Chapter 6

Magnetized Bianchi Type-IX Cosmological Model with Bilinear Deceleration Parameter in Sen-Dunn Theory of Gravitation

6.1 Introduction

The deceleration parameter is one of the most significant cosmological parameters for understanding the dynamic behaviour of the universe. Mishra and Chand [Mishra & Chand(2016)] proposed another form of deceleration parameter, a bilinear function of cosmic time ”t”. Further, researchers [Mishra et al.(2018), Mishra & Chand(2020)] discussed a cosmological model with various cosmological contexts considering the bilinear varying deceleration parameter. Some researchers [Sofuoğlu(2016), Pradhan et al.(2005), Bali & Dave(2003), Bali & Yadav(2005)] have discussed and studied Bianchi type-IX cosmological model with various contexts in different framework. Recently, Sharma and Poonia [Sharma & Poonia(2021)] investigated the cosmic inflation in Bianchi type-IX with bulk viscosity.

The researchers [Tyagi & Chhajed(2012), Ghate & Sontakke(2014)] studied anisotropic Bianchi type-IX cosmological model with an electromagnetic field. Sahoo et al. [Sahoo et al.(2018)] also investigated magnetized strange quark matter considering bilinear varying deceleration parameters in the f(R, T) gravity framework. Thus, this chapter dealt with the magnetized Bianchi type-IX cosmological model in the Sen-Dunn gravitational theory. Here, another form of bilinear deceleration parameter proposed by Mishra and Chand [Mishra & Chand(2016)] is assumed to investigate the derived model's physical and geometrical properties with the recent observations.

6.2 Metric and Field Equation

Bianchi Type IX metric is expressed as

$$ds^2 = -dt^2 + A^2dx^2 + B^2dy^2 + (B^2sin^2y + A^2cos^2y)dz^2 - 2A^2cosy\,dx\,dz \tag{6.1}$$

where A, B are the function of cosmic time t.

The Sen-Dunn's field equation (in natural units $c = 1$, 8πG = 1) is defined as

$$R_{ij} - \frac{1}{2}Rg_{ij} = \omega\phi^{-2}(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,k}\phi^{,k}) - \phi^{-2}T_{ij} \tag{6.2}$$

The Energy momentum tensor is given as

$$T_{ij} = (\rho + p)u_iu_j + pg_{ij} + E_{ij} \tag{6.3}$$

where E_{ij} is the electromagnetic field tensor satisfying Maxwell's equation and is given as

$$E_{ij} = \frac{1}{4\pi}[g^{kl}F_{ik}F_{jl} - \frac{1}{4}F_{kl}F^{kl}g_{ij}] \tag{6.4}$$

From Maxwell's equation, it follows that

$$\frac{\partial}{\partial x^j}\left(F^{ij}\sqrt{-g}\right) = 0 \tag{6.5}$$

Considering the incident magnetic field along x-axis. The only non-vanishing component of F_{ij} is F_{23}, which leads to

$$F_{23} = K siny(constant) \tag{6.6}$$

Thus, the field equation for the model is as follows

$$2\frac{\ddot{B}}{B}+\frac{\dot{B}^2}{B^2}+\frac{1}{B^2}-\frac{3A^2}{4B^4} = \frac{\omega}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2-\phi^{-2}\left(p+\frac{K^2}{8\pi b^4}\right) \tag{6.7}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}+\frac{A^2}{4B^4} = \frac{\omega}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2-\phi^{-2}\left(p-\frac{K^2}{8\pi b^4}\right) \tag{6.8}$$

$$2\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}^2}{B^2}+\frac{1}{B^2}-\frac{A^2}{4B^4} = \phi^{-2}\left(\rho-\frac{K^2}{8\pi b^4}\right)-\frac{\omega}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2 \tag{6.9}$$

where overdot represents the ordinary differentiation with t.
The average scale factor for Bianchi type-IX space-time is as follows

$$a(t) = (AB^2)^{\frac{1}{3}} \tag{6.10}$$

The physical parameters are spatial volume (V), expansion scalar(θ), Hubble parameter (H), anisotropic parameter (A_m), shear scalar (σ), and the deceleration parameter are defined as

$$V = a^3 \tag{6.11}$$

$$\theta = 3H \tag{6.12}$$

$$H = \frac{\dot{a}}{a} \tag{6.13}$$

$$A_m = \frac{1}{2}\sum_{1}^{3}\left(\frac{\Delta H_i}{H}\right)^2 = \frac{2\sigma^2}{3H^2} \tag{6.14}$$

$$\sigma^2 = \frac{1}{2}\left(\sum_{n=1}^{3} H_i^2 - \frac{1}{3}\theta^2\right) \tag{6.15}$$

$$q = -\frac{a\ddot{a}}{\dot{a}^2} \tag{6.16}$$

where $i = x, y, z$ represents the directional Hubble parameter.

6.3 Solutions of the field equation

The independent field equations (6.7)-(6.9) contain five unknown parameters, i.e., A, B, p, ρ, and ϕ. For the deterministic exact solution of the obtained field equations, consider two or more plausible conditions that are as follows:

To examine the characteristic behaviour of the model, assuming that the deceleration parameter is a postulated bilinear function of space-time "t" [Mishra & Chand(2016)].

$$q = -\frac{\alpha t}{1+t} \tag{6.17}$$

where $\alpha \geq 0$. The relation between $q(t)$ and $H(t)$ on integration is given as

$$H = \frac{1}{\int[1+q(t)]dt + c_1} \tag{6.18}$$

Thus on integration, it is given as

$$H = \frac{1}{(1-\alpha)t + \alpha log(t+1) + c_1} \tag{6.19}$$

From here it can be defined that at $t=0, H\rightarrow\infty$, thus $c_1=0$

$$H=\frac{1}{(1-\alpha)t+\alpha log(1+t)} \tag{6.20}$$

On expanding the above term

$H=\frac{1}{(1-\alpha)t+\alpha(t-\frac{t^2}{2}+\frac{t^3}{3}-\frac{t^4}{4}+...)}$

$H=\frac{1}{t}\left[1+\alpha t\left(-\frac{1}{2}+\frac{t}{3}-\frac{t^2}{4}\right)\right]^{-1}$

The scale factor and Hubble parameter have the following relationship as given below:

$H=\frac{\dot{a}}{a}$

On simplifying, it follows

$H=\frac{1}{t}+\frac{\alpha}{2}+\frac{(-4\alpha+3\alpha^2)}{12}t+\frac{(6\alpha-8\alpha^2+3\alpha^3)}{24}t^2+\frac{(-144\alpha+260\alpha^2-180\alpha^3+45\alpha^4)}{720}t^3+....$

By integrating the above equation, it is obtained as

$log\, a-log\, a_0=logt+\frac{\alpha}{2}t+\frac{(-4\alpha+3\alpha^2)}{24}t^2+\frac{(6\alpha-8\alpha^2+3\alpha^3)}{72}t^3+\frac{(-144\alpha+260\alpha^2-180\alpha^3+45\alpha^4)}{2880}t^4+$ ...

Thus resulting in the scale factor as

$$a(t)=a_0te^{F(t)} \tag{6.21}$$

where a_0 is constant of integration and

$F(t)=\frac{\alpha}{2}t+\frac{\alpha(-4+3\alpha)}{24}t^2+\frac{(6\alpha-8\alpha^2+3\alpha^3)}{72}t^3+\frac{(-144\alpha+260\alpha^2-180\alpha^3+45\alpha^4)}{2880}t^4+.....$

Using the constraint that the shear scalar is proportional to the expansion scalar [Thorne(1967)], it can derive the relationship between the metric potential as

$$A=B^m \tag{6.22}$$

where m is an arbitrary constant.

Further, assuming that the scale factor $a(t)$ and scalar field ϕ have a power-law relationship [Johri & Sudharsan(1989)]

$$\phi(t) = \phi_0 a^f(t) \tag{6.23}$$

where ϕ_0 and f being an ordinary constant.
Using the above equation (6.21), (6.22) in (6.21), determining the metric potential of the model

$$A = \left(a_0 t e^{F(t)}\right)^{\frac{3m}{m+2}} \tag{6.24}$$

$$B = \left(a_0 t e^{F(t)}\right)^{\frac{3}{m+2}} \tag{6.25}$$

Thus, the model for the metric (6.1) for $a_0 = 1$ is expressed as

$$\begin{aligned} ds^2 = -dt^2 + \left(te^{F(t)}\right)^{\frac{6m}{m+2}} dx^2 + \left(te^{F(t)}\right)^{\frac{6}{m+2}} dy^2 \\ + \left[\left(te^{F(t)}\right)^{\frac{6m}{m+2}} sin^2 y + \left(te^{F(t)}\right)^{\frac{6}{m+2}} cos^2 y\right] dz^2 - 2\left(te^{F(t)}\right)^{\frac{6m}{m+2}} cosy\, dx\, dz \end{aligned} \tag{6.26}$$

6.4 Cosmological parameters of the model

The energy density and the pressure for the model (6.26) are expressed in terms of "H" as

$$\rho = \frac{\phi^2}{2}\left[\frac{45m}{(m+2)^2}H^2 - \frac{3}{(m+2)}\dot{H} + \left(a_0 t e^{F(t)}\right)^{\frac{-3}{m+2}} + \frac{3}{2}f^2H^2\right] \tag{6.27}$$

$$-p = \frac{\phi^2}{2}\left[\frac{9(m^2+m+4)}{(m+2)^2}H^2 + \frac{3(m+3)}{(m+2)}\dot{H} + \left(a_0 t e^{F(t)}\right)^{\frac{-6}{m+2}} + \frac{1}{2}\left(a_0 t e^{F(t)}\right)^{\frac{6(m-2)}{m+2}} - \frac{3}{2}f^2H^2\right] \tag{6.28}$$

Also, the other cosmological parameters that plays a vitol role in identifying the physical description of the evolution of the universe for the model are as follows
Spatial volume (V)

$$V = a_0^3 t^3 e^{3F(t)} \tag{6.29}$$

Expansion scalar (θ)

$$\theta = \frac{3}{(1-\alpha)t + \alpha log(1+t)} \tag{6.30}$$

Shear scalar (σ)

$$\sigma = \frac{\sqrt{3}H(m-1)}{m+2} \tag{6.31}$$

Anisotropic parameter(A_m)

$$A_m = \frac{2(m-1)^2}{(m+2)^2} \tag{6.32}$$

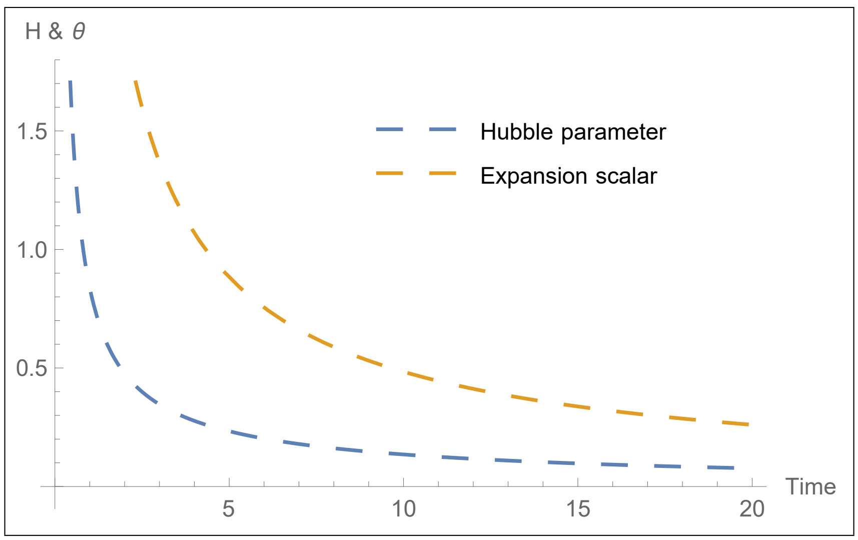

Figure 6.1: Variation of H and θ $vs.$ t, For α = 0.5

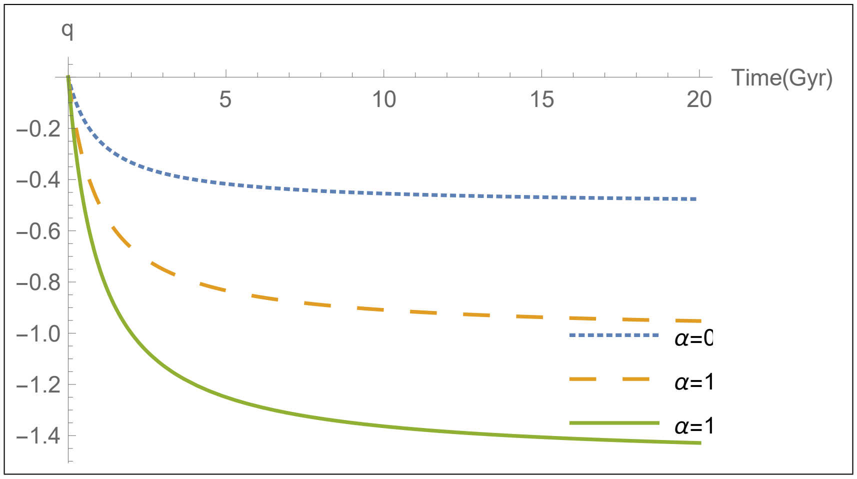

Figure 6.2: Variation of q *vs*. *t*

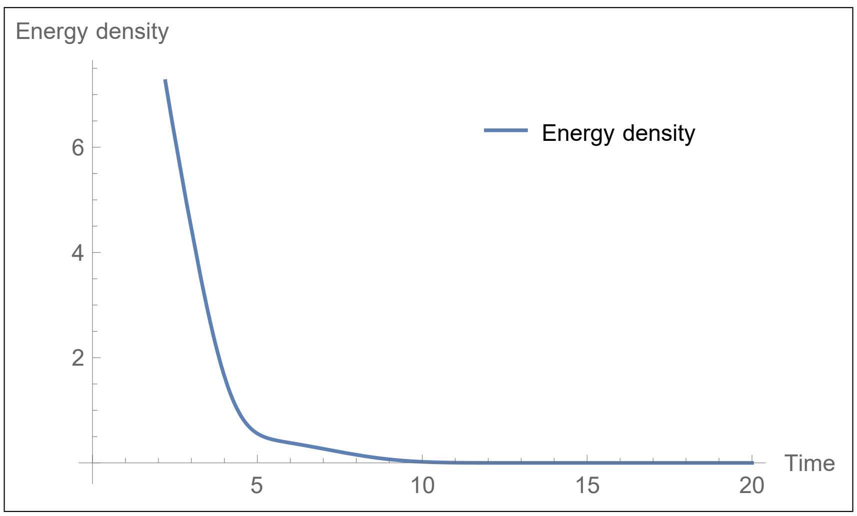

Figure 6.3: Variation of ρ *vs*. *t*, For a_0 = ϕ_0 = 1, m = 1.5, α = 0.5, f = 0.5

The dynamical behaviour of the energy conditions for the model (6.26), in terms of energy density and pressure for the increasing cosmic time, is observed as follows

(i) Null energy condition (NEC): $\rho + p \leq 0$

(ii) Weak energy condition (WEC): $\rho \geq 0; \rho + p \leq 0$

(iii) Dominant energy condition (DEC): $\rho + p \leq 0; \rho - p \geq 0$

(iv) Strong energy condition (SEC): $\rho + 3p \leq 0$

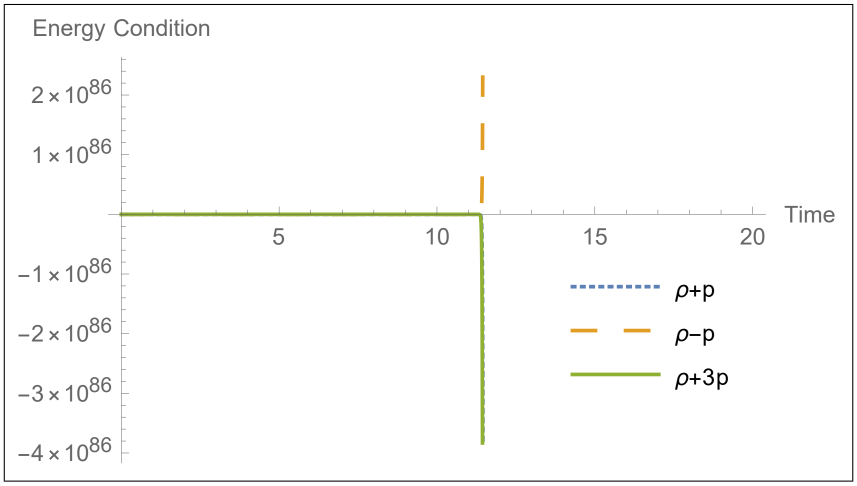

Figure 6.4: Variation of *energy condition vs. t*, For a_0 = ϕ_0 = 1, m = 1.5, α = 0.5, f = 0.5

Here, the energy condition for the model is violated with a very different deviation. Initially, all the energy conditions obey for a certain period and then are violated as time increases, $t \to \infty$. As, the null energy condition is violated after some period, the remaining other conditions are also violated except dominant energy condition (DEC). The strong energy condition (SEC) for the model is also violated. The scenario of the energy condition for the model might exist as per the phantom model of dark energy for EoS parameter $\omega = -1.1$ with negative pressure, [Yadav(2011)].

6.5 Conclusion

This chapter presents the dynamical behaviour of the magnetized Bianchi type-IX cosmological model in Sen-Dunn's gravitational theory with a bilinear deceleration parameter. The volume for model (6.24) indicates the same result as in the previous chapter that there is a singularity at $t = 0$, and as $t \to \infty$ increases, the volume exponentially. An exponential expansion for $t > 0$ and $0 < \alpha < 1$ with $-1 < q < 0$ and super-exponential expansion for $t > 0$ and $\alpha > 1$ with $0 < q < -2$ is observed. Furthermore the model is always accelerating as shown in Fig 6.2. The Hubble parameter and expansion scalar of the derived model (6.24) gradually tends to zero as $t \to \infty$ as plotted in Fig. 6.1. It can be seen that the model is anisotropic throughout the cosmic time, attaining isotropy only for the particular case when $m = 1$. The model thus obtained is shearing, non-rotating and expanding. In the presence of magnetic field, the model's energy density decreases with time when $f \geq 0.45$ as shown in Fig. 6.3, but for $f < 0.45$, the deviation of energy density increases as $t \to \infty$. The energy condition for the model is satisfied only for some particular range of time and later violated, as seen from Fig.6.4. The same depiction of energy condition is studied for the phantom model of dark energy [Yadav(2011)]. Cosmology might handle the co-existence with this condition as well. Thus, further study of this model may be beneficial in understanding the evolution of the dark energy universe.