Chapter 1

Introduction

1.1 General Relativity

General Relativity (GR), first proposed by Albert Einstein in 1915 is considered as the foundational theory in modern astrophysics. Three general principles guided Einstein; the principle of covariance, the principle of equivalence, and Mach's principle in developing the general theory of relativity. Also, Einstein's theory of gravity generalizes special relativity. Redefining Newton's law presented the unification of gravitation geometrically in four-dimensional space-time. The curvature of the space-time geometry is mainly related to the universe's energy and momentum of matter and radiation. Beyond Newton's law of universal gravitation in general relativity, which best describes classical gravity, there are specific predictions over time. The geometry of space, the motion of bodies in free fall, light propagation, gravitational lensing, the redshift of light, gravitational time dilation, singularity, and black holes. The foundation of cosmology and its evolution is depicted by the time-dependent solution of the general theory of relativity, which led to the discovery of the big bang and CMBR. Einstein, publishing the special theory of relativity in 1905, focused on the inclusion of gravity with his new relativistic framework. After numerous attempts, Einstein in 1915 presented his new theory at the Prussian Academy of Science, known as Einstein's theory of general relativity. Einstein noticed the fundamental relation between gravitation and space-time. Einstein, in his theory, described the space-time curvature with the energy-momentum tensor within that space-time for any matter. Einstein's theory described space-time in a Pseudo Riemannian metric as

$$ds^2 = g_{ij}dx^i dx^j \tag{1.1}$$

where $i, j = 1, 2, 3$ and 4 and the symmetric tensor constituents g_{ij} serve as gravitational potential.

The equation below defines Einstein's field equation in general relativity led by the gravitational field:

$$R_{ij} - \frac{1}{2}g_{ij}R + \Lambda g_{ij} = -8\pi T_{ij} \tag{1.2}$$

where R_{ij} is the Ricci tensor, R is the Ricci scalar, and T_{ij} is the energy-momentum tensor by the matter, and Λ is the cosmological constant. Due to the non-divergence of the Einstein tensor $G_{ij} = R_{ij} - \frac{1}{2}g_{ij}R$ the field equation conserve the energy-momentum. Thus leading

$$T^{ij}_{;j} = 0 \tag{1.3}$$

which is termed as the energy-momentum conservation equation, formulating the equations of motion of matter.

The three general principles that guided Einstein are:

Principle of Covariance: The principle of covariance highlights the use of only those physical quantities in improving physical laws, measurements of which might be made by observers with diverse points of view and still have evidence. Thus the mathematical representation of the set of coordinate transformations must be represented covariantly, irrespective of the space-time coordinate system.

Principle of Equivalence: The equivalence principle generalizes the well-known fact that observers falling freely are not affected by gravitation because gravitational and inertial mass are equivalent. According to the Equivalence Principle, at any spacetime point in an arbitrary gravitational field there is a "locally inertial" coordinate system in which the effects of gravitation are absent in a sufficiently small spacetime neighborhood of that point.

Mach's Principle: The Mach Principle asserts that the distribution of matter in the cosmos determines a body's inertial properties. Because the gravitational field interacts with all matter, it could express the Mach principle relationship between inertial and remote value in terms of the gravitational field. The ratio of a body's inertial mass to its active gravitational mass is regarded to express this in a unitless manner.

1.2 Cosmological Constant

Einstein in 1917 added the cosmological constant to his field equation to establish a static cosmic solution. He altered his field equation to bring a static universe by containing repulsive components which balance the usual gravitational attraction. Einstein observing his field equation, discovered that an additional term can naturally accommodate the geometry side of his field equation. With this alteration, he added a coefficient Λ termed the cosmological constant. This cosmological constant is simply narrated as energy density or vacuum energy. Later Einstein stated this term as his biggest blunder soon after Hubble's observation in 1929, which indicated the universe was not static but expanding. After that, Einstein ditched the concept of Λ in 1931. Since the discovery of the accelerating universe until the late 1990s, the value of the cosmological constant has a positive non-zero value [Perlmutter et al.(1998),Perlmutter et al.(1999),Riess et al.(1998),Schmidt et al.(1998),Weinberg(2015)]. The estimated value of Λ is minimal, decreasing with cosmic expansion, which states that the universe's age is old. It is also associated with the term so-called dark energy, which is the present candidate for the universe's evolution. However, many scientists continue to be interested in theorizing about and conducting empirical studies on the cosmological constant.

1.3 Cosmology and Cosmological Model

Cosmology is a branch of astronomy that investigates the universe's evolution from its beginning to the near future, considering the big bang and the initial formation of the universe. The Greek term "kosmos" is where the word "cosmology" originated, which means the study of the universe. Scientifically, this study of the universe's large-scale structure is termed cosmology. The elements that make up the universe's large-scale structure are many stars, a group of stars called galaxies, clusters of galaxies called superclusters, voids, and many mysterious characters still invisible to human knowledge. Cosmology includes general relativity, theoretical cosmology, observational astrophysics, quantum mechanics, particle physics, and plasma physics.

As dealing mainly with observational facts, Physical cosmology is a sub-branch of astronomy that portrays the universe's immense figure and dynamics. The Big-Bang theory governs modern cosmology to determine observational astronomy and particle physics together to understand the universe's evolution. More precisely, the ΛCDM model is the current standard model of cosmology that is parameterized both with DE and DM.

The fundamental queries about the universe's evolution and the production of large-scale structures are addressed by a mathematical description of the universe, which is a cosmological model of the observed cosmos. The cosmological principle represents the geometry of homogeneous and isotropic space-time expressing the Robertson-Walker metric.

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 sin^2\theta d\phi^2\right] \tag{1.4}$$

Here $a(t)$ is considered the cosmological scaling factor. The parameter k holds the values $+1, -1, 0$, according to which it can define the description of the universe. According to cosmological study, the universe with the value $k = +1$ positively curved space is a closed universe, $k = -1$ negatively curved space is an open universe, $k = 0$ is a flat universe. In addition, Friedmann [Friedmann(1922)] also defined the evolution of the scale factor $a(t)$ using Einstein's field equation for all three curvatures. As in large-scale observations, the universe is spatially homogeneous and isotropic, which is well demonstrated by the FRW model. Although the FRW model best defines the homogeneity and isotropy of the universe. The recent CMBR evidence shows the anisotropic behaviour and small magnetic field. Thus, a significant deviation from the FRW hypothesis occurred in the early phase of cosmic evolution. Thus, understanding the anisotropic cosmological model is beneficial to understanding the isotropic cosmological model. To bridge the gap between the FRW model and the inhomogeneous and anisotropic cosmos, Bianchi models are crucial in modern cosmology that is homogeneous and anisotropic .

1.4 Bianchi Space-time

The Bianchi-type cosmological model plays an essential role in modern cosmology. Bianchi-type cosmological models are homogeneous and anisotropic, studying the universe's isotropy as time passes. The anisotropic universe is theoretically more general than the isotropic universe. Bianchi space-time is crucial when building spatially homogenous and anisotropic models using the basic field equations and their solutions.

Bianchi [Bianchi(1898)] classified the relevant three-dimensional space-time. Bianchi is structured as the nine sets of structural constants for the three-parameter automorphism group, algebraically used to define space-time uniformly. Therefore, a three-parameter group of movements with a limited number of degrees of freedom is admissible in Bianchi space-time. To the present day, there has been a significant amount of work executed on the anisotropic universe. The simplest Bianchi models of nine types are spatially homogeneous [Taub(1951)] and have been investigated by many. Bianchi type-I, V, VII_0 and VII_h models tend towards isotropy at an arbitrarily large time, attributing to the formation of galaxies [Collins & Hawking(1973)]. The Bianchi models that represent the generalized flat FRW model are type-I and VII_0, and types-V and VII_h represent the generalized open FRW models. And the generalized non-flat models are the types II, VI, VIII, and IX.

For this research, the particular Bianchi type-III, VI_0, and IX have been used to determine the cosmological models of the universe. The relativistic Bianchi type-III, VI_0 and IX cosmological models, which are anisotropic and homogeneous in space-time, are as follows:

Bianchi type-III metric is expressed as

$ds^2 = -dt^2 + A^2dx^2 + B^2e^{-2mx}dy^2 + C^2dz^2$

where the metric potentials (directional scale factors) A, B, C act as a function of cosmic time and m as a constant.

Bianchi type-VI_0 metric is defined by

$ds^2 = -dt^2 + A^2dx^2 + B^2e^{-2mx}dy^2 + C^2e^{2mx}dz^2$

here also, m acts as a constant, and A, B, C as a function of cosmic time.

Bianchi type-IX metric is given by

$ds^2 = -dt^2 + A^2dx^2 + B^2dy^2 + (B^2sin^2y + A^2cos^2y)dz^2 - 2A^2cosy\ dx\ dz$

where A and B are characterized as a function of time t.

1.5 Energy-Momentum Tensors

The stress-energy tensor, often known as the energy-momentum tensor, is an extension of Newtonian physics' stress tensor. It is a physical tensor quantity that characterises the density and flux of energy and momentum in space-time attributed to matter, radiation, and the field of non-gravitational force. In Einstein's field equation of general relativity, the gravitational field is made up of this density and flow of energy and momentum.

1.5.1 Perfect Fluid

The frictionless, homogeneous, and incompressible fluid where the normal force acts between the neighbouring fluid layers cannot support any tangential stress or shear action.

The pressure of the fluid in every location is equal in all directions, whether in rest or motion. In general relativity, perfect fluids simulate idealized matter distributions, such as the inside of a star or an isotropic cosmos. The EoS for the ideal fluid can be utilized to represent the evolution of the cosmos in Friedmann-Lemaître-Robertson-Walker equations.

The formulation for a perfect fluid's stress-energy tensor in general relativity can be written as

$$T_{ij} = \left(\rho + \frac{p}{c^2}\right) u_i u_j + p g_{ij} \tag{1.5}$$

where ρ is the fluid's energy density, u its four-velocity vector, and p its pressure.

1.5.2 Viscous fluid

Viscosity is the resistance between the fluid's layers to the fluid's movement. The fluids with higher viscosity are known as viscous fluids. The viscous fluid's energy-momentum tensor is given as

$$T_{ij} = \rho u_i u_j + \left(\frac{p}{c^2} - \xi\theta\right) H_{ij} - 2\eta\sigma_{ij} \tag{1.6}$$

Here, u_i is the four-velocity vector, p is the isotropic pressure, ρ is the fluid density, and η, ξ are coefficients of bulk viscosity and shear viscosity, respectively.

where $H_{ij} = g_{ij} - u_i u_j$ denotes projection tensor and σ_{ij} denotes shear tensor given by

$$\sigma_{ij} = \frac{1}{2}\left(u^i_{,\mu} H^{\mu j} + u^i_{,\mu} H^{i\mu}\right) - \frac{1}{3}\Theta H_{ij} \tag{1.7}$$

1.5.3 Electromagnetic field

The physical field that is produced by an electrically charged object is known as an electromagnetic field. The electromagnetic field is generated by the charge on matter particles in space. The electromagnetic field created by the passage of charge in space has an energy-momentum tensor with proper density ρ with a four-dimensional velocity u_i and current density vector j_i is given as

$$T_{ij} = -F_{i\alpha}F^{\alpha j} + \frac{1}{4}g_{ij}F^{\alpha\beta}F_{\alpha\beta} \tag{1.8}$$

where the electromagnetic field F obeys the Maxwell equation:

$$F^{ij}_{;j} = \frac{4\pi}{c}j^i \tag{1.9}$$

$$F_{[ij,k]} = 0 \tag{1.10}$$

1.6 Lyra's Geometry

Einstein added the cosmological constant term Λ into the field equations to geometrize gravity using general relativity. But later, after discovering the large-scale recession of galaxies or expansion of the universe by Hubble, Einstein called the term his most significant blunder in his life. Even though Einstein's theory was used to describe cosmological models of the cosmos, it falls back on other forces defined in classical physics, such as electromagnetic forces, which is also a geometric phenomenon in cosmology. Weyl [Weyl(1918)] proposed a unified theory with the geometrizing theory of gravitation with electromagnetism. But due to the lack of non-integrability of the length transfer, it did not take the approach. Later on, Lyra [Lyra(1951)] modified the Riemannian geometry by removing the non-integrability requirement and adding the gauge function to the structureless manifold in Wely's unified theory resulting in the formation of the displacement vector.

Lyra's geometry is considered the extension of Einstein's theory of gravitation. Lyra [Lyra(1951)] modified Riemannian geometry by defining the displacement vector PQ between two neighbouring point P(x^i) and Q($x^i + dx^i$) involving the components $\zeta = x^0(x^i)dx^i$, where $x^0(x^i)$ is non-zero gauge function. Lyra included both gauge function

$x^0(x^i)$ and reference system x^i. Thus Lyra's geometry under the transformation of the co-ordinate under metric tensor is given as

$$ds^2 = g_{ij}x^0 dx^i x^0 dx^j \tag{1.11}$$

which is invariant under both the co-ordinate and gauge transformation and with g_{ij} as the fundamental tensor of rank two.

In contrast to Riemannian geometry, the components of the affine connection are no longer symmetric in the lower indices and cannot be recognized by the Christoffel symbols. As in Lyra geometry, the functions of Christoffel symbols and the functions of ϕ_i are the components of the affine connection. The Lyra curvature tensor $\bar{R}^h_{ij}$, the contracted curvature tensor R_{ij} and the scalar curvature can be defined in the same way as defined in the Riemannian geometry.

The scalar curvature $\bar{R}$ is defined as

$$\bar{R} = R(x^0)^{-2} + 3(x^0)^{-1}\phi^i_{;i} + \frac{3}{2}\phi^i\phi_i \tag{1.12}$$

Here R is the Riemannian curvature scalar. Also, ϕ_i is defined as

$$\phi_i = (x^0)^{-1}\frac{\partial}{\partial x^i}\{log(x^0)^2\} \tag{1.13}$$

The invariant volume integral in Lyra's geometry by considering the normal gauge $x^0 = 1$ and is given as

$$I = \int \bar{R}\sqrt{-g}d^4x \tag{1.14}$$

where d^4x is the element of volume in four-dimensional space.

Considering the field equations may be derivable from the variational principle

$$\delta(I+J) = 0 \tag{1.15}$$

where J is defined by

$$J = \int L\sqrt{-g}d^4x \tag{1.16}$$

The L here represents matter's Lagrangian density. The field equation obtained by Sen [Sen(1957)] using the above variational principle is defined as

$$R_{ij} - \frac{1}{2}R_{ij} + \frac{3}{2}\phi_i\phi^j - \frac{3}{4}g_{ij}\phi^k\phi_k = -8\pi T_{ij} \tag{1.17}$$

1.7 Sen-Dunn Theory

In the last decades, many gravitational theories alternating with Einstien's theory of gravitation have been proposed by many cosmologists. Einstein's theory of general relativity, or as said in his novel idea, introduced the physics-based geometrization principle, which is identified by the Riemannian space-time metric tensor as with the presence of gravitational potential. Brans-Dicke's theory [Brans & Dicke(1961)] is one of the straightforward scalar-tensor theories of gravitation that geometrize a tensor field. A scalar field is still foreign to the geometry, however. Brans-Dicke's phenomenon started with Mach's idea that inertia comes from the acceleration of the general masses distributed around the universe. Also, Lyra proposed a theory by adding a gauge function to the less manifold structure in Riemannian geometry, which nearly resembles Wely's geometry. Later, Sen and Dunn [Sen & Dunn(1971)] formulated changing Einstein's field equation and proposed a new scalar-tensor theory of gravitation based on Lyra geometry. The scalar field is characterized by a function defined by $x^0 = x^0(x^i)$ where it is pointed out that in a four-dimensional Lyra manifold, x^i is a co-ordinate, and the metric tensor g_{ij} represents the manifold's metric tensor field.
Sen and Dunn [Sen & Dunn(1971)] presented the equation where both scalar and tensor are present and are expressed as:

$$R_{ij}-\frac{1}{2}g_{ij}R=\omega(x^0)^{-2}\left(x^0_{,i}x^0_{,j}-\frac{1}{2}g_{ij}x^0_{,k}x^{0,k}\right)-8\pi G(x^0)^{-2}T_{ij} \tag{1.18}$$

The aforementioned equation may alternatively be rewritten in terms of ϕ as $\phi=\phi(x^i)$

$$R_{ij}-\frac{1}{2}g_{ij}R=\omega\phi^{-2}\left(\phi_{,i}\phi_{,j}-\frac{1}{2}g_{ij}\phi_{,k}\phi^{,k}\right)-8\pi G\phi^{-2}T_{ij} \tag{1.19}$$

In natural units for $c=1, 8\pi G=1$ the above equation transfoms to

$$R_{ij}-\frac{1}{2}g_{ij}R=\omega\phi^{-2}\left(\phi_{,i}\phi_{,j}-\frac{1}{2}g_{ij}\phi_{,k}\phi^{,k}\right)-\phi^{-2}T_{ij} \tag{1.20}$$

where

$\omega=\frac{3}{2}$

R_{ij} = Ricci tensor

R = Ricci scalar

T_{ij} = Energy-momentum tensor

1.8 Mathematical Formulation of Sen-Dunn Theory

Sen and Dunn [Sen & Dunn(1971)] considered a Lyra manifold in four dimensions that possesses a hyperbolic metric tensor and serves as the foundation for a scalar-tensor theory of gravity. To obtain the exterior (vacuum) field equations, it is necessary to use the variational principle.

$$\delta\int W dx^1 ... dx^4 = 0 \tag{1.21}$$

where an absolute invariant serves as the integrand. The curvature tensor's simplest variational principle is

$$\delta\int K(-g)^{\frac{1}{2}}x^0 dx^1 ... x^0 dx^4 = 0 \tag{1.22}$$

Under coordinate and gauge translation, gauge factors turn the integrand into an ultimate integral. The other potential constraints include

$$W = W(-g)^{\frac{1}{2}}(x^0)^4 \tag{1.23}$$

Here the curvature scalar is provided as

$$K = g^{ij}K_{ij} = (x^0)^{-2}R + 3(x^0)^{-1}\phi^i_{;i} + \frac{3}{2}\phi_i\phi^i + \frac{3}{2}\overset{\circ}{\phi}_i\phi^i \tag{1.24}$$

Sen and Dunn considered (1.15) and have iteratively looked into its effects. Substitution of (1.17) in (1.15) and independent alterations of g_{ij} and ϕ_i arise in the equations for the external fields.

$$R_{ij} - \frac{1}{2}g_{ij}R + \frac{3}{2}(x^0)^2\phi_i\phi_j - \frac{3}{4}(x^0)^2 g_{ij}\phi_k\phi^k - \frac{3}{4}(x^0)^2 g_{ij}\overset{\circ}{\phi}_k\phi^k + \overset{\circ}{\phi}_k\phi_j = 0 \tag{1.25}$$

$$3\phi_i + \frac{3}{2}\overset{\circ}{\phi}_i = 0 \tag{1.26}$$

The two pairs of equations are combined to create the final set of equations:

$$R_{ij} - \frac{1}{2}g_{ij}R - \omega(x^0)^{-2}x^0_{,i}x^0_{,j} + \frac{1}{2}\omega(x^0)^{-2}g_{ij}x^0_{,k}x^{0,k} = 0 \tag{1.27}$$

Where the set $\overset{\circ}{\phi}_i = (x^0)^{-1}[log(x^0)^2]_{,i}$ and $\omega = \frac{3}{2}$. The equation for the interior (matter) component is then written as

$$R_{ij} - \frac{1}{2}g_{ij}R - \omega(x^0)^{-2}x^0_{,i}x^0_{,j} + \frac{1}{2}\omega(x^0)^{-2}g_{ij}x^0_{,k}x^{0,k} = -[8\pi G/(x^0)^2]T_{ij} \tag{1.28}$$

Comparing the equation (1.21) with Brans and Dicke's interior field equations. Thus obtaining the equation

$$R_{ij} - \frac{1}{2}g_{ij}R - \omega\phi^{-2}\phi_{,i}\phi_{,j} + \frac{1}{2}\omega\phi^{-2}g_{ij}\phi_{,k}\phi^{,k} = -(8\pi/c^2)T_{ij} + \phi^{-1}(\phi_{,i;j} - g_{ij}\Box\phi) \tag{1.29}$$

Thus the Sen-Dunn theory is considered a particular case (1.22), wherein Brans-Dicke scalar function ϕ meets the condition

$$\phi_{i;j} - g_{ij}\Box\phi = 0 \tag{1.30}$$

where $\omega = 3/2$ is Brans-Dicke's coupling constant. The contrast between the Sen-Dunn theory and the Brans-Dicke theory isn't quite clear-cut, though, because the equations of motion are different.

1.9 Work related to Sen-Dunn theory

Halford [Halford(1972)] obtained an exact closed-form solution of the field equations corresponding to a scalar-tensor theory similar to the B-D approach. The scalar-tensor treatment based on Lyra's geometry predicts the same effect within observational limits as Einstein's theory.

Reddy [Reddy(1973)] demonstrated that Birkhoff's Theorem of general relativity holds for all scalar fields, regardless of their nature, in the gravitational scalar-tensor theory of Sen-Dunn. On the other hand, Birkhoff's Theorem is only valid in the B-D theory for the scalar field independent of time.

Reddy [Reddy(1977)] demonstrated that Birkhoff's Theorem of general relativity is also true in the presence of an electromagnetic field by using a time-independent scalar field in the scalar-tensor theory proposed by Sen and Dunn [Sen & Dunn(1971)]. As a result, he suggested that the Sen-Dunn gravitational theory be deemed an enhanced concept of the Brans-Dicke theory.

Krori and Nandy [Krori & Nandy(1977)] obtained a non-static solution and showed that the Birkhoff theorem does not hold for Ross or Sen-Dunn theory when the scalar field remains a function of t only. Also, the solution obtained in Sen-Dunn's theory contains Mach's principle, which states an improvement of Brans-Dicke's theory regarding the

Mach principle. One of the components of Marder's metric is influenced by the existence of scalar interaction x^0.

Roy and Chatterjee [Roy & Chatterjee(1980)] obtained a class of exact solutions for a non-static cylindrically symmetric space time of Marder in line with vacuum field equations in Sen-Dunn gravitational theory. They discovered that the wave aspect of Einstein's theory is preserved via solutions.

Roy and Chatterjee [Roy& Chatterjee(1981)] studied plane-symmetric static charged dust distribution in Sen-Dunn Theory. By considering the relation between the four components of the metric tensor, the electromagnetic potential ψ and the scalar interaction x^0 and obtained that the ratio $\frac{\sigma}{\rho}$ is related to the scalar interaction x^0 giving that the charge density far exceeds the mass density if the values are small and also the function x^0 and the electromagnetic potential ψ becomes singular at spatial infinity.

Singh & Rai [Singh & Rai(1983)] studied and derived a field equation by taking gravitation as a scalar and tensor field, considering Einstein's theory's mathematical and physical foundation.

Reddy and Venkateswarlu [Reddy & Venkateswarlu(1987)] obtained a homogeneous and anisotropic cosmological model considering the gravitational fluid as a perfect fluid with the pressure equal to energy density describing the universe's expansion.

Mukherjee [Mukherjee(2003)] studied and obtained a class of exact solutions of the non-vacuum field equation of Sen-Dunn gravitational theory considering the relativistic magneto-fluid proposed by Lichnerowicz for the cylindrically symmetric Einstien Rosen metric.

Mukherjee [Mukherjee(2004)] determined the solution obtained from the study of static spherically symmetric anisotropic fluid distribution dispersion when an electromagnetic field is present in the Sen-Dunn gravitational theory with the Reissner-Nordstrom solution over the boundary, obtaining a non-negative expression for the mass density and pressure.

Venkateswarlu et al. [Venkateswarlu et al.(2011)] studied the Bianchi type-I space-time with the context of cosmic string in the scalar-tensor theory proposed by Sen-Dunn, obtaining the existence of the cosmic string in the Sen-Dunn theory of gravitation.

Venkateswarlu et al. [Venkateswarlu et al.(2013a)] used Bianchi type-VI_0 cosmological model in the presence of a cosmic string in the framework of Sen-Dunn gravitational theory, identifying the existence of cosmic string in the geometry.

Venkateswarlu et al. [Venkateswarlu et al.(2013b)] discussed three parts of the cosmic string considering higher dimensional FRW space-time in Sen-Dunn's theory of gravitation. They observed that the scalar field in Sen-Dunn's theory causes the reduction of extra dimension in the context of the cosmic string with a different range of metric potential.

Venkateswarlu et al. [Venkateswarlu et al.(2013c)] studied spatially isotropic and homogeneous LRS Bianchi type-III space-time with the presence of a string cosmological model where they obtained the existence of the string in the scalar field and also found the interlink in a gravitational scalar-tensor theory derived by Sen and Dunn.

Ghate and Sontakke [Ghate & Sontakke(2014)] studied the universe when filled with dark energy taking the cosmological model Bianchi type-IX in the Sen-Dunn scalar-tensor theory, taking two cases of scale factor where they obtained a matter-dominated and dark energy era of the universe.

1.10 Work related to Bianchi space-time

[Bianchi(1918)] obtained the structural constants of nine different types of Bianchi type I-IX space-time. Later, Taub [Taub(1951)], Ellis & MacCallum [Ellis & MacCallum(1969)] and many other authors applied and studied Bianchi type cosmological models in general relativity and alternative theories of general relativity. Some related works on Bianchi type-III, VI_0, and IX cosmological models are mentioned below.

Ftaclas and Cohen [Ftaclas & Cohen(1979)] studied tilted Bianchi type-III space-time, obtaining an anisotropic solution that is either rotating or tilted and non-rotating.

Lorenz [Lorenz(1982a)] obtained the barrel type singularity solving the exact solution for the stiff matter and electromagnetic field in the Bianchi type-III model.

Chakraborty [Chakraborty(1991)] obtained a class of cosmological models of strings both in the presence of a magnetic field and without a magnetic field in Bianchi type-VI_0 space-time.

Latifi et al. [Latifi et al.(1994)] showed that the Bianchi type-IX cosmological models are non-integrable in the Painleve sense and have no vacuum solutions.

Romano and Pavón [Romano & Pavón(1994)] studied the evolution of a Bianchi type-III cosmological model for both the truncated and the full version of the casual thermodynamic theory of the non-equilibrium phenomenon, describing the FRW solutions as unstable and that de-Sitter solution are stable. The initial isotropy gets rapidly extinguished in either version of the numerical analysis.

Bali and Dave [Bali & Dave(2002)] examined the bulk viscosity component of the Bianchi type-III string cosmological model in general relativity, obtaining the cosmological parameters for the model. The model received is expanding, shearing, non-rotating and anisotropic for the large values of the time.

Xing-Xiang [Xing-Xiang(2006)] obtained a string model universe in the presence of both bulk viscosity and magnetic field in Bianchi type-III space-time which was found to be inflationary, shearing, and non-rotating.

Pradhan and Amirhashchi [Pradhan & Amirhashchi(2011)] for the anisotropic Bianchi type-III space-time with changeable EoS parameter, an accurate solution of Einstein's field equations were studied. The universe was matter-dominated in the early stages with a positive EoS parameter, but the universe is accelerating with a negative EoS parameter in the present epoch. The outcome also included cosmological constant, quintessence, and phantom fluid-dominated universes, which represented the many stages of the cosmos during cosmic history [Caldwell(2002)].

Yadav and Yadav [Yadav & Yadav(2011)] analysed DE models with adjustable EoS parameters following two cases. In the first case, matter dominated the cosmos with a positive equation of state, but this has changed to a negative equation of state in the present epoch. As for the second case, various stages of the universe's evolution are investigated, allowing for anisotropic DE in the Bianchi type-III model for both cases.

Shamir [Shamir(2011)] deliberately examined the exact solution for Bianchi type-III cosmology in f(R) gravity, considering some plausible assumptions. The model represented that the universe's expansion for a considerable time will cease and attain isotropy.

Reddy et al. [Reddy et al.(2012)] evaluated a homogeneous Bianchi type-III cosmological model with perfect fluid in $f(R,T)$ gravitational theory by Harko et al. [Harko et al.(2011)], assuming that Hubble's parameter follows Berman's special rule of variation [Berman(1983)] obtaining universe's rapid late-time expansion without initial singularity. The model's physical parameters diverge in the initial epoch, which tends to zero for large t.

In modified $f(R,T)$ gravity and considering Berman's special rule of variation for Hubble's parameter, Reddy et al. [Reddy et al.(2013)] produced an anisotropic Bianchi type-III DE cosmic model with variable EoS parameter. The model's EoS and skewness parameters are all functions of t, showing the accelerating, expanding, a non-rotating model with no initial singularity.

Santhi et al. [Santhi et al.(2018)] studied the universe with HDE components and employed proportionality between expansion and shear scalar and hybrid expansion laws for the average scale factor. The outcome states that the faster expansion of the current epoch may be caused by the HDE density dominating the universe's evolution. Even though the space-time is anisotropic, the HDE model eventually becomes isotropic. Additionally, the model depicts the transition from the decelerated to the accelerated phase even at low redshifts that match the quintessence scalar field concept.

Korunur [Korunur(2019)] considered the Bianchi type-III space-time model to investigate the new Tsallis holographic dark energy (TDHE) model with some well-known scalar field models, such as tachyon, quintessence, and K-essence.

In Bianchi type-III space-time with imperfect fluid, Akarsu and Kılınç [Akarsu &

Kılınç(2010)] found a precise solution to Einstein's field equation, taking into account exponential and power-law expansion. The model demonstrates that throughout the accelerated expansion period, the universe monotonically approaches isotropy, even with the existence of anisotropic fluid.

Bali et al. [Bali et al.(2010)] assuming the barotropic condition and shear proportional to expansion, studied the Bianchi type-III model for the perfect fluid with variable G and Λ. Behera et al. [Behera et al.(2010)] also discussed bulk viscous anisotropic Bianchi type-III cosmological models in Einstein's relativity with time-dependent gravitational and cosmological constants.

Sharif and Kausar [Sharif & Kausar(2011a)] explored Bianchi type-III space-time for exponential and power-law volumetric expansion in f(R) gravity. Here the Bianchi type-III model maintains anisotropic even in late times, whereas in general relativity, it tends to isotropy.

The interacting dark energy is studied in Bianchi type-III space-time, Amirhashchi [Amirhashchi(2014)] considering cases when DE and DM interact and when they do not interact. When DE is considered dense, its EoS parameter crosses the PDL line for both cases, but in the absence of viscosity, it varies in the quintessence region.

Hatkar et al. [Hatkar et al.(2019)] studied the Bianchi type-III model with bulk viscosity and heat flux in a simplified gravitational theory and discovered the relevance of bulk viscosity early in the universe's history, with the heat function being hostile throughout the transition.

[Xing-Xiang(2005), Yadav et al.(2007), Bali & Pradhan(2007), Upadhaya & Dave(2008), Tripathy et al.(2008), Pradhan et al.(2010), Amirhashchi et al.(2011), Kiran& Reddy(2013), Vidya Sagar et al.(2014), Sahoo & Mishra(2015), Sahoo et al.(2017), Dixit et al.(2020)] are some authors who investigated the Bianchi type-III string cosmological model with the existence of viscosity and with various contexts.

Amirhashchi et al. [Amirhashchi et al.(2013)] studied a spatially homogeneous and anisotropic Bianchi type-III space-time filled with barotropic fluid and dark energy. The solution that fulfills the most recent observation is derived by considering a scaling factor that produces a time-dependent deceleration parameter and a scalar expansion proportionate to the shear scalar.

Kiran et al. [Kiran et al.(2015)] obtained a minimally interacting HDE model in Brans-Dicke theory applying linearly varying deceleration parameters as derived by Akarsu and Dereli [Akarsu & Dereli(2012)]. The results depicted the finite lifetime scenario of the universe with HDE density, pressure, and scale factor diverging in a finite period showing big rip behaviour.

Mishra et al. [Mishra et al.(2018)] investigated different physical parameters on the Bianchi type-III cosmological model using BVDP in modified $f(R,T)$ theory with the expansion scalar being proportional to the shear scalar, demonstrating that their results are consistent with recent observations.

Pawar and Shahare [Pawar & Shahare(2019)] stated universe's development under $f(R,T)$ gravity for a tilted Bianchi type-III cosmic model is expanding, shearing, and anisotropic, beginning with a Big Bang singularity at the first epoch.

Naidu et al. [Naidu et al.(2012)] discussed the anisotropic Bianchi type-III dark energy model in Seaz-Ballester theory with the variational law for generalized Hubble's parameter. The model obtained is non-singular and expanding, pointing to DE as the reasonable contender for the universe's accelerated expansion.

In the context of Lyra geometry, Samanta [Samanta(2013)] investigated the significance of anisotropic DE in the Bianchi type-III cosmological model.

Mahanta et al. [Mahanta et al.(2014)] constructed and discussed the dark energy model with a variable equation of state (EoS) parameter in an anisotropic Bianchi type-III cosmological model in the self-creation theory of gravitation. The observation shows the consistency of the time-varying ω and is found to be negative for the first model. Still, it is more significant than zero for the second model defining the matter-dominated universe.

Umadevi and Ramesh [Umadevi & Ramesh(2015)] stated the early inflation and late-period acceleration for the model obtained using variational Hubble's parameter with the existence of HDE and weakly interacting dark matter in Brans-Dicke theory of gravitation.

Raju et al. [Raju et al.(2020)] analyzed the Bianchi type III space-time with a massive scalar field model with anisotropic dark energy. The decreasing effect of the scalar field (SF) for the model over time is observed, and the physical parameters thus are independent of it at late times.

Bali and Jain [Bali & Jain(2007)] investigated the physical features and behaviour of the Bianchi type-III cosmic model in general relativity with and without a magnetic field for perfect fluid distribution.

Adhav et al. [Adhav et al.(2011b)] considered the wet, dark fluid behaviour of dark energy in the Bianchi type-III cosmological model in the absence and presence of a magnetic field where current flows along the z-axis. Anisotropy is well preserved with or without the magnetic field for the model.

Singh and Ram [Singh & Ram(1996)] presented the work on Bianchi type-III cosmological model with massive strings. They obtained the exact solution in the magnetic field's presence and absence, motivated by the work of Tikekar and Patel [Tikekar & Patel(1992)]. The models obtained represented the evolution from initial singularity to empty space-time for an incredibly long period.

Dunn and Tupper [Dunn & Tupper(1976)] generalized the solution of Einstein-Maxwell field equations for the perfect fluid in the Bianchi type-VI model with the absence and presence of an electromagnetic field.

Lorenz [Lorenz(1982b)] examined the solution of Einstein-Maxwell equations for Bianchi type-VI_0 model with matter and electromagnetic field. The study showed the behaviour of the point type singularity, and the models obtained approaches isotropy for the extensive time t.

Barrow [Barrow(1984)] presented Bianchi type-VI_0 space-time as the better simplification of the universe with the sense that the universe sometimes isotropizes. Cosmological problems such as primordial helium abundance are described.

Roy et al. [Roy et al.(1985a), Roy et al.(1985b)] discussed the Bianchi type-I and VI_0 cosmological models with the magnetic field. The expansion and contraction of the Bianchi Type-I and VI_0 model rely on the values of the cosmological constant that are both negative and positive, describing the perfect fluid distribution.

Ribeiro & Sanyal [Ribeiro & Sanyal(1987)] also discussed the Bianchi type-VI_0 with viscous fluid and magnetic field pointing to the contracting model in the absence of magnetic field but in presence does not change the nature of singularity.

Patel and Koppar [Patel & Koppar(1991)] studied the viscous Bianchi type-VI_0 cosmic model, assuming some of the mathematical assumptions to solve the exact field equation.

Mohanty and Mishra [Mohanty & Mishra(2002)] expressed that the non-singular Bianchi type-VI_1 cosmological model does not exhibit either the big bang or big crunch even in the initial epoch or in infinite time but evolves with constant volume representing a flat model.

For all three models used in the study of Bianchi type-VI_0 space-time for a massive string with a viscous fluid in the existence and non-existence of magnetic field in general relativity, Bali et al. [Bali et al.(2008a)] concluded a point type singularity at $t = 0$ for the models.

Bali et al. [Bali et al.(2008b)] considering material distribution as magnetized bulk viscous fluid in the massive string, obtained exact solutions for an accelerating Bianchi type-VI_0 cosmic model in general relativity.

Bali et al. [Bali et al.(2009)] imposed different conditions over the free gravitational field to obtain the LRS Bianchi type-VI_0 cosmic model in Einstein's theory of gravity.

Sharif and Zubair [Sharif & Zubair(2010)] discussed the dynamics and effects of electromagnetic field on Bianchi Type-VI_0 cosmic model in the existence of anisotropic dark energy, which favours the ΛCDM model.

Adhav et al. [Adhav et al.(2011a)] studied Bianchi type-VI_0 cosmological models with anisotropic dark energy and stated that the importance of anisotropic dark energy could not be left unnoticed even though accelerating models isotropize the anisotropic fluid at late times. The deceleration parameter represents the characteristic behaviour of decelerating to accelerating at late times, which confirms with the current scenario.

Pradhan et al. [Pradhan et al.(2012)] considering time-dependent deceleration parameter, observed the behaviour of anisotropic DE models represented in Bianchi type-VI_0 space-time which on suitable conditions approaches isotropy.

Mishra and Sahoo [Mishra & Sahoo(2014)] used the scale-invariant theory of gravity to build a non-singular Bianchi type-VI_1 model with moist, dark fluid. The model increases spatially with a constant volume, compresses briefly throughout evolution, and does not demonstrate a huge explosion or great crunch in the early and future.

Reddy et al. [Reddy et al.(2016)] established the precise solution of Bianchi type-VI_0 space-time in the Seaz-Ballester scalar-tensor theory of gravity by investigating two marginally correlated forces, namely matter and anisotropic holographic dark energy field. In addition, they acquired a non-singular model that extends spatially in the scalar-tensor field.

Hegazy [Hegazy(2019)] investigated different cases of solution for Bianchi type-VI cosmological model in the framework of self-creation theory for prefect fluid, indicating the consistency of anisotropic universe throughout the evolution.

Satish and Venkateswarlu [Satish & Venkateswarlu(2019)] studied two fluid anisotropic Bianchi type VI_0 cosmological models considering the coupled massless scalar field and time-varying G and Λ in Einstein's theory of gravitation.

By taking the exponential scale factor and variable Λ parameter, Dewri [Dewri(2018)] obtained the exact solutions of the field equation for Bianchi type-VI model accommodated with dark energy describing an accelereated expansion model of the universe.

Misner [Misner(1969)] described the behaviour of the Bianchi type-IX "mixmaster universe" due to its chaotic and unusual singularity approach.

Waller [Waller(1984)] investigated and discussed the dynamical behaviour of a spatially homogeneous and anisotropic Bianchi type IX universe within the case of an electromagnetic field.

According to Uggla and Zur-Muhlen [Uggla & Zur-Muhlen(1990)], an LRS Bianchi type-IX with ideal fluid models exhibits compactification and decreased dynamics.

King [King(1991)] interpreted the Bianchi type-IX cosmological model as a Friedmann universe with complex dynamics and gravitational-wave scenario compatible with a closed universe.

Burd et al. [Burd et al.(1990)] analyzed the chaotic behaviour of the Bianchi type-IX model numerically with a perfect fluid. In addition, they calculated and studied the Lyapunov exponent and its effect in cosmological models.

Bali and Dave [Bali & Dave(2001)] discussed the cosmological parameters of the Bianchi type- IX string cosmological model in general relativity, assuming the rest energy density is equal to the string tension density. Later, Bali and Dave [Bali & Dave(2003)] presented the work on the exact cosmic string model with the existence of bulk viscous fluid.

De Oliveira et al. [De Oliveira et al.(2002)] determined the dynamical aspect of the Bianchi type-IX model with dust and the cosmological constant, which is highly complex and chaotic, presumably for the homoclinic origin.

Bali and Dave [Bali & Dave(2003)] claimed the Bianchi Type-IX universe with bulk viscous fluid expands with the big bang and decreases as the time increases for the massive string in general relativity.

Pradhan et al. [Pradhan et al.(2005)] For the presence of viscous fluid, presented a homogeneous Bianchi type-IX model with viscosity claiming shear, non-rotating and anisotropic model throughout the cosmic time resulting in physically decaying law for the cosmological constant.

Considering some plausible mathematical assumptions, Bali and Yadav [Bali & Yadav(2005)] discussed the behaviour of the Bianchi type-IX cosmological model in the existence and non-existence of viscosity in GR.

Three types of models of in corresponding to geometric string, Tabayasi string and Barotropic string are discussed in the framework of scalar-tensor theory for Bianchi type-

IX space-time. Reddy and Naidu [Reddy & Naidu(2007)] stated that for the geometric and barotroic string, the models are non-singular models where the physical parameters are possible for the understanding of the ultimate universe. Whereas, for the Takabayasi string model it is not possible to discussed the physical properties due to unobtainable explicit form of the model.

For the homogeneous Bianchi type-IX cosmological model for perfect fluid with an electromagnetic field, Tyagi and Chhajed [Tyagi & Chhajed(2012)] interpreted the model as expanding with shearing and non-rotating in general.

In the context of Seaz-Ballester scalar-tensor theory, Ghate and Sontakke [Ghate & Sontakke(2014)] arrived at the answer by utilising the specific rule of variation for Hubble's parameter suggested by Bermann in the magnetized Bianchi type-IX dark energy model. The model is anisotropic throughout development and devoid of a big-bang singularity.

With studying the dynamics of the expansion and rotation of the Bianchi type-IX model, Sofuoğlu [Sofuoğlu(2016)] investigated and explored using the enhanced $f(R,T)$ gravity theory, identifying shear-free does not counterpart in the assumed theory.

Assuming the context of bulk viscosity and flat potential, Sharma and Poonia [Sharma & Poonia(2021)] formulated and obtained the solution of the Bianchi type-IX universe, indicating the continuous expansion of the universe and entering isotropy for certain specific values.

1.11 Hubble's law and Parameter

According to Hubble's findings, the speed at which galaxies move away from the Earth is related to their distance or that of the recession. Recessional velocity is equal to distance times (Hubble's constant)

$$i.e. V = HD \tag{1.31}$$

V denotes the galaxy's measured motion away from Earth, usually in km/sec.

H is Hubble's constant in km/sec/Mpc.

D is the distance of galaxies in MPc.

According to the relationship above, the Hubble parameter or Hubble constant H specifies the pace of cosmic expansion. $H = \frac{V}{D}$ calculated the recession velocity V of an object at a distance D. Also it is the logarithmic derivative of the scale factor $a(t)$ i.e $H = \frac{\dot{a}}{a}$. Since the mean recessional speed of the cluster is computed from the motions of these individual members, having a significant random motion may introduce substantial uncertainty in the computation of the mean recessional speed.

When this rate is used to compute the value H, it should accept the value with a reservation that has devotedly worked for many decades for the correct evaluation of H. In 1929, Hubble calculated that the Hubble constant, also known as the expansion factor, was worth approximately $500 km/sec/Mpc$. Two groups of astronomers have been at odds for many years. Alan Sandage and his coauthors make a claim by noting that $H = 50 km/sec/Mpc$. Also, Vaucauleurs and his coauthors argue that the value should be around $10 km/sec/Mpc$. But many outsiders thought the geometric mean of their value $H = 71 km/sec/Mpc$ was a good compromise. The controversy persists while authors often work with some intermediate value of H. In the sixties and seventies, authors had done lots of work with $H = 75 km/sec/Mpc$. Considering the problem's numerous facets and any inherent uncertainty in the decision. Dressler contends that a more fair value for H should be $H = 70 km/sec/Mpc$. Many researchers are, however, currently working with the value of $50 km/sec/Mpc$. But from the latest source the Hubble space, Telescope key team came up with the answer

$$H = 75 + / - 8 km/sec/Mpc. \tag{1.32}$$

And finally, WMAP came up with

$$H = 71 +/- 3.5 km/sec/Mpc \tag{1.33}$$

where $1MPc = 3.26$ million light years.

1.12 Deceleration Parameter

The dimensionless parameter used in relativity and cosmology to calculate the cosmic acceleration of the universe's expansion is referred to as the deceleration parameter.

$$q = -\frac{a\ddot{a}}{\dot{a}^2} \tag{1.34}$$

where $a(t)$ is the universe's average scale factor, and the above dots represent the derivatives to the appropriate period. When q is negative, the universe expands more quickly than when it is positive because the value of q depends on whether $\ddot{a}$ is positive or negative. Deceleration parameter q was first defined with the assumption that it would have a positive value. However, it has been determined through current research and empirical results that our universe is speeding at the moment rather than slowing down. It is thought that dark energy, which rules the cosmos with positive energy density and negative pressure, causes the necessary acceleration in the universe's late-time development.
In addition, the deceleration parameter q is described in terms of Hubble's parameter H as

$$q = -\frac{d}{dt}\left(\frac{1}{H}\right) - 1 \tag{1.35}$$

Riess et al. [Riess et al.(1998)] team was the first to propose the accelerating universe. The deceleration parameter q is more significant than -1 for all proposed kinds of matter, although phantom dark energy violates all energy criteria. As a result, for the expanding universe, Hubble's parameter H should be decreasing, causing the local expansion of space to slow down. The cosmological model presented by Berman and Gomide [Berman & Gomide(1988)] has a constant slowing value, which is well developed into cosmological models with variable deceleration parameters [Mishra et al.(2016), Tiwari et al.(2016b), Sahoo et al.(2018)], linearly varying deceleration parameter [Adhav(2011), Akarsu & Dereli(2012), Sarkar et al.(2014)] and bilinear deceleration parameter [Mishra & Chand(2016), Mishra & Chand(2017), Mishra & Dua(2019)].

1.13 Expansion Scalar

The volume expansion scalar measures how the fractional rate of volume expansion of a tiny spherical object changes over time. It is expressed as

$$\theta = 3H = 3\frac{\dot{a}}{a} \tag{1.36}$$

1.14 Anisotropic Parameter

The implementation of variable anisotropic parameters leads to finding a suitable EoS in such a way that precisely confirms observations, compared to a constant anisotropic model that has shown only a constant EoS versus cosmic time.

$$A_m = \frac{1}{2}\sum_{1}^{3}\left(\frac{\Delta H_i}{H}\right)^2 = \frac{2\sigma^2}{3H^2} \tag{1.37}$$

1.15 Shear Tensor:

The rate of shape deformation is described by the shear σ, constructed from the difference between the expansion rates of the scale factors. It also determines the distortion in the fluid flow, leaving the volume invariant. It is defined by:

$$\sigma_{ij} = \frac{1}{2}\left(u_{i;j} - u_{j;i}\right) + \frac{1}{2}\left(\dot{u}_i u_j - u_j \dot{u}_i\right) \tag{1.38}$$

The magnitude of σ_{ij} is the shear σ^2 defined by

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} \tag{1.39}$$

1.16 Energy condition

The energy conditions are crucial for comprehending the universe's various characteristics, including its current phase of accelerated expansion and the potential existence of so-called phantom fields. Also, the cosmic models are further constrained by energy constraints. These conditions may be critical in determining cosmological development, notably the acceleration or slowdown of cosmic fluid and, as a result, the creation of Big Rip singularities. The cosmological solutions and associated equations of state may also be categorised based on energy conditions. Transformations from the Jordan framework to the Einstein frame show a qualitative difference in certain energy circumstances. Initially, the energy condition is derived from the well known Raychoudhuri equation which describe the null and timelike geodesics with positve energy density [Raychaudhuri(1979)].

The energy conditions for the perfect fluid distribution can be interpreted in terms of ρ and p as follows:

- Null Energy Condition (NEC): $\rho + p \geq 0$.
- Weak Energy Condition (WEC): $\rho + p \geq 0$ and $\rho \geq 0$.
- Strong Energy Condition (SEC): $\rho + p \geq 0$ and $\rho + 3p \geq 0$.
- Dominant Energy Condition (DEC): $\rho + p \geq 0$ and $|\rho| \geq |p|$.

1.17 ΛCDM model

The findings of modern cosmology stated the ΛCDM model as the standard cosmological model that provides the current scenario of cosmic evolution, considerably the accelerating universe. There exist three components: first, the cosmological constant denoted by Λ, which is considered the component associated with the dark energy; second, the cold dark matter marked as CDM; and third, the ordinary matter. It is viewed as the parameterization of the Big Bang cosmological model. The existence of the cosmic microwave background anisotropies [Page et al.(2003), Hu & Dodelson(2002)], the large-scale structure distribution of galaxies [Bernardeau et al.(2002), Bull et al.(2016)], the existence of the abundances of hydrogen, helium, and lithium [Steigman(2007), Cyburt et al.(2016)], the accelerating expansion of the universe [Riess et al.(1998), Perlmutter et al.(1999)], all can be considered as the reasonable properties of the ΛCDM model.

1.18 Dark Energy and Dark Matter

The tremendous success in modern cosmology is the discovery of cosmic acceleration. The idea of the accelerating universe came to existence after the observation presented by Supernovae Type Ia (SNe Ia) [Riess et al.(1998), Riess et al.(2004)], High-redshift Supernova Search Team [Perlmutter et al.(1999)], WMAP observations [Spergel & Steinhardt(2000)] and BAO [Eisenstein et al.(2005)]. Dark energy thus is one of the most important and widely studied research areas due to its involvement in cosmic acceleration and its repulsive force nature which acts as anti-gravity. Dark energy is the source of the universe's accelerated expansion. Dark energy is thought to exert negative pressure, causing the cosmos to expand faster when it interacts with gravity. According to the findings, dark energy occupies 73% of the cosmos. Dark matter is pressureless stuff that interacts exceptionally weakly with conventional matter particles. Zwicky demonstrated the presence of dark matter in the 1930s by comparing the coma cluster's dispersion velocities of the galaxies to the observed star mass. Because dark matter does not reflect electromagnetic forces, its existence is discovered by gravitational impacts on visible matter. Also, dark matter is understood as the remnants of stars of the previous cycle which is deteced as the microwave background. Dark matter could be in two possible forms, hot and cold [Narlikar(2002)][p.363,407]. Dark matter is thought to have an important role in developing large-scale structures like galaxies and galactic clusters. Dark matter is hypothetical matter that does not emit or reflect light or electromagnetic radiation. Dark matter is a commonly recognized theory by the cosmologist as there must be a particular matter that interacts with ordinary matter gravitationally, helping structure formation. The present universe is governed by 23% of dark matter with 4% of usual baryons in the universe. Many cosmologists reviewed their work in particle detection of dark matter [Smith & Lewin(1990), Bertone et al.(2005)], which may consist of weakly-interacting massive particles [Jungman et al.(1996), Pospelov et al.(2008), Arkani-Hamed et al.(2009)], cold dark matter [Dubinski & Carlberg(1991), Liddle & Lyth(1993), Navarro(1996), Spergel & Steinhardt(2000)] and many other candidates governing the dark matter.

Einstein in 1917 introduced the cosmological constant in his field equation which he later considered non-efficient. However, it is considered by many as a natural or most straightforward candidate for dark energy for the model that best describes the accelerating universe. Thus the best standard model in cosmology is known as the ΛCDM model, where cosmological constant Λ is cold dark matter. The dynamical behaviour of dark energy with the cosmological constant is well developed and with outstanding observations [Weinberg(1989), Peebles & Ratra(1988), Sahni & Starobinsky(2000), Sahni & Starobinsky(2006), Carroll(2001), Padmanabhan(2003), Peebles & Ratra(2003), Copeland et al.(2006), Frieman et al.(2008), Bamba et al.(2012), Li et al.(2011)]. Some of the essential scalar-field models of dark energy are quintessence [Steinhardt et al.(1999), Zimdahl et al.(2001), Tsujikawa(2013)], tachyons [Sen(2002), Padmanabhan(2002)], chaplain gas [Gorini et al.(2003), Bento et al.(2002), Debnath et al.(2004), Guo & Zhang(2007), Elmardi et al.(2016)], phantom [Caldwell(2002), Nojiri & Odintsov(2006), Ludwick(2017)], k-essence [Armendariz-Picon et al.(2000), Armendariz-Picon et al.(2001), Malquarti et al.(2003),Tian & Zhu(2021)], quintom [Wei et al.(2005),Guo et al.(2005),Feng et al.(2006), Leon et al.(2018)],h-essence [Wei & Cai(2005)] and ghost condensate [Arkani-Hamed et al.(2004), Piazza & Tsujikawa(2004)]. The above literature described the universe with a negative pressure executed by dark energy. Although the cosmic acceleration of the universe with dark energy is widely observed and studied by many, it still lacks a piece of factual information or cosmological data that can precisely distinguish the natural background and boundary of the evolution of the universe.

1.19 Jerk and Statefinder parameter:

The jerk parameter studies the rate of change of deceleration parameter q. The measure of this jerk shows the transition of the deceleration parameter or the universe's evolution from decelerating to the accelerating phase. Mathematically it is defined as:

$$j = \frac{\dddot{a}}{aH^3} = q + 2q^2 - \frac{\dot{q}}{H} \tag{1.40}$$

A pair of the statefinder parameter is introduced by Sahni et al. [Sahni et al.(2003)] to characterize the dark energy models, particularly the interacting models. The parameter (r,s) is as mentioned below:

$$r = \frac{\dddot{a}}{aH^3} \tag{1.41}$$

$$s = \frac{r-1}{3(q-\frac{1}{2})} \tag{1.42}$$

As for the ΛCDM model and CDM the value limit for $(r,s) = (1,0)$ and $(r,s) = (1,1)$ respectively. For the limit $r < 1$ and $s > 0$ the phantom and quintessence dark energy region is distinguished. Also, the Chaplygin gas is identified under the limit $r > 1$ and $s < 0$. [Farajollahi et al.(2011), Singh & Kumar(2016), Santhi et al.(2017a), Panotopoulos &

Rincón(2018),Naidu et al.(2019),Varshney et al.(2019),Saleem & Imtiaz(2020)] are some authors who investigated different cosmological models with the satefinder parameter.

1.20 Aims and Objective of the Research Work

The aims and objectives of the research proposal are as under:

- To discuss Bianchi Type-III Dark Energy Model with Charged Fluid Distribution in Sen-Dunn Theory.
- To discuss Bianchi Type-VI Dark Energy Model with Variable Deceleration Parameter in Sen-Dunn Theory
- To investigate Viscous Bianchi-IX Dark Energy Model in Sen-Dunn Theory of Gravitation.
- To investigate Bianchi Type-III Cosmological Model with Viscous Fluid in Sen-Dunn theory of Gravitation.
- To investigate Magnetized Bianchi Type-VI Model with Variable Deceleration Parameter in Sen-Dunn Theory.
- To discuss Bianchi Type- IX Cosmological Model Universe filled with Radiation in Sen-Dunn Theory.

1.21 Methodology and Tools

In this thesis, the research primarily deals with the gravitational scalar-tensor theory proposed by Sen and Dunn [Sen & Dunn(1971)]. In recent years, the study of the alternative theories of gravitation has gained momentum in understanding the cosmological models of the universe. The most comprehensive theories of gravitation that are widely used to research the cosmic model of the universe are Einstein's general theory of relativity and the alternate gravitational theory. Modern physics theories typically consist of a mathematical model defined by a particular set of differential equations and supported by a mathematical conclusion that makes some significant claims about the nature of the physical universe. Therefore, the construction of the cosmological model plays a vital role in asserting the universe's evolution and shape and the ultimate fate of the universe in modern cosmology. The collection of differential equations that are not linear is theoretically generated from the mathematical formalism and the energy-momentum tensor, and these equations are solved for the universe's physical properties. Then, the solutions are interpreted by plotting these parameters. Next, the material and dynamical properties of the cosmological parameters are compared with the recent astrophysical and cosmological observational data of different experimental probes. For this research work a secondary data is used with reference to journal publication and books in this field which are available in the library and internet. Finally, the plotting of graphs is executed using software like Mathematica, Sci-lab, Python, and Reduce-algebra to solve the differential equations of the field equations.

1.22 Importance of the study

Since the introduction of the scalar-tensor theory of gravitation a few decades ago, it has become the alternative theory to Einstein's general theory of relativity since it displays the lower level of a more fundamental theory and its phenomenon of explaining the recent remark of accelerating universe and solving the cosmological constant problem. The scalar-tensor theory was highly unfolded after Brans-Dicke proposed the theory. Sen-Dunn is one of the scalar-tensor theories of gravitation, considered a special case of Brans-Dicke's theory. But in Sen-Dunn's theory, both the tensor and scalar field are intrinsically geometrized, whereas Brans-Dicke's scalar field remains alien with the tensor field being geometrized. Sen-Dunn theory describes the accelerating universe which is consistant with the recent observations also it can unify gravity with other fundamental forces, such as electroweak forces. Bianchi Type cosmological model is important since it is homogeneous and anisotropic, and the isotropy is studied over time. Bianchi Type-III, VI, and XI space-time are considered in this research to obtain the cosmological model of the universe within the framework of Sen-Dunn's theory of gravitation. Therefore, this research seeks to find and solve problems in Sen-Dunn gravitational theory with cosmic factors and observe the physical findings with the recent day cosmological observation.