimensional FRW radiating model with the physical parameters viz. volume *V*, Hubble's parameter *H*, energy density ρ and isotropic pressure *p*, which are all important in cosmology.

$$V = a^4 = [(\frac{\frac{n^2}{2} + n + 4}{n{\phi_0}^{\frac{n+2}{2}}})t]^{\frac{4}{(\frac{n^2}{2}+n+4)}} \tag{3.19}$$

$$H = (\frac{1}{\frac{n^2}{2} + n + 4})\frac{1}{t} \tag{3.20}$$

$$\rho = \frac{1}{8\pi}[\frac{(\omega n^2 - 12)}{2t^2(\frac{n^2}{2} + n + 4)^2}] - 6[(\frac{\frac{n^2}{2} + n + 4}{n{\phi_0}^{\frac{n+2}{2}}})t]^{-\frac{2}{(\frac{n^2}{2}+n+4)}} \tag{3.21}$$

$$p = \frac{1}{8\pi}[\frac{(\omega n^2 + 6)}{2t^2(\frac{n^2}{2} + n + 4)^2} + 3(\frac{\frac{n^2}{2} + n + 4}{n{\phi_0}^{\frac{n+2}{2}}}t)^{-\frac{2}{(\frac{n^2}{2}+n+4)}} - 3(\frac{\frac{n^2}{2} + n + 3}{\frac{n^2}{2} + n + 4})] \tag{3.22}$$

3.3.2 Case (II): For $k = -1$ (i.e., Open model)

In this case, the model is

$$ds^2 = dt^2 - [(\frac{\frac{n^2}{2}+n+4}{n{\phi_0}^{\frac{n+2}{2}}})t]^{\frac{2}{(\frac{n^2}{2}+n+4)}}[\frac{dr^2}{1+r^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) + (1+r^2)d\psi^2] \tag{3.23}$$

The energy density is given by

$$\rho = \frac{1}{8\pi}[\frac{(\omega n^2 - 12)}{2t^2(\frac{n^2}{2}+n+4)^2}] + 6[(\frac{\frac{n^2}{2}+n+4}{n{\phi_0}^{\frac{n+2}{2}}})t]^{-\frac{2}{(\frac{n^2}{2}+n+4)}} \tag{3.24}$$

and the pressure is defined by

$$p = \frac{1}{8\pi}[\frac{(\omega n^2 + 6)}{2t^2(\frac{n^2}{2}+n+4)^2} - 3(\frac{\frac{n^2}{2}+n+4}{n{\phi_0}^{\frac{n+2}{2}}}t)^{-\frac{2}{(\frac{n^2}{2}+n+4)}} - 3(\frac{\frac{n^2}{2}+n+3}{\frac{n^2}{2}+n+4})] \tag{3.25}$$

3.3.3 Case (III): For $k = 0$ (i.e., flat model)

In this case, the model is becomes

$$ds^2 = dt^2 - [(\frac{\frac{n^2}{2}+n+4}{n{\phi_0}^{\frac{n+2}{2}}})t]^{\frac{2}{(\frac{n^2}{2}+n+4)}}[dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) + d\psi^2] \tag{3.26}$$

The energy density is

$$\rho = \frac{1}{8\pi}[\frac{(\omega n^2 - 12)}{2t^2(\frac{n^2}{2}+n+4)^2}] \tag{3.27}$$

and the pressure is

$$p = \frac{1}{8\pi}[\frac{(\omega n^2 + 6)}{2t^2(\frac{n^2}{2}+n+4)^2} - 3(\frac{\frac{n^2}{2}+n+3}{\frac{n^2}{2}+n+4})] \tag{3.28}$$

For all the cases (i.e, closed, open and flat), the scalar field ϕ, volume *V* and the Hubble's parameter *H* are same, as given by equations (3.18), (3.19) and (3.20).

Also, the deceleration parameter *q* is the same for all the cases and it is defined as follows:

$$q = (\frac{n^2}{2} + n + 3) \tag{3.29}$$

All graphs are drawn using $\pi = 3.14$, $n = 1$, $\omega = 500$, $\phi_0 = .001$

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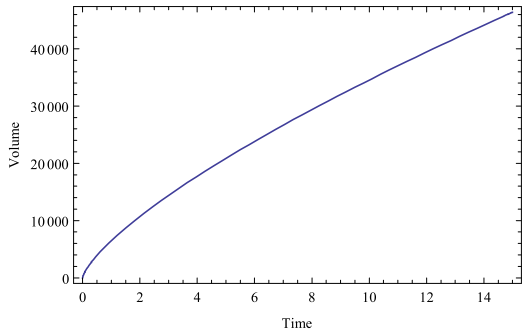

Figure 3.1: *V* vs. *t* (billion years)

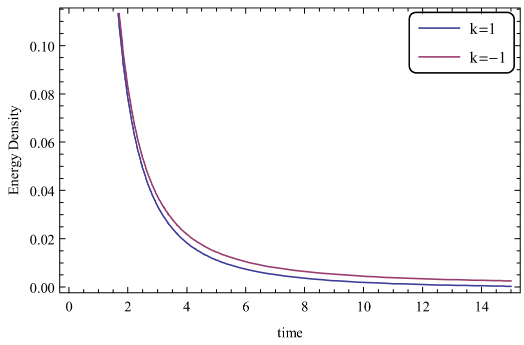

Figure 3.2: ρ for $k = 1, -1$ vs. *t* (billion years)

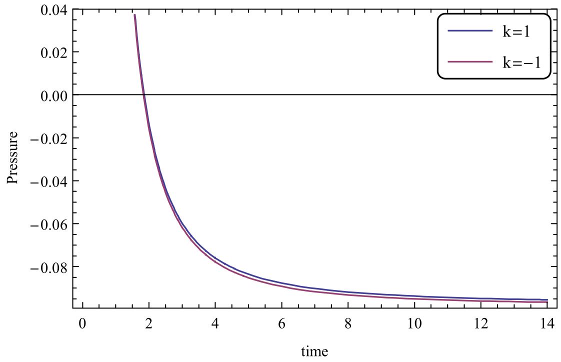

Figure 3.3: *p* for $k = 1, -1$ vs. *t* (billion years)

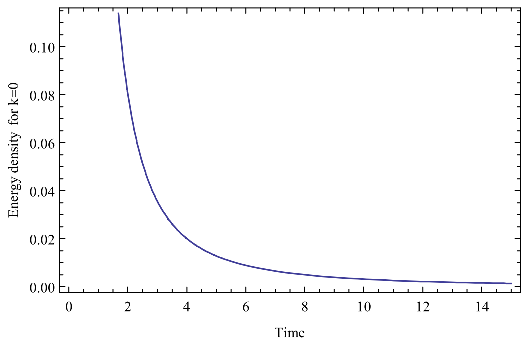

Figure 3.4: ρ for $k = 0$ vs. *t* (billion years)

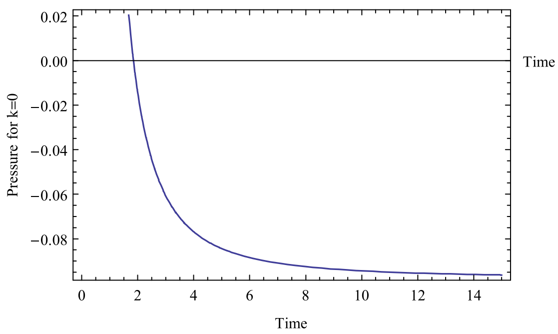

Figure 3.5: *p* for $k = 0$ vs. *t* (billion years)

3.4 Physical and geometrical interpretation

Equations (3.17), (3.23) and (3.26) in Sáez-Ballester theory represents FRW five-dimensional radiating closed, open and flat models. It should be noted that none of the models contain an initial singularity. The energy density and the pressure of closed and open models diverges at $t = 0$ and decreases throughout time. They will vanishes as $t \to \infty$ [shown in fig-3.2, fig-3.3]. Also in the flat model, the energy density and the pressure decreases with time and eventually vanishes for infinitely large time (i.e. $t \to \infty$)[as seen in fig-3.4 and fig-3.5]. At the initial epoch, they also diverges. All the models have the same spatial volume, which increases with time and eventually reaches infinity when $t \to \infty$. At initial epoch (i.e., $t = 0$) volume vanishes. [as illustrated in fig-3.1]. The average Hubble's parameter, given by equation (3.20), will diverge at the initial epoch, i.e. when $t = 0$ and will vanishes when time *t* increases indefinitely. The scalar field increases with time in all the models. In each case, the deceleration parameter is $q = (\frac{n^2}{2} + n + 3)$. The model represents a standard decelerating universe because the deceleration parameter $q > 0$ for all values of *t*. Despite the fact that the universe decelerates in this case, it will accelerate in infinite time due to cosmic re-collapse, in which the universe inflates "decelerates and then accelerates" (Nojiri and Odintsov, 2003; Rao et al., 2015a; Reddy and Naidu, 2007; Vishwakarma, 2003).