Chapter 4

Magnetized Bianchi Type-VI_0 Cosmological Model with Variable Deceleration Parameter in Sen-Dunn Theory of Gravitation

4.1 Introduction

The authors [Melvin(1975), Pudritz & Silk(1989), Tsagas & Barrow(1997)] discussed the primordial magnetic field at the early stage of the universe and large galactic scale structure field. The homogeneous and anisotropic Bianchi cosmological model rises to provide greater detail of the enormous large-scale universe. Bianchi type-VI_0 cosmological model is studied for its significant role in evaluating anisotropic space-time. Dark energy has been tremendously investigated in recent years as it is considered the main elements causing the universe's accelerated expansion. Some researchers [Adhav et al.(2011a), Pradhan et al.(2012), Reddy et al.(2016)] investigated the anisotropic Bianchi type-VI_0 dark energy cosmological model in various cosmological contexts.

The Bianchi type-VI_0 cosmic space-time in the modified gravitational theories [Sharif & Kausar(2011b), Rao & Neelima(2013)] is well discussed. Many authors have discussed a study of the Bianchi type-VI_0 cosmological model in the existence of a magnetic field [Ribeiro & Sanyal(1987), Sharif & Zubair(2010), Abdel-Megied & Hegazy(2016)]. Here, in this chapter, an exact solution of the field equations with the presence of a magnetic field in Sen-Dunn theory for the anisotropic Bianchi type-VI_0 geometry is investigated. The intermediate inflationary scale factor proposed by Barrow [Barrow(1990)] is considered to analyse the accelerating universe. The effect of the magnetic field displays the dynamic behaviour of dark energy with negative pressure.

4.2 Metric and Field Equation

Here the Bianchi Type-VI_0 metric is considered as

$$ds^2 = -dt^2 + A^2dx^2 + B^2e^{2x}dy^2 + C^2e^{-2x}dz^2 \tag{4.1}$$

where the metric A, B, and C are a cosmic function of time t.
Sen and Dunn's field equation (in natural units, c = $8\pi G$ = 1) for the coupled scalar and tensor fields is as given below:

$$R_{ij} - \frac{1}{2}Rg_{ij} = \omega\phi^{-2}(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,k}\phi^{,k}) - \phi^{-2}T_{ij} \tag{4.2}$$

The energy-momentum tensor is considered as

$$T_{ij} = (\rho + p)u_iu_j + pg_{ij} + E_{ij} \tag{4.3}$$

here p being pressure, ρ as density and $u_i = (0,0,0,1)$ is the four-velocity vector satisfying $g_{ij}u^iu^j = -1$ for the line element (4.1).

The electromagnetic field tensor is expressed as

$$E_{ij} = -F_{il}F^{jl} + \frac{1}{4}g_i^j F_{lm}F^{lm} \tag{4.4}$$

Here F_{il} is the electromagnetic field. The non-vanishing electromagnetic tensor is derived as

$$E_1^1 = -E_2^2 = -E_3^3 = E_4^4 = -\frac{1}{2}g^{11}g^{44}F_{10}^2 = \frac{1}{2}\frac{F_{10}^2}{A^2} \tag{4.5}$$

From the above, the following set of equations results from the field equation (4.2) for the line element (4.1) and equation (4.3).

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} + \frac{1}{A^2} = \frac{\omega}{2}(\frac{\dot{\phi}}{\phi})^2 - \phi^{-2}(p + \frac{F_{10}^2}{2A^2}) \tag{4.6}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} - \frac{1}{A^2} = \frac{\omega}{2}(\frac{\dot{\phi}}{\phi})^2 - \phi^{-2}(p - \frac{F_{10}^2}{2A^2}) \tag{4.7}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} - \frac{1}{A^2} = \frac{\omega}{2}(\frac{\dot{\phi}}{\phi})^2 - \phi^{-2}(p - \frac{F_{10}^2}{2A^2}) \tag{4.8}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} - \frac{1}{A^2} = \phi^{-2}(\rho + \frac{F_{10}^2}{2A^2}) - \frac{\omega}{2}(\frac{\dot{\phi}}{\phi})^2 \tag{4.9}$$

$$\frac{\dot{B}}{B} - \frac{\dot{C}}{C} = 0 \tag{4.10}$$

The differentiation concerning time t is shown here by the dot. The conservation law $T^i_{j;i}$ = 0 for the energy-momentum tensor yields the equation.

$$\dot{\rho} + (\rho + p)\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = 0 \tag{4.11}$$

It is specified that the average scale factor for a space-time of Bianchi Type-VI_0 is

$$a(t) = (ABC)^{\frac{1}{3}} \tag{4.12}$$

The Hubble parameter (H), Expansion scalar(θ), Shear Scalar(σ), Anisotropy parameter(A_m) and the deceleration parameter(q) is defined as

$$H = \frac{\dot{a}}{a} \tag{4.13}$$

$$\theta = 3H \tag{4.14}$$

$$\sigma^2 = \frac{1}{2}(\Sigma_{i=1}^{3} H_i^2 - \frac{1}{3}\theta^2) \tag{4.15}$$

$$A_m = \frac{1}{3}\Sigma_{i=1}^{3}(\frac{H_i - H}{H})^2 \tag{4.16}$$

where $i = 1, 2, 3$ represent the directional parameters in x, y, z direction respectively.

$$q = -\frac{a\ddot{a}}{\dot{a}^2} \tag{4.17}$$

4.3 Solutions of the field equation

From (4.10), the following equation is obtained

$$B = mC \tag{4.18}$$

where m is a constant of integration. Here considering $m = 1$ without the loss of generality.

So, equations (4.6)-(4.9) and (4.11) reduces to

$$2\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} + \frac{1}{A^2} = \frac{\omega}{2}(\frac{\dot{\phi}}{\phi})^2 - \phi^{-2}(p + \frac{F_{10}^2}{2A^2}) \tag{4.19}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} - \frac{1}{A^2} = \frac{\omega}{2}(\frac{\dot{\phi}}{\phi})^2 - \phi^{-2}(p - \frac{F_{10}^2}{2A^2}) \tag{4.20}$$

$$2\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2} - \frac{1}{A^2} = \phi^{-2}(\rho + \frac{F_{10}^2}{2A^2}) - \frac{\omega}{2}(\frac{\dot{\phi}}{\phi})^2 \tag{4.21}$$

$$\dot{\rho} + (\rho + p)\left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right) = 0 \tag{4.22}$$

The above non-linear equation (4.19)-(4.22) contents six different unknown parameters A, B, ρ, p, ϕ, F_{10} . For the deterministic solution, two essential conditions for further observation are assumed. To solve the field equation, assuming that the shear tensor and the expansion scalar are proportional is well described by the work of [Thorne(1967)]. The physical cause of assuming the condition deals with the redshift relation for extra-galactic source observation. Thus suggesting the ratio of shear σ to Hubble's constant H to limit $\frac{\sigma}{H} \leq 0.3$ [Kantowski & Sachs(1966), Kristian & Sachs(1966)]. The normal conformity to the uniform hypersurface fulfils the requirement that $\frac{\sigma}{\theta}$ is constant for spatially homogeneous metrics [Collins & Hawking(1973)].

$$A = C^n \tag{4.23}$$

where the constant n is positive with $n \neq 1$, preserving the anisotropy of the model. The following condition gives the metric potential to find the deterministic solution. Secondly, considering the power law relation by Johri and Sudharsan [Johri & Sudharsan(1989)] for the simplicity of determining the exact solutions, the scalar field ϕ can be expressed as a function of time and scale factor $a(t)$.

$$\phi = \phi_0 a^{\alpha}(t) \tag{4.24}$$

where ϕ_0 is a constant and α is an ordinary constant.

To discuss inflation, the scale factor during the intermediate era has the form as proposed by Barrow [Barrow(1990)]

$$a(t) = exp(Kt^f) \tag{4.25}$$

where $K > 0$ and $0 < f < 1$.

With the help of the equations (4.12), (4.18), (4.23), and (4.25), the metric potential is obtained as

$$A = exp\left(\frac{3nKt^f}{n+2}\right) \tag{4.26}$$

$$B = exp\left(\frac{3Kt^f}{n+2}\right) \tag{4.27}$$

$$C = exp\left(\frac{3Kt^f}{n+2}\right) \tag{4.28}$$

The obtained model for the metric (4.1) is as follows

$$ds^2 = -dt^2 + exp\left(\frac{6nKt^f}{n+2}\right)dx^2 + exp\left(\frac{6Kt^f}{n+2}\right)e^{2x}dy^2 + exp\left(\frac{6Kt^f}{n+2}\right)e^{-2x}dz^2 \tag{4.29}$$

4.4 Model's Physical and Geometrical Characteristics

The physical and dynamical parameters for the model that provides the well description of the physical universe are as follows

Hubble Parameter (H)

$$H = Kft^{(f-1)} \tag{4.30}$$

Expansion Scalar (θ)

$$\theta = 3Kft^{(f-1)} \tag{4.31}$$

Spatial Volume (V)

$$V = exp(3Kt^f) \tag{4.32}$$

Anisotropy Parameter (A_m)

$$A_m = \frac{2(n-1)^2}{(n+2)^2} \tag{4.33}$$

Shear Scalar (σ)

$$\sigma = \frac{3(n-1)Kft^{f-1}}{\sqrt{t}(n+2)} \tag{4.34}$$

Deceleration Parameter (q)

$$q = -1 + \frac{(1-f)}{Kft^f} \tag{4.35}$$

Using the results above and solving the field equations (4.19)-(4.22), the pressure (p),

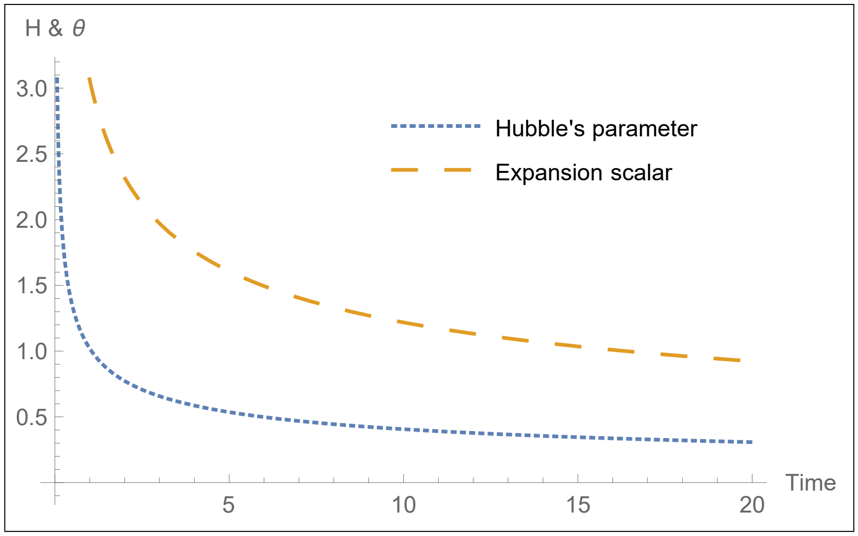

Figure 4.1: Variation of H and θ *vs*. t, For K = 2 and f = 0.5

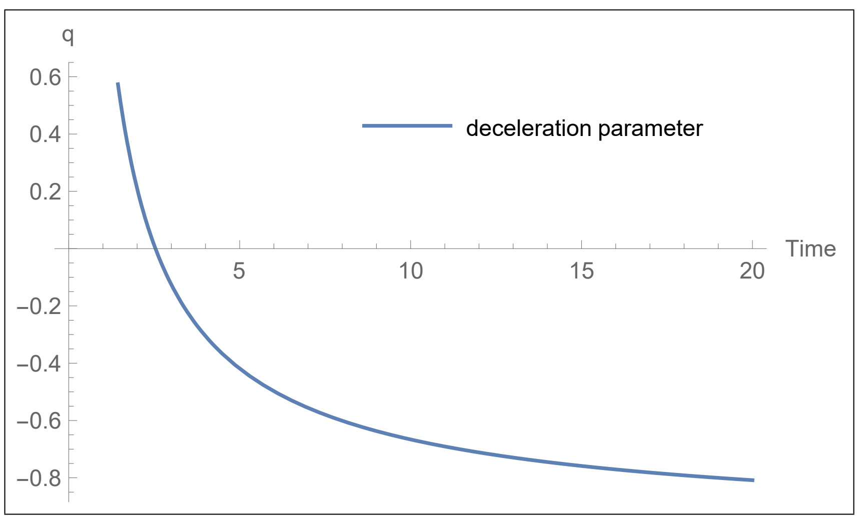

Figure 4.2: Variation of q *vs*. t, For K = 2 and f = 0.5

energy density (ρ), EoS parameter (w), and the electromagnetic field (F_{10}) are obtained as

$$p = -\phi_0^2 exp\left(2K\alpha t^f\right)\Bigg[\frac{27K^2 f^2 t^{2(f-1)}}{(n+2)^2} + \frac{6K(f-1)ft^{(f-2)}}{n+2} + \frac{3(n-1)}{2(n+2)}\left(K(f-1)t^{f-2} + 3K^2 f^2 t^{2(f-1)}\right)\Bigg] \tag{4.36}$$

$$\rho = \phi_0^2 exp(2K\alpha t^f)\Bigg[\frac{9(2n+1)K^2 f^2 t^{2(f-1)}}{(n+2)^2} + \frac{3}{4}K^2\alpha^2 f^2 t^{2(f-1)} - \frac{3(n-1)}{2(n+2)}\left(K(f-1)t^{f-2} + 3K^2 f^2 t^{2(f-1)}\right)\Bigg] \tag{4.37}$$

$$F_{10}^2 = \phi_0^2 exp(2K\alpha t^f)\left[\frac{(n-1)}{(n-2)}exp\left(\frac{6nKt^f}{n+2}\right)\left[9k^2 f^2 t^{f-2} + 3K(f-1)t^{f-2}\right] - 2\right] \tag{4.38}$$

$$w = -\frac{104K^2 f^2 t^{2(f-1)} + 24K(n+2)(f-1)ft^{f-2} + 3(n-1)(n-2)I}{36(2n+1)f^2 t^{2(f-1)} + 3K^2\alpha^2(n+2)^2 f^2 t^{2(f-1)} - 6(n-1)(n+2)I} \tag{4.39}$$

where, $I = (K(f-1)t^{f-2} + 3K^2 f^2 t^{2(f-1)})$

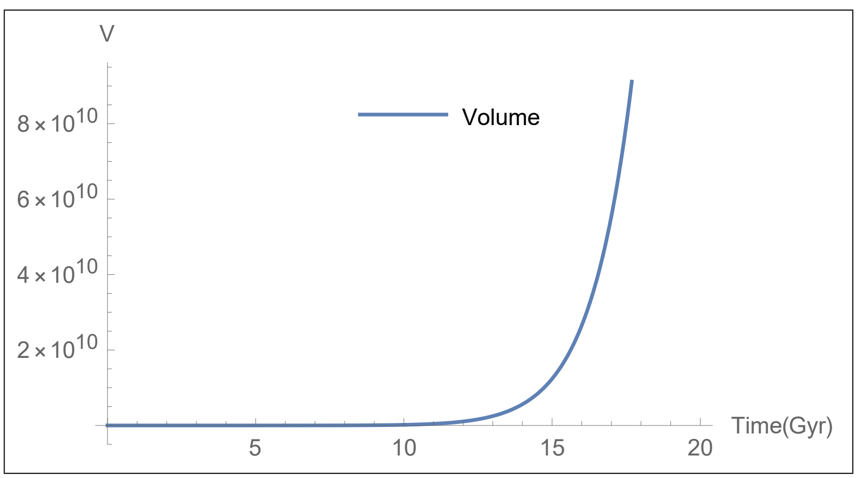

Figure 4.3: Variation of V $vs.$ t, For K = 2 and f = 0.5

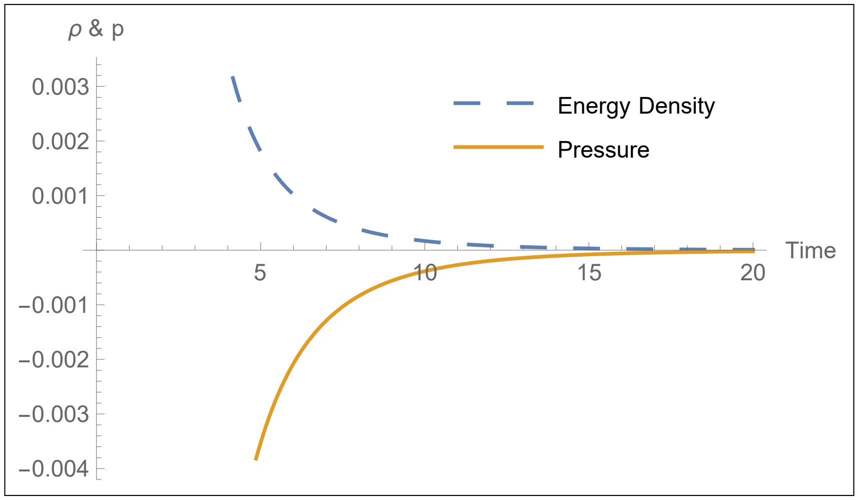

Figure 4.4: Variation of ρ *and* p *vs.* t, For ϕ_0 = 0.7, K =2, f = 0.5, α = -0.5 and n = 2

4.4 Energy Conditions, Statefinder Parameter

The model's energy conditions in terms of energy density and pressure are respectively obtained as

$$\rho + p = \phi_0^2 exp(2K\alpha t^f)\Bigg[\frac{3}{4}K^2\alpha^2 f^2 t^{2(f-1)} + \frac{18(n-1)K^2 f^2 t^{2(f-1)}}{(n+2)^2} - \frac{6(n-1)K^2 f^2 t^{2(f-1)} + 6(f-1)Kft^{(f-2)}}{(n+2)}\Bigg] \geq 0 \tag{4.40}$$

$$\rho + 3p = \phi_0^2 exp(2K\alpha t^f)\Bigg[\frac{3}{4}K^2\alpha^2 f^2 t^{2(f-1)} + \frac{18(n-4)K^2 f^2 t^{2(f-1)}}{(n+2)^2} - \frac{15(n-1)K^2 f^2 t^{2(f-1)} + 3(f-1)(n-1+6f)Kt^{(f-2)}}{(n+2)}\Bigg] \leq 0 \tag{4.41}$$

$$\rho - p = \phi_0^2 exp(2K\alpha t^f)\Bigg[\frac{3}{4}K^2\alpha^2 f^2 t^{2(f-1)} + \frac{3(n+5)K^2 f^2 t^{2(f-1)} + 3K(f-1)(n-1+2f)t^{(f-2)}}{(n+2)}\Bigg] \geq 0 \tag{4.42}$$

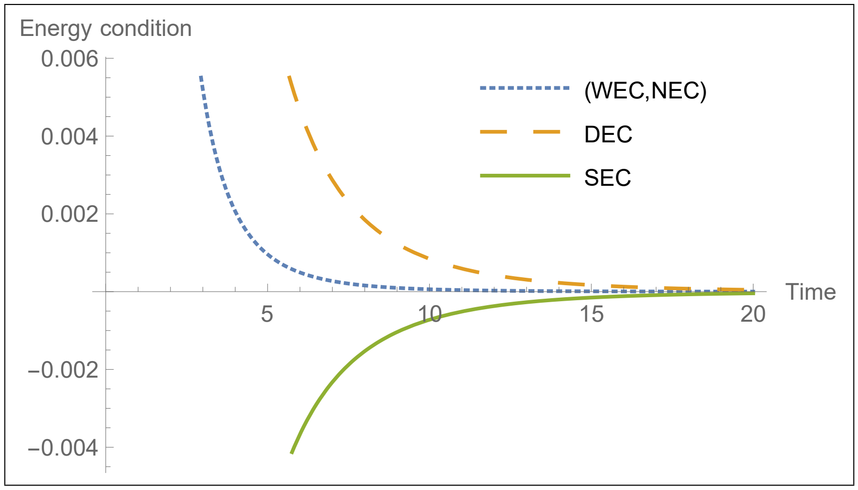

Figure 4.5: Variation of energy condition vs. t, For ϕ_0 = 0.7, K =2, f = 0.5, α = -0.5 and n = 2

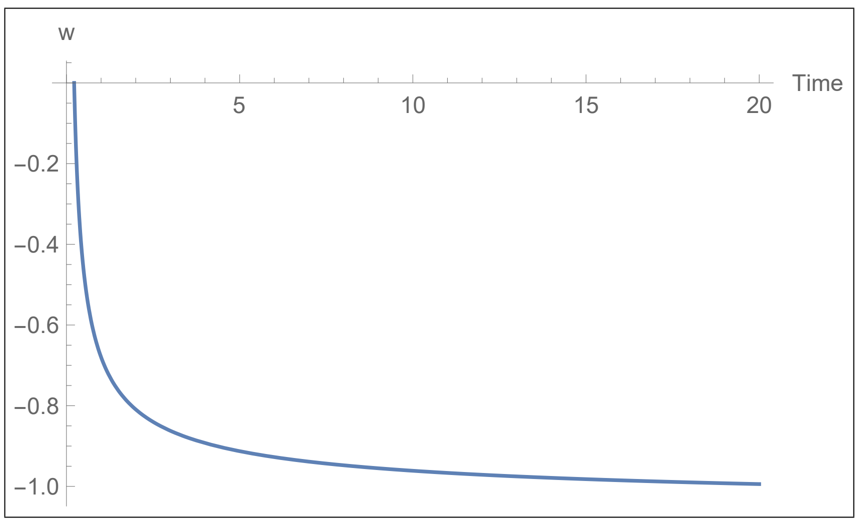

Figure 4.6: Variation of w vs. t, For ϕ_0 = 0.7, K =2, f = 0.5, α = -0.5 and n = 2

The graphical representation of the energy conditions is presented in Fig. 4.5. The behaviour of the energy changes dynamically as cosmic inflation evolves. The NEC and WEC are always satisfied with the model. The DEC is also satisfied with the model. However, the SEC for the model is violated after some time, $t = 0.6$, leading to the anti-gravitational phenomenon known as dark energy and the universe's jolt. The outcome favours the universe's accelerated expansion [Caldwell et al.(2006)].

Statefinder Parameters{r,s} :

According to Sahni et al. [Sahni et al.(2003)], the statefinder parameter $\{r, s\}$ is given as

$$r = \frac{\dddot{a}}{aH^3} = 1 + 3\frac{\dot{H}}{H^2} + \frac{\ddot{H}}{H^3} \tag{4.43}$$

$$s = \frac{(r-1)}{3(q-\frac{1}{2})} \tag{4.44}$$

The statefinder parameter $\{r, s\}$ helps separate various types of dark energy and improves the model's accuracy concerning the fundamental observational data. The statefinder parameter is the dimensionless and time derivative of the cosmic scale factor. In this case, the deceleration parameter is q, and the Hubble parameter is H.

The following result with the help of equations (4.30) and (4.35):

$$r = 1 + \frac{3(f-1)}{Kft^f} + \frac{(f-1)(f-2)}{K^2 f^2 t^{2f}} \tag{4.45}$$

and,

$$s = \frac{6(f-1)Kft^f + 2(f-1)(f-2)}{6(1-f)Kft^f - 9K^2 f^2 t^{2f}} \tag{4.46}$$

From the above result, when $t \to \infty$ the pair $\{r, s\} \to \{1, 0\}$ which implies that the model progresses from the Einstein static era to the standard ΛCDM model.

4.5 Conclusion

In this chapter, the Bianchi type-VI_0 cosmological model filled with the electromagnetic field is investigated in the Sen-Dunn scalar-tensor theory of gravitation. The scale factor generating a variable deceleration parameter proposed by Barrow [Barrow(1990)], which shows the behaviour of "intermediate" inflationary universes expanding at a rate intermediate between that of power-law and exponential inflation, is considered to obtain the model along with some plausible conditions. The proper volume of the model increases exponentially from zero volume at $t = 0$ to an infinite as time $t \to \infty$, indicating that the model expands with time. The model exhibits anisotropic dynamics and does not tends to isotropy for an immense value of time except for the particular case $n = 1$. The electromagnetic field for the model in this study shows that it slowly decreases as time tends to infinity. The Hubble parameter and the expansion scalar decrease finitely and tend to zero as $t \to \infty$. The model's energy density decreases positively, whereas the pressure is negative and tends to zero for $t \to \infty$. The deceleration parameter holds $-1 < q < 0$, indicating the transit from early decelerating to accelerating universe. For $f = 1$, the model shows de-Sitter expansion, and for $f = 0$, the expansion is at a constant rate. The SEC for the model is violated, thus exhibiting the dark energy property [Visser & Barcelo(2000)], which is considered the primary cause of the universe's accelerated expansion. The EoS parameter tends to the constant phantom limit or says vacuum energy for $w = -1$ (ΛCDM model) as $t \to \infty$ favouring the dark energy paradigm. The statefinder parameter for the model also concludes with the universe's standard ΛCDM model [Naidu et al.(2019)]. The results obtained for the model help better understand the role of dark energy in the universe's evolution.