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List of Publications and Presentations

List of Publications

1. Basumatary, J., Basumatary, B., and Broumi, S. (2022). On the structure of number of neutrosophic clopen topological space. *International Journal of Neutrosophic Science*, 18(4):192-203.
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3. Basumatary, B. Basumatary, J., and Nordo, G. (2023). On the Number of Clopen Topological Spaces on a Finite Set. *Afrika Matematika*, 34(3):1-14.

List of Papers Communicated to Journal

1. Basumatary, B. and Basumatary J., “A note on the representation of the number of neutrosophic clopen bi & tri topological space”.
2. Basumatary, B. and Basumatary, J., “A study on cardinalities of neutrosophic set and neutrosophic crisp set”.

Paper Presentations in Conference

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2. Basumatary, J., A Brief Review of the Number of Topological Spaces, *National Conference on Science & Technology for Sustainable Development (STSD-2022)*, Science College, Kokrajhar, Assam, India, held on September 9-10, 2022.
3. Basumatary, J., A Comparative Study of the Number of Neutrosophic Topological Spaces and Neutrosophic Clopen Topological Spaces having $k \leq 4$ Open Sets on a Finite Set, *National Conference on Advances in Mathematical Sciences*, Department of Mathematics, Gauhati University, Guwahati -781014, India, held on December 22-23, 2022.

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On the Structure of Number of Neutrosophic Clopen Topological Space

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Abstract

Let \mathcal{X} be a finite set having n elements. The formula for giving the number of topologies $T(n)$ is still not obtained. If the number of elements n of a finite set is small, we can compute it by hand. However, the difficulty of finding the number of the topology increases when n becomes large. A topology describes how elements of a set are spatially related to each other, and the same set can have different topologies. Studying this particular area is also a highly valued part of the topology, and this is one of the fascinating and challenging research areas. Note that the explicit formula for finding the number of topologies is undetermined till now, and many researchers are researching this particular area. This paper is towards the formulae for finding the number of neutrosophic clopen topological spaces having two, three, four, and five open sets. In addition, some properties related to formulae are determined.

Keywords: Combinatorics; Neutrosophic Set; Neutrosophic Clopen Topological Space; Number of Neutrosophic Clopen Topological Space

1 Introduction

The notion of the NS (neutrosophic set) was originally introduced by Smarandache¹ by the generalization of fuzzy set²² and intuitionistic fuzzy sets (IFSs).² Salama and Alblowi³ introduced the concept of NTS (neutrosophic topological space) after Coker⁴ introduced IFTS (intuitionistic fuzzy topological space). Nowadays, several researchers have contributed to NS and NTS (^{5,67}). Neutrosophic semi-open sets were introduced by Ishwarya and Bageerathi⁸ in NTS. Dhavaseelan and Jafari⁹ introduced generalized Neutrosophic closed sets. Later, Shanthy, Chandrasekar, and Safina¹⁰ generalized semi-closed sets in NTS. Many authors (^{11,12}) also studied the notion of NSs and discussed various properties.

Recall that a topology in which every open set is closed will be called a clopen topology (CLT). Many results for finding the number of topologies (NoTs) have been developed by several researchers (^{13,14,15,16,17}). Authors in (^{18,19,21}) also studied FTS (fuzzy topological spaces) and computed some formulas for finding NoTs and the number of FTS on a finite set. Recently, Francis, and Adenji²⁰ studied on clopen sets in topological spaces (TS). In this paper we focus on number of neutrosophic clopen topologies (NCLTs) and related results.

2 Preliminaries

In the following, \mathcal{X} denotes a non-empty finite set of cardinality n , M is a totally ordered set of cardinality $m \geq 2$, and let $\mathcal{N}_{\mathcal{X}}$ be the collection of all neutrosophic subsets of \mathcal{X} whose membership values lies in M . Also, $0^{NCL} = \langle \frac{x}{(0,1,1)} : x \in \mathcal{X} \rangle$ and $1^{NCL} = \langle \frac{x}{(1,0,0)} : x \in \mathcal{X} \rangle$ are taken to denote neutrosophic empty set and neutrosophic universal set respectively. The following definitions will be useful in the results. Moreover, open set is used to mean neutrosophic open set (NOS) in this paper.

Definition 2.1.¹ A NS A^{NCL} on a universe of discourse \mathcal{X} is defined as $A^{NCL} = \langle \frac{x}{(T(x),I(x),F(x))} : x \in \mathcal{X} \rangle$ where $T, I, F : \mathcal{X} \rightarrow]-0, 1^+[$. Note that $-0 \leq T(x) + I(x) + F(x) \leq 3^+$; $T(x), I(x)$ and $F(x)$ represents degree of membership function, degree of indeterminacy and degree of non-membership function respectively.

Definition 2.2.³ Let $\tau^{NCL} \subseteq \mathcal{N}_{\mathcal{X}}$ then τ^{NCL} is called a NT (neutrosophic topology) on \mathcal{X} if

- (i) $0^{NCL}, 1^{NCL} \in \tau^{NCL}$
- (ii) $A_1^{NCL} \cap A_2^{NCL} \in \tau^{NCL}$, for any $A_1^{NCL}, A_2^{NCL} \in \tau^{NCL}$.
- (iii) $\bigcup A_i^{NCL} \in \tau^{NCL}$, for arbitrary family $\{A_i^{NCL} : i \in I\} \in \tau^{NCL}$.

The pair $(\mathcal{X}, \tau^{NCL})$ is called NTS and any NS in τ^{NCL} is called NOS in \mathcal{X} .

Definition 2.3.¹⁶ The Stirling number of the second kind is the number of partitions of a finite set with n elements into k blocks. It is denoted by $S(n, k)$ or $S_{n,k}$ and its explicit formula is

$$S(n, k) = S_{n,k} = \frac{1}{k!} \sum_{j=0}^k (-1)^j \binom{k}{j} (k-j)^n.$$

3 Results

Definition 3.1. A NT τ^{NCL} on a non-empty set \mathcal{X} is said a NCLT if it consists of neutrosophic clopen sets, i.e., if every its NOSs is closed too. The pair $(\mathcal{X}, \tau^{NCL})$ is called neutrosophic clopen topological space (NCLTS) and if τ^{NCL} contains k -open sets then $(\mathcal{X}, \tau^{NCL})$ is called NCLTS having k -open sets.

Example 3.2. Let $\mathcal{X} = \{u_1, v_1, w_1\}$ and consider the family $\tau^{NCL} = \{0^{NCL}, 1^{NCL}, A_1^{NCL}, A_2^{NCL}\}$, Where $0^{NCL} = \langle \frac{u_1}{(0,1,1)}, \frac{v_1}{(0,1,1)}, \frac{w_1}{(0,1,1)} \rangle$, $1^{NCL} = \langle \frac{u_1}{(1,0,0)}, \frac{v_1}{(1,0,0)}, \frac{w_1}{(1,0,0)} \rangle$, $A_1^{NCL} = \langle \frac{u_1}{(0.2,0.5,0.3)}, \frac{v_1}{(0.3,0.6,0.5)}, \frac{w_1}{(0,0.7,0.4)} \rangle$, $A_2^{NCL} = \langle \frac{u_1}{(0.3,0.5,0.2)}, \frac{v_1}{(0.5,0.4,0.3)}, \frac{w_1}{(0.4,0.3,0)} \rangle$. Then τ^{NCL} is a NCLT on \mathcal{X} and so $(\mathcal{X}, \tau^{NCL})$ is a NCLTS on \mathcal{X} .

Proposition 3.3. Arbitrary intersection of NCLTs on \mathcal{X} is clopen.

Proof. Let

$$\mathcal{N}_{\tau}^{NCL} = \bigcap_{i \in \lambda} \tau_i^{NCL},$$

where λ is an index set and $\tau_i^{NCL} \in \mathcal{N}_{\mathcal{X}}^{NCL}$, collection of all NCLTs on \mathcal{X} . Clearly $\mathcal{N}_{\tau}^{NCL} \neq \emptyset$ as $0^{NCL}, 1^{NCL} \in \mathcal{N}_{\tau}^{NCL}$.

Let A^{NCL}, B^{NCL} be any two members of \mathcal{N}_{τ}^{NCL} . Then

$$\begin{aligned} & A^{NCL}, B^{NCL} \in \mathcal{N}_{\tau}^{NCL} \\ \implies & A^{NCL}, B^{NCL} \in \bigcap_{i \in \lambda} \tau_i^{NCL} \\ \implies & A^{NCL}, B^{NCL} \in \tau_i^{NCL} \quad ; \forall i \\ \implies & A^{NCL} \cap B^{NCL} \in \tau_i^{NCL} \text{ and } A^{NCL} \cup B^{NCL} \in \tau_i^{NCL} \quad ; \forall i \\ \implies & A^{NCL} \cap B^{NCL} \in \bigcap_{i \in \lambda} \tau_i^{NCL} \text{ and } A^{NCL} \cup B^{NCL} \in \bigcap_{i \in \lambda} \tau_i^{NCL} \quad ; \forall i \\ \implies & A^{NCL} \cap B^{NCL} \in \mathcal{N}_{\tau}^{NCL} \text{ and } A^{NCL} \cup B^{NCL} \in \mathcal{N}_{\tau}^{NCL} \end{aligned}$$

Therefore \mathcal{N}_{τ}^{NCL} is a NT on \mathcal{X} .

Let A^{NCL} be any element of \mathcal{N}_{τ}^{NCL} , then

$$\begin{aligned} A^{NCL} \in \mathcal{N}_{\tau}^{NCL} & \implies A^{NCL} \in \bigcap_{i \in \lambda} \tau_i^{NCL} \\ & \implies A^{NCL} \in \tau_i^{NCL} \quad ; \forall i \\ & \implies C(A^{NCL}) \in \tau_i^{NCL} \quad ; \forall i \\ & \implies C(A^{NCL}) \in \bigcap_{i \in \lambda} \tau_i^{NCL}; \\ & \implies C(A^{NCL}) \in \mathcal{N}_{\tau}^{NCL} \end{aligned}$$

Since A^{NCL} is arbitrary element of \mathcal{N}_{τ}^{NCL} and $C(A^{NCL}) \in \mathcal{N}_{\tau}^{NCL}$ implies that every element of \mathcal{N}_{τ}^{NCL} is NCLS. Hence arbitrary intersection of NCLTs is clopen. \square

Remark 3.4. Union of NCLTs on \mathcal{X} is not clopen.

Illustration: Let $\mathcal{X} = \{u_1, v_1\}$ and $M = \{(0, 1, 1), (0.5, 0.5, 0.5), (1, 0, 0)\}$. Then number of elements in $\mathcal{N}_{\mathcal{X}}$, i.e., $|\mathcal{N}_{\mathcal{X}}| = 3^2 = 9$. These are

$$\begin{aligned} 0^{NCL} &= \langle \frac{u_1}{(0,1,1)}, \frac{v_1}{(0,1,1)} \rangle, 1^{NCL} = \langle \frac{u_1}{(1,0,0)}, \frac{v_1}{(1,0,0)} \rangle, A_1^{NCL} = \langle \frac{u_1}{(0,1,1)}, \frac{v_1}{(0.5,0.5,0.5)} \rangle, \\ A_2^{NCL} &= \langle \frac{u_1}{(0,1,1)}, \frac{v_1}{(1,0,0)} \rangle, A_3^{NCL} = \langle \frac{u_1}{(0.5,0.5,0.5)}, \frac{v_1}{(0,1,1)} \rangle, \\ A_4^{NCL} &= \langle \frac{u_1}{(0.5,0.5,0.5)}, \frac{v_1}{(0.5,0.5,0.5)} \rangle, A_5^{NCL} = \langle \frac{u_1}{(0.5,0.5,0.5)}, \frac{v_1}{(1,0,0)} \rangle, \\ A_6^{NCL} &= \langle \frac{u_1}{(1,0,0)}, \frac{v_1}{(0,1,1)} \rangle, A_7^{NCL} = \langle \frac{u_1}{(1,0,0)}, \frac{v_1}{(0.5,0.5,0.5)} \rangle. \end{aligned}$$

Here $n = 2, m = 3$, so $\mathcal{J}_{\mathcal{N}_{\mathcal{X}}}(2, 3, 3) = 3^2 - 2 = 9 - 2 = 7$.

In this case

$$\begin{aligned} \tau_1^{NCL} &= \{0^{NCL}, 1^{NCL}, A_4^{NCL}\} \text{ and} \\ \tau_2^{NCL} &= \{0^{NCL}, 1^{NCL}, A_1^{NCL}, A_2^{NCL}, A_6^{NCL}, A_7^{NCL}\} \text{ are NCLTs on } \mathcal{X}. \end{aligned}$$

But $\tau_1^{NCL} \cup \tau_2^{NCL} = \{0^{NCL}, 1^{NCL}, A_1^{NCL}, A_2^{NCL}, A_4^{NCL}, A_6^{NCL}, A_7^{NCL}\}$ is not a NCLT on \mathcal{X} as $A_4^{NCL} \cap A_6^{NCL} = A_3^{NCL} \notin \tau_1^{NCL} \cup \tau_2^{NCL}$.

Proposition 3.5. Union of two NCLTs is again a NCLT if one is contained in the other.

Proof. Let A^{NCL} and B^{NCL} be two NCLTs on \mathcal{X} . Let $A^{NCL} \subseteq B^{NCL}$, then $A^{NCL} \cup B^{NCL} = B^{NCL}$, which is a NCLT on \mathcal{X} . Similarly, if $B^{NCL} \subseteq A^{NCL}$, then $A^{NCL} \cup B^{NCL} = A^{NCL}$, which is a NCLT on \mathcal{X} . This shows that union of two NCLTs is again a NCLT if one is contained in the other. \square

Number of Neutrosophic clopen topology having 2-open Sets:

1. If $\mathcal{X} = \{u_1\}$ or $|\mathcal{X}| = 1$ whose neutrosophic values lies in M .

Case I: If $M = \{(0, 1, 1), (1, 0, 0)\}$.

Then, $|\mathcal{N}_{\mathcal{X}}| = 2^1 = 2$. These are

$$0^{NCL} = \langle \frac{u_1}{(0,1,1)} \rangle, 1^{NCL} = \langle \frac{u_1}{(1,0,0)} \rangle.$$

In this case we will get only one NT which is $\tau_1^{NCL} = \{0^{NCL}, 1^{NCL}\}$. This NT is also NCLT.

Case II: If $M = \{(0, 1, 1), (0.5, 0.5, 0.5), (1, 0, 0)\}$

Then, $|\mathcal{N}_{\mathcal{X}}| = 3^1 = 3$. These are

$$0^{NCL}, 1^{NCL}, A_1^{NCL} = \langle \frac{u_1}{(0.5,0.5,0.5)} \rangle.$$

In this case also we will get only one NT having 2-open set which is $\tau_1^{NCL} = \{0^{NCL}, 1^{NCL}\}$ and hence a NCLT.

Case III: If $M = \{(0, 1, 1), (T, I, F), (F, 1 - I, T), (1, 0, 0)\}; T, I, F \in [0, 1]$

In this case, $|\mathcal{N}_{\mathcal{X}}| = 4^1 = 4$. These are

$$0^{NCL}, 1^{NCL}, A_1^{NCL} = \langle \frac{u_1}{(T,I,F)} \rangle, A_2^{NCL} = \langle \frac{u_1}{(F,1-I,T)} \rangle.$$

In this case also we will get only one NT having 2-open set which is $\tau_1^{NCL} = \{0^{NCL}, 1^{NCL}\}$ and therefore a NCLT.



Figure 1: NCLT having 2-open sets

2. If $\mathcal{X} = \{u_1, v_1\}$ or $|\mathcal{X}| = 2$ whose neutrosophic values lies in M .

Case I: If $M = \{(0, 1, 1), (1, 0, 0)\}$

Then, $|\mathcal{N}_{\mathcal{X}}| = 2^2 = 4$. These are

$$\begin{aligned} 0^{NCL} &= \langle \frac{u_1}{(0,1,1)}, \frac{v_1}{(0,1,1)} \rangle, 1^{NCL} = \langle \frac{u_1}{(1,0,0)}, \frac{v_1}{(1,0,0)} \rangle, \\ A_1^{NCL} &= \langle \frac{u_1}{(0,1,1)}, \frac{v_1}{(1,0,0)} \rangle, A_2^{NCL} = \langle \frac{u_1}{(1,0,0)}, \frac{v_1}{(0,1,1)} \rangle. \end{aligned}$$

In this case, NCLT having 2-open set is one i.e., $\tau_1^{NCL} = \{0^{NCL}, 1^{NCL}\}$.

Case II: If $M = \{(0, 1, 1), (0.5, 0.5, 0.5), (1, 0, 0)\}$

Then, $|\mathcal{N}_{\mathcal{X}}| = 3^2 = 9$. These are

$$\begin{aligned} 0^{NCL}, 1^{NCL}, A_1^{NCL} &= \langle \frac{u_1}{(0,1,1)}, \frac{v_1}{(0.5,0.5,0.5)} \rangle, A_2^{NCL} = \langle \frac{u_1}{(1,0,0)}, \frac{v_1}{(0,1,1)} \rangle, \\ A_3^{NCL} &= \langle \frac{u_1}{(0.5,0.5,0.5)}, \frac{v_1}{(0,1,1)} \rangle, A_4^{NCL} = \langle \frac{u_1}{(0.5,0.5,0.5)}, \frac{v_1}{(0.5,0.5,0.5)} \rangle, \\ A_5^{NCL} &= \langle \frac{u_1}{(0.5,0.5,0.5)}, \frac{v_1}{(1,0,0)} \rangle, A_6^{NCL} = \langle \frac{u_1}{(1,0,0)}, \frac{v_1}{(0,1,1)} \rangle, \\ A_7^{NCL} &= \langle \frac{u_1}{(1,0,0)}, \frac{v_1}{(0.5,0.5,0.5)} \rangle. \end{aligned}$$

In this case also NCLT having 2-open set is one i.e., $\tau_1^{NCL} = \{0^{NCL}, 1^{NCL}\}$.

Case III: If $M = \{(0, 1, 1), (T, I, F), (F, 1 - I, T), (1, 0, 0)\}; T, I, F \in [0, 1]$

In this case, $|\mathcal{N}_X| = 4^2 = 16$. These are

$$\begin{aligned} 0^{NCL}, 1^{NCL}, A_1^{NCL} &= \langle \frac{u_1}{(0,1,1)}, \frac{v_1}{(T,I,F)} \rangle, A_2^{NCL} = \langle \frac{u_1}{(0,1,1)}, \frac{v_1}{(1,0,0)} \rangle, \\ A_3^{NCL} &= \langle \frac{u_1}{(T,I,F)}, \frac{v_1}{(0,1,1)} \rangle, A_4^{NCL} = \langle \frac{u_1}{(T,I,F)}, \frac{v_1}{(T,I,F)} \rangle, \\ A_5^{NCL} &= \langle \frac{u_1}{(T,I,F)}, \frac{v_1}{(1,0,0)} \rangle, A_6^{NCL} = \langle \frac{u_1}{(1,0,0)}, \frac{v_1}{(0,1,1)} \rangle, \\ A_7^{NCL} &= \langle \frac{u_1}{(1,0,0)}, \frac{v_1}{(T,I,F)} \rangle, A_8^{NCL} = \langle \frac{u_1}{(0,1,1)}, \frac{v_1}{(F,1-I,T)} \rangle, \\ A_9^{NCL} &= \langle \frac{u_1}{(T,I,F)}, \frac{v_1}{(F,1-I,T)} \rangle, A_{10}^{NCL} = \langle \frac{u_1}{(F,1-I,T)}, \frac{v_1}{(0,1,1)} \rangle, \\ A_{11}^{NCL} &= \langle \frac{u_1}{(F,1-I,T)}, \frac{v_1}{(F,1-I,T)} \rangle, A_{12}^{NCL} = \langle \frac{u_1}{(F,1-I,T)}, \frac{v_1}{(1,0,0)} \rangle, \\ A_{13}^{NCL} &= \langle \frac{u_1}{(F,1-I,T)}, \frac{v_1}{(T,I,F)} \rangle, A_{14}^{NCL} = \langle \frac{u_1}{(1,0,0)}, \frac{v_1}{(F,1-I,T)} \rangle. \end{aligned}$$

In this case also NCLT having 2-open set is one i.e., $\tau_1^{NCL} = \{0^{NCL}, 1^{NCL}\}$.

Proposition 3.6. For $|\mathcal{X}| = n, |M| = m, M$ is the any set of neutrosophic values containing $(1, 0, 0)$ and $(0, 1, 1)$, then number of NCLT having 2-open sets is one.

Proof. The NT having 2-open sets is indiscrete NT only i.e., $\tau^{NCL} = \{0^{NCL}, 1^{NCL}\}$. This NT is NCLT as 0^{NCL} and 1^{NCL} are complements of each other. Therefore, the number of NCLTS having 2-open sets is one. □

Number of Neutrosophic clopen topology having 3-open Sets:

1. If $\mathcal{X} = \{p\}$ or $|\mathcal{X}| = 1$ whose neutrosophic values lies in M .

Case I: If $M = \{(0, 1, 1), (1, 0, 0)\}$

In this case, $|\mathcal{N}_X| = 2^1 = 2$. We will get only one NCLT which is

$\tau_1^{NCL} = \{0^{NCL}, 1^{NCL}\}$ and so no NCLT having 3-open sets. Therefore, the number of NCLT having 3-open sets is zero.

Case II: If $M = \{(0, 1, 1), (0.5, 0.5, 0.5), (1, 0, 0)\}$

In this case, $|\mathcal{N}_X| = 3^1 = 3$, we will get only one NCLT having 3-open sets, which is

$\tau_1^{NCL} = \{0^{NCL}, A_1^{NCL}, 1^{NCL}\}$ as complement of A_1^{NCL} i.e., $C(A_1^{NCL}) = A_1^{NCL}$. Therefore, the number of NCLT having 3-open sets is one.

Case III: If $M = \{(0, 1, 1), (T, I, F), (F, 1 - I, T), (1, 0, 0)\}; T, I, F \in [0, 1]$

In this case, $|\mathcal{N}_X| = 4^1 = 4$. We will get no NT having 3-open sets. Therefore, the number of NCLT having 3-open sets is zero.

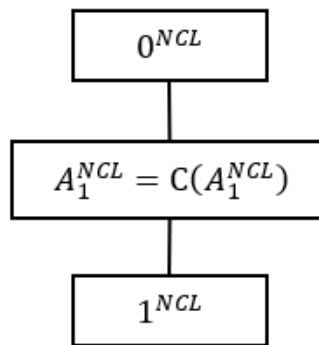


Figure 2: NCLT having 3-open sets

2. If $\mathcal{X} = \{p, q\}$ or $|\mathcal{X}| = 2$ whose neutrosophic values lies in M .

Case I: If $M = \{(0, 1, 1), (1, 0, 0)\}$

In this case, $|\mathcal{N}_X| = 2^2 = 4$. We will get no NCLT having 3-open sets. Therefore, the number of NCLT having 3-open sets is zero.

Case II: If $M = \{(0, 1, 1), (0.5, 0.5, 0.5), (1, 0, 0)\}$

Then, $|\mathcal{N}_X| = 3^2 = 9$. We will get only one NT having 3-open sets, which is $\tau_1^{NCL} = \{0^{NCL}, A_1^{NCL}, 1^{NCL}\}$. Therefore, the number of NCLT having 3-open sets is one.

Case III: If $M = \{(0, 1, 1), (T, I, F), (F, 1 - I, T), (1, 0, 0)\}$, where $T, I, F \in [0, 1]$

In this case, $|\mathcal{N}_X| = 4^2 = 16$. We will get no NT having 3-open sets. Therefore, the number of NCLT having 3-open sets is zero.

Proposition 3.7. For $|\mathcal{X}| = n$, the number of NCLT having 3-open sets is always one for M containing $(0, 1, 1)$, $(1, 0, 0)$ and $(T, 0.5, F)$, $T = F$; $T, F \in [0, 1]$, and zero for M that does not include neutrosophic values which are complement to each other i.e., $(T, 0.5, F)$, $T = F$; $T, F \in [0, 1]$.

Proof. Let, M be the set containing $(0, 1, 1)$, $(1, 0, 0)$, $(T, 0.5, F)$, $T = F$; $T, F \in [0, 1]$, and other neutrosophic values.

Case I: Let $|\mathcal{X}| = 1$, say $\mathcal{X} = \{a\}$

In this case the NCLT having 3-open sets is $\{0^{NCL}, A_1^{NCL}, 1^{NCL}\}$, where $A_1^{NCL} = \langle \frac{a}{(T, 0.5, F)} \rangle$.

Case II: Let $|\mathcal{X}| = 2$, say $\mathcal{X} = \{a, b\}$

In this case the NCLT having 3-open sets is $\{0^{NCL}, 1^{NCL}, A_2^{NCL}\}$, where $A_2^{NCL} = \langle \frac{a}{(T, 0.5, F)}, \frac{b}{(T, 0.5, F)} \rangle$.

Case III: Let $|\mathcal{X}| = 3$, say $\mathcal{X} = \{a, b, c\}$

In this case the NCLT having 3-open sets is $\{0^{NCL}, 1^{NCL}, A_3^{NCL}\}$, where $A_3^{NCL} = \langle \frac{a}{(T, 0.5, F)}, \frac{b}{(T, 0.5, F)}, \frac{c}{(T, 0.5, F)} \rangle$.

Case IV: Let $|\mathcal{X}| = n$ (finite), say $\mathcal{X} = \{a_1, a_2, a_3, \dots, a_n\}$

In this case the NCLT having 3-open sets is $\{0^{NCL}, 1^{NCL}, A_n^{NCL}\}$, where

$$A_n^{NCL} = \langle \frac{a_1}{(T, 0.5, F)}, \frac{a_2}{(T, 0.5, F)}, \frac{a_3}{(T, 0.5, F)}, \dots, \frac{a_n}{(T, 0.5, F)} \rangle.$$

It is seen that there exists only one NCLT having 3-open sets. This NCLT contains 0^{NCL} , 1^{NCL} and A_n^{NCL} . Note that, in A_n^{NCL} every member of \mathcal{X} has neutrosophic membership value as $(T, 0.5, F)$, $T = F$; $T, F \in [0, 1]$, whose complement is itself i.e., $A_n^{NCL} = C(A_n^{NCL})$.

On the other hand, if M does not contain $(T, 0.5, F)$, $T = F$; $T, F \in [0, 1]$, then there exist no neutrosophic subset of \mathcal{X} of the form A_n^{NCL} , such that $A_n^{NCL} \neq C(A_n^{NCL})$. Hence there exists no NCLT having 3-open sets. □

Number of Neutrosophic clopen topology having 4-open Sets:

1. If $\mathcal{X} = \{p\}$ or $|\mathcal{X}| = 1$ whose neutrosophic values lies in M .

Case I: If $M = \{(0, 1, 1), (1, 0, 0)\}$

In this case, there exists no NCLT having 4-open sets. Therefore, the number of NCLT having 4-open sets is zero.

Case II: If $M = \{(0, 1, 1), (0.5, 0.5, 0.5), (1, 0, 0)\}$

In this case, $|\mathcal{N}_{\mathcal{X}}| = 3^1 = 3$, we will get no NT having 4-open set. Therefore, the number of NCLT having 4-open sets is zero.

Case III: If $M = \{(0, 1, 1), (T, I, F), (F, 1 - I, T), (1, 0, 0)\}$; $T, I, F \in [0, 1]$.

In this case, $|\mathcal{N}_{\mathcal{X}}| = 4^1 = 4$. We will get one NCLT having 4-open set. Therefore, the number of NCLT having 4-open sets is one.

2. If $\mathcal{X} = \{p, q\}$ or $|\mathcal{X}| = 2$ whose neutrosophic values lies in M .

Case I: If $M = \{(0, 1, 1), (1, 0, 0)\}$

In this case, $|\mathcal{N}_{\mathcal{X}}| = 2^2 = 4$, we will get one NCLT having 4-open sets. Therefore, the number of NCLT having 4-open sets is one i.e., $\tau_1^{NCL} = \{0^{NCL}, A_1^{NCL}, A_2^{NCL}, 1^{NCL}\}$.

Case II: If $M = \{(0, 1, 1), (0.5, 0.5, 0.5), (1, 0, 0)\}$

In this case, $|\mathcal{N}_{\mathcal{X}}| = 3^2 = 9$. We will get three NCLTs having 4-open set. These are

$$\tau_1^{NCL} = \{0^{NCL}, A_1^{NCL}, A_7^{NCL}, 1^{NCL}\}, \quad \tau_2^{NCL} = \{0^{NCL}, A_2^{NCL}, A_6^{NCL}, 1^{NCL}\},$$

$$\tau_3^{NCL} = \{0^{NCL}, A_3^{NCL}, A_5^{NCL}, 1^{NCL}\}.$$

Therefore, the number of NCLTs having 4-open sets is three.

Case III: If $M = \{(0, 1, 1), (T, I, F), (F, 1 - I, T), (1, 0, 0)\}$; $T, I, F \in [0, 1]$.

In this case, $|\mathcal{N}_{\mathcal{X}}| = 4^2 = 16$. We will get four NCLT having 4-open sets. These are

$$\tau_1^{NCL} = \{0^{NCL}, A_1^{NCL}, A_{14}^{NCL}, 1^{NCL}\}, \quad \tau_2^{NCL} = \{0^{NCL}, A_4^{NCL}, A_{11}^{NCL}, 1^{NCL}\},$$

$$\tau_3^{NCL} = \{0^{NCL}, A_2^{NCL}, A_6^{NCL}, 1^{NCL}\}, \quad \tau_4^{NCL} = \{0^{NCL}, A_3^{NCL}, A_{12}^{NCL}, 1^{NCL}\}.$$

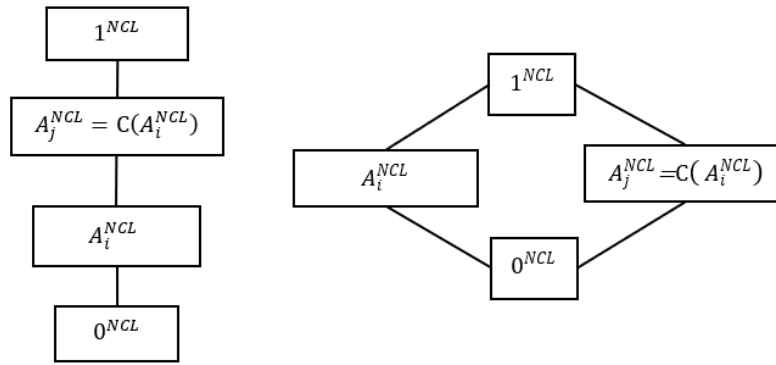


Figure 3: NCLT having 4-open sets

Number of Neutrosophic clopen topology having 5-open Sets:

1. If $\mathcal{X} = \{p\}$ or $|\mathcal{X}| = 1$ whose neutrosophic values lies in M .

Case I: If $M = \{(0, 1, 1), (1, 0, 0)\}$

In this case, $|\mathcal{N}_{\mathcal{X}}| = 2^1 = 2$, we will get only one NT which is $\tau_1^{NCL} = \{0^{NCL}, 1^{NCL}\}$ and so, there exists no NCLT having 5-open sets. Therefore, the number of NCLT having 5-open sets is zero.

Case II: If $M = \{(0, 1, 1), (0.5, 0.5, 0.5), (1, 0, 0)\}$

In this case, $|\mathcal{N}_{\mathcal{X}}| = 3^1 = 3$, we will get only two NCLTs having 2 and 3-open sets. Therefore, the number of NCLTs having 5-open sets is zero.

Case III: If $M = \{(0, 1, 1), (T, I, F), (F, 1 - I, T), (1, 0, 0)\}; T, I, F \in [0, 1]$

In this case, $|\mathcal{N}_{\mathcal{X}}| = 4^1 = 4$. We will get no NT having 5-open sets. Therefore, the number of NCLT having 5-open sets is zero.

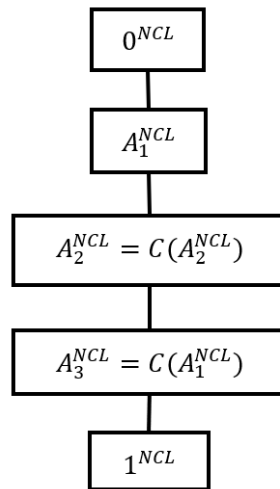


Figure 4: NCLT having 5-open sets

2. If $\mathcal{X} = \{p, q\}$ or $|\mathcal{X}| = 2$ whose neutrosophic values lies in M .

Case I: If $M = \{(0, 1, 1), (1, 0, 0)\}$

In this case, $|\mathcal{N}_{\mathcal{X}}| = 2^2 = 4$, we will get no NCLT having 5-open sets. Therefore, the number of NCLT having 5-open sets is zero.

Case II: If $M = \{(0, 1, 1), (0.5, 0.5, 0.5), (1, 0, 0)\}$

Then $|\mathcal{N}_{\mathcal{X}}| = 3^2 = 9$. We will get two NCLTs having 5-open sets. These are $\tau_1^{NCL} = \{0^{NCL}, A_1^{NCL}, A_4^{NCL}, A_7^{NCL}, 0^{NCL}\}$, $\tau_2^{NCL} = \{0^{NCL}, A_3^{NCL}, A_4^{NCL}, A_5^{NCL}, 0^{NCL}\}$.

Therefore, the number of NCLTs having 5-open sets is two.

Case III: If $M = \{(0, 1, 1), (T, I, F), (F, 1 - I, T), (1, 0, 0)\}; T, I, F \in [0, 1]$

In this case, $|\mathcal{N}_{\mathcal{X}}| = 4^2 = 16$. We will get no NCLT having 5-open sets. Therefore, the number of NCLT having 5-open sets is zero.

Proposition 3.8. For $|\mathcal{X}| = n, |M| = m, M$ is the any finite set of neutrosophic values containing $(1, 0, 0), (0, 1, 1)$

and other neutrosophic values which are complement to each other, then number of NCLTs having m^n -open sets is one and for any other M it is zero.

Proof. In this case, the NT having m^n -open sets is discrete NT. This NCLT is also clopen as it contains all neutrosophic subsets and complement of each subset is also in that NT. Therefore, the number of NCLTs having m^n -open sets is one.

Further, it is found that any other M , with $|M| = m > 2$, is equivalent to $M = \{(1, 0, 0), (0, 1, 1)\}$ for NCLT. But for $M = \{(1, 0, 0), (0, 1, 1)\}$, the maximum number of open sets in NCLT is 2^n . Since $m^n \geq 2^n$, the number of NCLTs having m^n -open sets is zero. \square

Corollary 3.9. *The minimum number of NOSs in a NCLT is 2 and the maximum number of NOSs in a NCLT is m^n , where n is the number of elements in \mathcal{X} and m is the number of elements in M .*

Proof. This result is obtained by using Proposition 3.8. \square

Proposition 3.10. *For $|\mathcal{X}| = n$, $M = \{(0, 1, 1), (T, I, F), (F, 1 - I, T), (1, 0, 0)\}$, where $T \neq F; T, I, F \in [0, 1]$, then the number of NCLTs having odd number of open sets is always zero.*

Proposition 3.11. *For $|\mathcal{X}| = n$, $M = \{(1, 0, 0), (0, 1, 1)\}$ and $\mathcal{N}_{\mathcal{X}}^{NCL}$ be the set of all NCLTs on \mathcal{X} whose membership values lies in M then*

- (i) $\mathcal{N}_{\mathcal{X}}^{NCL}$ contains only NCLTs having 2^k -open sets where $k = 1, 2, 3, \dots, n$.
- (ii) Number of NCLTs having 2^k -open sets is $S(n, k)$, $k = 1, 2, \dots, n$.

Proof. Let, $|\mathcal{X}| = n$, $M = \{(1, 0, 0), (0, 1, 1)\}$ and η_k be the number of NCLTs having k -open sets.

Case I: If $|\mathcal{X}| = 1$ say, $\mathcal{X} = \{u_1\}$ whose neutrosophic values lies in M . Then $|\mathcal{N}_{\mathcal{X}}| = 2^1 = 2$. These are $0^{NCL} = \langle \frac{u_1}{(0,1,1)} \rangle, 1^{NCL} = \langle \frac{u_1}{(1,0,0)} \rangle$.

In this case we will get only one NCLT which is the indiscrete NT i.e., $\tau_1^{NCL} = \{0^{NCL}, 1^{NCL}\}$. This shows that for $|\mathcal{X}| = 1$,

- (a) there exists only NCLT having $2^n, n = 1$ open sets.
- (b) there exists 1 NCLT having 2^1 -open sets i.e., $\eta_{2^1} = S(1, 1)$.

Case II: If $|\mathcal{X}| = 2$ say, $\mathcal{X} = \{u_1, v_1\}$ whose neutrosophic values lies in M . Then $|\mathcal{N}_{\mathcal{X}}| = 2^2 = 4$. These are $0^{NCL} = \langle \frac{u_1}{(0,1,1)}, \frac{v_1}{(0,1,1)} \rangle, 1^{NCL} = \langle \frac{u_1}{(1,0,0)}, \frac{v_1}{(1,0,0)} \rangle,$

$$A_1^{NCL} = \langle \frac{u_1}{(0,1,1)}, \frac{v_1}{(1,0,0)} \rangle, A_2^{NCL} = \langle \frac{u_1}{(1,0,0)}, \frac{v_1}{(0,1,1)} \rangle.$$

These NCLTs are

$$\tau_1^{NCL} = \{0^{NCL}, 1^{NCL}\}, \tau_2^{NCL} = \{0^{NCL}, A_1^{NCL}, A_2^{NCL}, 1^{NCL}\}.$$

This gives that the number of open sets in τ_1^{NCL} is 2 and in τ_2^{NCL} is 4 = 2^2 . Therefore, for $|\mathcal{X}| = 2$,

- (a) there exist only NCLTs having $2^n, n = 1, 2$ open sets i.e., τ_1^{NCL} and τ_2^{NCL} .
- (b) there exists 1 NCLT having 2^1 -open set and 1 NCLT having 2^2 -open sets i.e., $\eta_{2^1} = 1 = S(2, 1)$ and $\eta_{2^2} = 1 = S(2, 2)$ respectively.

Case III: If $|\mathcal{X}| = 3$ say, $\mathcal{X} = \{u_1, v_1, w_1\}$ whose neutrosophic values lies in M . Then $|\mathcal{N}_{\mathcal{X}}| = 2^3 = 8$. These are

$$0^{NCL} = \langle \frac{u_1}{(0,1,1)}, \frac{v_1}{(0,1,1)}, \frac{w_1}{(0,1,1)} \rangle, 1^{NCL} = \langle \frac{u_1}{(1,0,0)}, \frac{v_1}{(1,0,0)}, \frac{w_1}{(1,0,0)} \rangle,$$

$$A_1^{NCL} = \langle \frac{u_1}{(1,0,0)}, \frac{v_1}{(0,1,1)}, \frac{w_1}{(0,1,1)} \rangle, A_2^{NCL} = \langle \frac{u_1}{(0,1,1)}, \frac{v_1}{(1,0,0)}, \frac{w_1}{(0,1,1)} \rangle,$$

$$A_3^{NCL} = \langle \frac{u_1}{(0,1,1)}, \frac{v_1}{(0,1,1)}, \frac{w_1}{(1,0,0)} \rangle, A_4^{NCL} = \langle \frac{u_1}{(1,0,0)}, \frac{v_1}{(1,0,0)}, \frac{w_1}{(0,1,1)} \rangle,$$

$$A_5^{NCL} = \langle \frac{u_1}{(1,0,0)}, \frac{v_1}{(0,1,1)}, \frac{w_1}{(1,0,0)} \rangle, A_6^{NCL} = \langle \frac{u_1}{(0,1,1)}, \frac{v_1}{(1,0,0)}, \frac{w_1}{(1,0,0)} \rangle.$$

In this case the NCLTs are

$$\tau_1^{NCL} = \{0^{NCL}, 1^{NCL}\}, \tau_2^{NCL} = \{0^{NCL}, A_1^{NCL}, A_6^{NCL}, 1^{NCL}\},$$

$$\tau_3^{NCL} = \{0^{NCL}, A_2^{NCL}, A_5^{NCL}, 1^{NCL}\},$$

$$\tau_4^{NCL} = \{0^{NCL}, A_3^{NCL}, A_4^{NCL}, 1^{NCL}\},$$

$$\tau_5^{NCL} = \{0^{NCL}, A_1^{NCL}, A_2^{NCL}, A_3^{NCL}, A_4^{NCL}, A_5^{NCL}, A_6^{NCL}, 1^{NCL}\}.$$

This gives that the number of open sets in τ_1^{NCL} is 2, in $\tau_2^{NCL}, \tau_3^{NCL}, \tau_4^{NCL}$ is 4 = 2^2 , and in τ_5^{NCL} is 8 = 2^3 . Therefore, for $|\mathcal{X}| = 2$,

- (a) there exist only NCLTs having $2^n, n = 1, 2, 3$ open sets.
- (b) there exists 1 NCLT having 2^1 -open set, 3 NCLT having 2^2 -open sets and 1 NCLT having 2^3 -open sets i.e., $\eta_{2^1} = 1 = S(3, 1), \eta_{2^2} = 3 = S(3, 2)$ and $\eta_{2^3} = 1 = S(3, 3)$ respectively.

Continuing in this way, it is seen that for $|\mathcal{X}| = n$ (finite), and

$$\mathcal{M} = \{(0, 1, 1), (1, 0, 0)\},$$

(i) there exist only NCLTs having $2^n, n = 1, 2, 3, \dots, n$ open sets.

(ii) $\eta_{2^1} = 1 = S(n, 1), \eta_{2^2} = S(n, 2), \eta_{2^3} = S(n, 3), \dots, \eta_{2^n} = S(n, n)$.

Hence, $\mathcal{N}_{\mathcal{X}}^{NCL}$ contains only NCLTs having 2^k -open sets where $k = 1, 2, 3, \dots, n$. and number of NCLTs having 2^k -open sets is $S(n, k), k = 1, 2, \dots, n$. □

Table 1: Number of NCLTSs on \mathcal{X}

$M = \{(0, 1, 1), (1, 0, 0)\}$	$ \mathcal{X} $	Number of NCLTSs having k -open sets					
		$k = 2^1$	$k = 2^2$	$k = 2^3$	$k = 2^4$...	$k = 2^n$
	1	$S(1, 1)$	-	-	-	-	-
	2	$S(2, 1)$	$S(2, 2)$	-	-	-	-
	3	$S(3, 1)$	$S(3, 2)$	$S(3, 3)$	-	-	-
	4	$S(4, 1)$	$S(4, 2)$	$S(4, 3)$	$S(4, 4)$	-	-
	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots
	n	$S(n, 1)$	$S(n, 2)$	$S(n, 3)$	$S(n, 4)$...	$S(n, n)$

Proposition 3.12. Let \mathcal{X} be a finite set with $|\mathcal{X}| = n$ and $M = \{(0, 1, 1), (1, 0, 0), (0.5, 0.5, 0.5)\}$. Then the number of NCLTSs having 4-open sets is obtained by

$$t_n = 0 + 3.2^0 + 3.2^1 + 3.2^2 + 3.2^3 + \dots + 3.2^{n-2} = 6.2^{n-2} - 3, \text{ where } t_n \text{ is the sum of first } n^{\text{th}} \text{ term and } t_1 = 0.$$

Proof. For $n = 1$, we have $t_1 = 0 = 6.2^{1-2} - 3$.

Let $\mathcal{X} = \{a\}$ then $|\mathcal{N}_{\mathcal{X}}| = 3^1 = 3$. So there is no NCLT having 4-open set.

Therefore, for $n = 1$ the result is true.

For $n = 2$, we have $t_2 = 0 + 3.2^0 = 3 = 6.2^{2-2} - 3$.

Let $\mathcal{X} = \{u_1, v_1\}$ then $|\mathcal{N}_{\mathcal{X}}| = 3^2 = 9$. In this case NCLTs having 4-open sets are

$$\begin{aligned} \tau_1^{NCL} &= \left\{ \left\langle \frac{u_1}{(0,1,1)}, \frac{v_1}{(0,1,1)} \right\rangle, \left\langle \frac{u_1}{(1,0,0)}, \frac{v_1}{(1,0,0)} \right\rangle, \left\langle \frac{u_1}{(0,1,1)}, \frac{v_1}{(0.5,0.5,0.5)} \right\rangle, \left\langle \frac{u_1}{(1,0,0)}, \frac{v_1}{(0.5,0.5,0.5)} \right\rangle \right\}, \\ \tau_2^{NCL} &= \left\{ \left\langle \frac{u_1}{(0,1,1)}, \frac{v_1}{(0,1,1)} \right\rangle, \left\langle \frac{u_1}{(1,0,0)}, \frac{v_1}{(1,0,0)} \right\rangle, \left\langle \frac{u_1}{(0.5,0.5,0.5)}, \frac{v_1}{(0,1,1)} \right\rangle, \left\langle \frac{u_1}{(0.5,0.5,0.5)}, \frac{v_1}{(1,0,0)} \right\rangle \right\}, \\ \tau_3^{NCL} &= \left\{ \left\langle \frac{u_1}{(0,1,1)}, \frac{v_1}{(0,1,1)} \right\rangle, \left\langle \frac{u_1}{(1,0,0)}, \frac{v_1}{(1,0,0)} \right\rangle, \left\langle \frac{u_1}{(0,1,1)}, \frac{v_1}{(1,0,0)} \right\rangle, \left\langle \frac{u_1}{(1,0,0)}, \frac{v_1}{(0,1,1)} \right\rangle \right\}. \end{aligned}$$

Let us consider, the result is true for $n = k$ i.e., $t_k = 6.2^{k-2} - 3$.

We now try to prove the result for $n = k + 1$.

$$\begin{aligned} \text{Therefore, } t_n = t_{k+1} &= t_k + 3.2^{(k+1)-2} \\ &= 6.2^{k-2} - 3 + 3.2^{k-1} \\ &= 3.2^{k-2}(2 + 2) - 3 \\ &= 3.4.2^{k-2} - 3 \\ &= 6.2^{k-1} - 3 \\ &= 6.2^{(k+1)-2} - 3. \end{aligned}$$

Thus, for $n = k + 1$ the result is true. Hence, for all the natural number the result is true .

Table 2: Number of NCLTSs having 4-open sets on \mathcal{X}

$ \mathcal{X} $	$M = \{(0, 1, 1), (0.5, 0.5, 0.5), (1, 0, 0)\}$						
	1	2	3	4	5	...	n
Number of NCLT having 4-open sets	0	3	9	21	45	...	$6.2^{n-2} - 3$

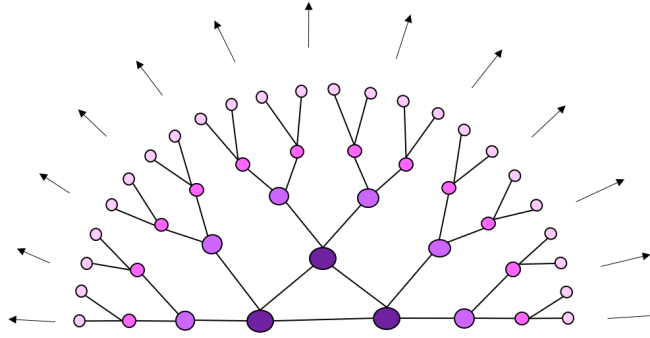


Figure 5: Representation of NCLTSs having 4-open sets on \mathcal{X}

□

Proposition 3.13. Let \mathcal{X} be a finite set, $|\mathcal{X}| = n$ and $M = \{(0, 1, 1), (1, 0, 0), (0.5, 0.5, 0.5)\}$. Then the number of NCLTs having 5-open sets is obtained by

$$t_n = 0 + 2^1 + 2^2 + 2^3 + \dots + 2^{n-1} = 2^n - 2, \text{ where } t_n \text{ is the sum of first } n^{\text{th}} \text{ term and } t_1 = 0.$$

Proof. For $n = 1$, we have $t_1 = 0 = 2^1 - 2$.

Let $\mathcal{X} = \{a\}$ then $|\mathcal{N}_{\mathcal{X}}| = 3^1 = 3$. So there is no NCLT having 4-open set.

Therefore, for $n = 1$ the result is true.

For $n = 2$, we have $t_2 = 2^1 = 2 = 2^2 - 2$.

Let $\mathcal{X} = \{u_1, v_1\}$ then $|\mathcal{N}_{\mathcal{X}}| = 3^2 = 9$. In this case NCLTs having 5-open sets are

$$\tau_1^{NCL} = \left\{ \left\langle \frac{u_1}{(0,1,1)}, \frac{v_1}{(0,1,1)} \right\rangle, \left\langle \frac{u_1}{(1,0,0)}, \frac{v_1}{(1,0,0)} \right\rangle, \left\langle \frac{u_1}{(0,1,1)}, \frac{v_1}{(0.5,0.5,0.5)} \right\rangle, \right. \\ \left. \left\langle \frac{u_1}{(1,0,0)}, \frac{v_1}{(0.5,0.5,0.5)} \right\rangle, \left\langle \frac{u_1}{(0.5,0.5,0.5)}, \frac{v_1}{(0.5,0.5,0.5)} \right\rangle \right\},$$

$$\tau_2^{NCL} = \left\{ \left\langle \frac{u_1}{(0,1,1)}, \frac{v_1}{(0,1,1)} \right\rangle, \left\langle \frac{u_1}{(1,0,0)}, \frac{v_1}{(1,0,0)} \right\rangle, \left\langle \frac{u_1}{(0.5,0.5,0.5)}, \frac{v_1}{(0,1,1)} \right\rangle, \right. \\ \left. \left\langle \frac{u_1}{(0.5,0.5,0.5)}, \frac{v_1}{(1,0,0)} \right\rangle, \left\langle \frac{u_1}{(0.5,0.5,0.5)}, \frac{v_1}{(0.5,0.5,0.5)} \right\rangle \right\}.$$

Let us consider, the result is true for $n = k$ i.e., $t_k = 2^k - 2$.

We now try to prove the result for $n = k + 1$.

Therefore, $t_n = t_{k+1} = t_k + 2^{(k+1)-1} = 2^k - 2 + 2^k = 2 \cdot 2^k - 2 = 2^{k+1} - 2$.

Hence, for $n = k + 1$ the result is true. So, for all the natural number the result is true.

□

Table 3: NCLTSs having 5-open sets on \mathcal{X}

$ \mathcal{X} $	$M = \{(0, 1, 1), (1, 0, 0), (0.5, 0.5, 0.5)\}$						
	1	2	3	4	5	...	n
Number of NCLT having 5-open sets	0	2	6	14	30	...	$2^n - 2$

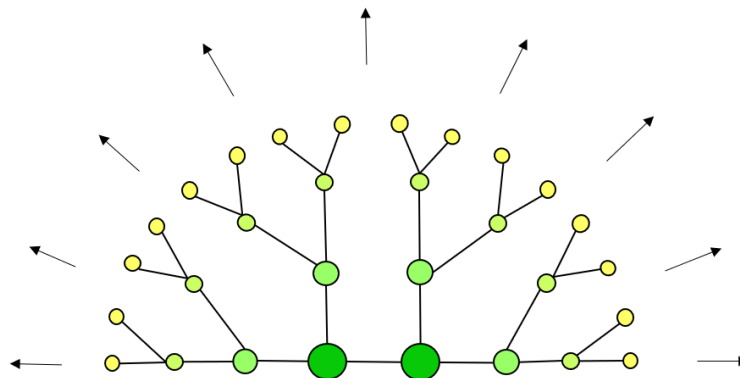


Figure 6: Representation of NCLTSs having 5-open sets on \mathcal{X}

Proposition 3.14. Let \mathcal{X} be a finite set, $|\mathcal{X}| = n$ and $M = \{(0, 1, 1), (T, I, F), (1, 0, 0)\}, T = F; I = 0.5$. Then the number of NCLTs having

(i) 4-open sets is obtained by

$$t_n = 0 + 3.2^0 + 3.2^1 + 3.2^2 + 3.2^3 + \dots + 3.2^{n-2} = 6.2^{n-2} - 3 \text{ and}$$

(ii) 5-open sets is obtained by

$$t_n = 0 + 2^1 + 2^2 + 2^3 + \dots + 2^{n-1} = 2^n - 2.$$

Proof. Prove is straightforward. □

Proposition 3.15. For $|\mathcal{X}| = n$ (finite) and $M = \{(0, 1, 1), (1, 0, 0), (T, I, F), (F, 1-I, T)\}$. Then the number of NCLTSs having 4-open sets is obtained by

$$t_n = 1 + 3.2^0 + 3.2^1 + 3.2^2 + \dots + 3.2^{n-2} = 3.2^{n-1} - 2,$$

where t_n is the sum of first n^{th} term and $t_1 = 1$.

Proof. For $n = 1$, we have $t_1 = 1 = 3.2^{1-1} - 2$.

Let $\mathcal{X} = \{u_1\}$ then $|\mathcal{N}_{\mathcal{X}}| = 4^1 = 4$. So there is one NCLT having 4-open set which is

$$\left\{ \left\langle \frac{u_1}{(0,1,1)}, \frac{u_1}{(1,0,0)} \right\rangle, \left\langle \frac{u_1}{(T,I,F)}, \frac{u_1}{(F,1-I,T)} \right\rangle \right\}.$$

Therefore, for $n = 1$ the result is true .

For $n = 2$, we have $t_2 = 1 + 3.2^0 = 4 = 3.2^1 - 2$.

Let $\mathcal{X} = \{u_1, v_1\}$ and $M = \{(0, 1, 1), (1, 0, 0), (T, I, F), (F, 1 - I, T)\}$.

Then $|\mathcal{N}_{\mathcal{X}}| = 4^2 = 16$. In this case NCLTs having 4-open sets are

$$\begin{aligned} \tau_1^{NCL} &= \left\{ \left\langle \frac{u_1}{(0,1,1)}, \frac{v_1}{(0,1,1)} \right\rangle, \left\langle \frac{u_1}{(1,0,0)}, \frac{v_1}{(1,0,0)} \right\rangle, \left\langle \frac{u_1}{(0,1,1)}, \frac{v_1}{(T,I,F)} \right\rangle, \left\langle \frac{u_1}{(1,0,0)}, \frac{v_1}{(F,1-I,T)} \right\rangle \right\}, \\ \tau_2^{NCL} &= \left\{ \left\langle \frac{u_1}{(0,1,1)}, \frac{v_1}{(0,1,1)} \right\rangle, \left\langle \frac{u_1}{(1,0,0)}, \frac{v_1}{(0,1,1)} \right\rangle, \left\langle \frac{u_1}{(T,I,F)}, \frac{v_1}{(T,I,F)} \right\rangle, \left\langle \frac{u_1}{(F,1-I,T)}, \frac{v_1}{(F,1-I,T)} \right\rangle \right\}, \\ \tau_3^{NCL} &= \left\{ \left\langle \frac{u_1}{(0,1,1)}, \frac{v_1}{(0,1,1)} \right\rangle, \left\langle \frac{u_1}{(1,0,0)}, \frac{v_1}{(1,0,0)} \right\rangle, \left\langle \frac{u_1}{(0,1,1)}, \frac{v_1}{(1,0,0)} \right\rangle, \left\langle \frac{u_1}{(1,0,0)}, \frac{v_1}{(0,1,1)} \right\rangle \right\}, \\ \tau_4^{NCL} &= \left\{ \left\langle \frac{u_1}{(0,1,1)}, \frac{v_1}{(0,1,1)} \right\rangle, \left\langle \frac{u_1}{(1,0,0)}, \frac{v_1}{(1,0,0)} \right\rangle, \left\langle \frac{u_1}{(T,I,F)}, \frac{v_1}{(0,1,1)} \right\rangle, \left\langle \frac{u_1}{(F,1-I,T)}, \frac{v_1}{(1,0,0)} \right\rangle \right\}. \end{aligned}$$

Let us consider, for $n = k$ the result is true i.e., $t_k = 3.2^{k-1} - 2$.

We now try to prove the result for $n = k + 1$.

Therefore,

$$t_n = t_{k+1} = t_k + 3.2^{(k+1)-2} = 3.2^{k-1} - 2 + 3.2^{k-1} = 2.3.2^{k-1} - 2 = 3.2^k - 2 = 3.2^{(k+1)-1} - 2$$

Hence, for $n = k + 1$ the result is true. So, for all the natural number the result is true. □

Table 4: Number of NCLTSs having 4-open sets on \mathcal{X}

$ \mathcal{X} $	$M = \{(0, 1, 1), (1, 0, 0), (T, I, F), (F, 1 - I, T)\}, I = 0.5$						n
	1	2	3	4	5	...	
Number of NCLT having 4-open sets	1	4	10	22	46	...	$3.2^{n-1} - 2$

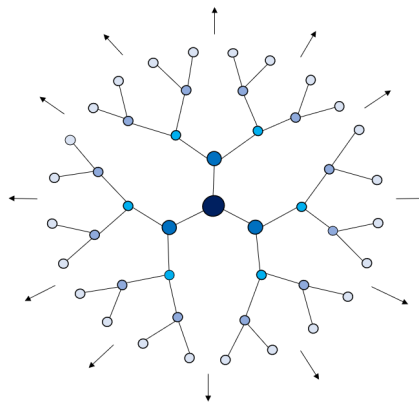


Figure 7: Representation of NCLTSs having 4-open sets on \mathcal{X}

Remark 3.16. Results obtained for $k = 4$ and $k = 5$ are true for all M provided $I = 0.5, T = F$, and $T < F$, $I < 0.5$. Moreover, in this case, the results obtained for $I = 0.5, T = F$ coincides with results obtained for $M = \{(0, 1, 1), (1, 0, 0), (0.5, 0.5, 0.5)\}$ and $M = \{(0, 1, 1), (1, 0, 0), (T, 0.5, F)\}, T = F$ and the results obtained for $T < F$ and $I < 0.5$ coincides with results obtained for $M = \{(0, 1, 1), (1, 0, 0)\}$.

Proposition 3.17. For $n \geq m \geq 2$, the number of NCLTs having k -open set where $m^n - m^{n-2} < k < m^n$ is 0.

4 Conclusion

In this paper, a number of formulae for finding the number of the NCLTSs are determined. Moreover, relevant propositions are observed, where we can draw logical pictures.

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Number of Neutrosophic Topological Spaces on Finite Set with $\mathfrak{k} \leq 4$ Open Sets

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Abstract. In this paper, the number of neutrosophic topological spaces having two, three, and four open sets are computed for a finite set \mathbb{X}^{NT} whose membership values lies in \mathbb{M}^{NT} . Further, the number of neutrosophic bitopological spaces and neutrosophic tritopological spaces having $\mathfrak{k}(\mathfrak{k} = 2, 3, 4)$ neutrosophic open sets on finite sets are computed.

Keywords: : Neutrosophic Set; Neutrosophic Topology; Two Open Set; Three Open Set; Four Open Set.

1. Introduction

Finding the number of topologies in a set is an interesting task. Many authors have done their work in this field. Krishnamurty [1] obtained a sharper bound namely $2^{n(n-1)}$ for the number of distinct topologies. Sharp [2] shows that only discrete topology has cardinal greater than $\frac{3}{4}2^n$ and derived bounds for the cardinality of topologies which are connected, non-connected, non- \mathcal{T}_0 , and some more. After obtaining all non-homeomorphic topologies with n points and $> \frac{7}{16}2^n$ open sets, Stanley [3] also determined which of these are \mathcal{T}_0 . The concept of partial chain topologies supported Kamel [4] to formulate a special case for computing the number of chain topologies and maximal elements with natural generalization. Ragnarsson *et al.* [5], have also studied obtainable sizes of topologies on a finite set. Benoumhani [6] computed the number of topologies having 2, 3, \dots , 12-open sets, and also \mathcal{T}_0 topologies having $n+4$, $n+5$, and $n+6$ open sets. These results are extended in [7].

Later on, Benoumhani *et al.* [8] extended their work to fuzzy topological spaces (FTS). They computed the number of FTS having 2, 3, 4, and 5-open sets and certain cases, where the number of open sets is large. Basumatary *et al.* [9] discussed the number of fuzzy bitopological spaces and gave some formulae.

After the generalization of the fuzzy set [10] from crisp set and intuitionistic fuzzy set [11], Smarandache discovered the concept of the neutrosophic set by combining the fuzzy set and intuitionistic fuzzy set. Since the introduction of the NS (Neutrosophic set) by Smarandache [12], several authors have contributed their work in science and technology by taking NS as a tool. Wang [13] studied single-valued NSs in multiset and multistructure. Salama *et al.* [14] studied the neutrosophic topological spaces (NTS). Lupiáñez [15–18] investigated NTS. Mwchahary *et al.* [19] studied neutrosophic bitopological space (NBTS). Devi *et al.* [20] and Ozturk *et al.* [21] also discussed NBTS. Kelly [22] and Kovar [23] introduced the notion of bitopological space and tritopological space respectively. The neutrosophic crisp tri-topological spaces are studied by Al-Hamido *et al.* [24].

Ishtiaq *et al.* [25, 26] studied fixed-point results in orthogonal neutrosophic metric spaces and also certain new aspects in fuzzy fixed-point theory. Ali *et al.* [27] discussed solving nonlinear fractional differential equations for contractive and weakly compatible mappings in neutrosophic metric spaces. Hussain *et al.* [28] worked on some new aspects of the intuitionistic fuzzy and neutrosophic fixed point theory. Javed *et al.* [29] studied the fuzzy b-metric-like spaces. Hussain *et al.* [30] studied the pentagonal controlled fuzzy metric spaces with an application to dynamic market equilibrium.

From the literature survey, it is observed that generally finding the number of topologies (NoTs) for a set is not an easy task. Because of this current authors started research work in this area. This article discusses formulae for calculating the NNTSs (number of NTSs) with 2, 3, or 4-open sets, as well as the NNBTSSs (number of NBTSs) and NNTRSs (number of neutrosophic tritopological spaces) with the same number of open sets in topologies.

Let \mathbb{X}^{NT} be a non-empty finite set, \mathbb{M}^{NT} be the finite totally ordered set with $|\mathbb{M}^{NT}| = m \geq 2$ and $\mathcal{N}_{\mathbb{X}}^{\mathcal{F}}$ be a set that contains all the neutrosophic subsets (NSubs) of \mathbb{X}^{NT} with membership values in \mathbb{M}^{NT} .

Note that in this paper $\mathcal{T}_{\mathbb{X}}^{NT}(n, m, \mathfrak{k})$ denotes NNTSs on \mathbb{X}^{NT} with $|\mathbb{X}^{NT}| = n$ and \mathfrak{k} -open sets, $(\mathcal{T}_i^{NT}, \mathcal{T}_j^{NT})_{\mathbb{X}}^{NT}(n, m, \mathfrak{k})$ and $(\mathcal{T}_i^{NT}, \mathcal{T}_j^{NT}, \mathcal{T}_{\mathfrak{k}}^{NT})_{\mathbb{X}}^{NT}(n, m, \mathfrak{k})$ denotes NNBTSSs and NNTRSs respectively on \mathbb{X}^{NT} consisting \mathfrak{k} -open sets in topologies at a time where $n, m, \mathfrak{k} \in \mathbb{N}$, $n \geq 1, m \geq 2$ and $\mathfrak{k} \geq 2$.

2. Preliminaries

Definition 2.1. [14] On a universe of discourse \mathbb{X}^{NT} a NS \mathfrak{U}^{NT} is defined as $\mathfrak{U}^{NT} = \langle \frac{u}{(T_{\mathfrak{U}}^{NT}(u), I_{\mathfrak{U}}^{NT}(u), F_{\mathfrak{U}}^{NT}(u))} : u \in \mathbb{X}^{NT} \rangle$, where $T_{\mathfrak{U}}^{NT}, I_{\mathfrak{U}}^{NT}, F_{\mathfrak{U}}^{NT} : \mathbb{X}^{NT} \rightarrow]-0, 1^+[$. Here $-0 \leq T_{\mathfrak{U}}^{NT}(u) + I_{\mathfrak{U}}^{NT}(u) + F_{\mathfrak{U}}^{NT}(u) \leq 3^+$; $T_{\mathfrak{U}}^{NT}(u)$ represents degree of membership function, $I_{\mathfrak{U}}^{NT}(u)$ degree of indeterminacy and $F_{\mathfrak{U}}^{NT}(u)$ degree of non-membership function.

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Definition 2.2. [14,15] Let $\mathcal{F}^{NT} \subseteq \mathcal{N}_{\mathbb{X}}^{NT}$ then \mathcal{F}^{NT} is called a neutrosophic topology (NT) on \mathbb{X}^{NT} if

- $0^{NT}, 1^{NT} \in \mathcal{F}^{NT}$
- $\mathfrak{U}_1^{NT} \cap \mathfrak{U}_2^{NT} \in \mathcal{F}^{NT}$ for any $\mathfrak{U}_1^{NT}, \mathfrak{U}_2^{NT} \in \mathcal{F}^{NT}$.
- $\cup \mathfrak{U}_i^{NT} \in \mathcal{F}^{NT}$, for arbitrary family $\{\mathfrak{U}_i^{NT} : i \in \mathbb{I}\} \in \mathcal{F}^{NT}$.

The pair $(\mathbb{X}^{NT}, \mathcal{F}^{NT})$ is called NTS and any NS in \mathcal{F}^{NT} is called NOS (neutrosophic open set) in \mathbb{X}^{NT} .

Definition 2.3. [19] Let \mathcal{F}_1^{NT} and \mathcal{F}_2^{NT} be the two NTs on \mathbb{X}^{NT} . Then $(\mathbb{X}^{NT}, \mathcal{F}_1^{NT}, \mathcal{F}_2^{NT})$ is called a NBTS.

Example 2.4. If $\mathbb{X}^{NT} = \{u, v, w\}$ and if $\mathcal{F}_1^{NT} = \{0^{NT}, 1^{NT}, \mathfrak{U}_1^{NT}\}$ and $\mathcal{F}_2^{NT} = \{0^{NT}, 1^{NT}, \mathfrak{U}_2^{NT}\}$, where

$$\mathfrak{U}_1^{NT} = \langle \overline{(0.7,0.1,0.5)}, \overline{(0.5,0.2,0.3)}, \overline{(0.3,0.4,0.4)} \rangle, \mathfrak{U}_2^{NT} = \langle \overline{(0.2,0.5,0.1)}, \overline{(0.1,0.2,0.3)}, \overline{(0.6,0.3,0.5)} \rangle.$$

Then $(\mathbb{X}^{NT}, \mathcal{F}_1^{NT})$ and $(\mathbb{X}^{NT}, \mathcal{F}_2^{NT})$ form NTS. Therefore, $(\mathbb{X}^{NT}, \mathcal{F}_1^{NT}, \mathcal{F}_2^{NT})$ is a NBTS.

Definition 2.5. [31] Let $\mathcal{F}_1^{NT}, \mathcal{F}_2^{NT}$ and \mathcal{F}_3^{NT} be the three NTs on \mathbb{X}^{NT} . Then $(\mathbb{X}^{NT}, \mathcal{F}_1^{NT}, \mathcal{F}_2^{NT}, \mathcal{F}_3^{NT})$ is called a neutrosophic tritopological space (NTRS).

Example 2.6. If $\mathbb{X}^{NT} = \{u, v, w\}$ and consider $\mathcal{F}_1^{NT} = \{0^{NT}, 1^{NT}, \mathfrak{U}_1^{NT}\}$, $\mathcal{F}_2^{NT} = \{0^{NT}, 1^{NT}, \mathfrak{U}_2^{NT}\}$ and $\mathcal{F}_3^{NT} = \{0^{NT}, 1^{NT}, \mathfrak{U}_3^{NT}\}$.

Here, $\mathfrak{U}_1^{NT} = \langle \overline{(0.7,0.1,0.5)}, \overline{(0.5,0.2,0.3)}, \overline{(0.3,0.6,0.2)} \rangle$, $\mathfrak{U}_2^{NT} = \langle \overline{(0.6,0.5,0.3)}, \overline{(0.7,0.0,2)}, \overline{(0.8,0.1,0.1)} \rangle$,
 $\mathfrak{U}_3^{NT} = \langle \overline{(0.5,0.2,0.3)}, \overline{(0.2,0.1,0.2)}, \overline{(0.1,0,0.1)} \rangle$.

Then $(\mathbb{X}^{NT}, \mathcal{F}_1^{NT})$, $(\mathbb{X}^{NT}, \mathcal{F}_2^{NT})$ and $(\mathbb{X}^{NT}, \mathcal{F}_3^{NT})$ form NTS.

Therefore $(\mathbb{X}^{NT}, \mathcal{F}_1^{NT}, \mathcal{F}_2^{NT}, \mathcal{F}_3^{NT})$ is a NTRS. In this case, $(\mathbb{X}^{NT}, \mathcal{F}_1^{NT}, \mathcal{F}_2^{NT}, \mathcal{F}_3^{NT})$ is a NTRS having 3-NOS in each of the topologies.

3. Results on NNTS

Proposition 3.1. *The NNTs (Number of Neutrosophic Topologies) on \mathbb{X}^{NT} , whose membership values lies in \mathbb{M}^{NT} , is finite if and only if both \mathbb{X}^{NT} and \mathbb{M}^{NT} are finite.*

Result 3.2. *The NNTSs having 2-NOS is one i.e., $\mathcal{F}_{\mathbb{X}}^{NT}(\mathbf{n}, \mathbf{m}, 2) = 1$.*

The NT having 2-open set is the indiscrete NT which is $\mathcal{F}_1^{NT} = \{0^{NT}, 1^{NT}\}$.

Result 3.3. *The NNTs having 3-NOS is $\mathbf{m}^{\mathbf{n}} - 2$ i.e., $\mathcal{F}_{\mathbb{X}}^{NT}(\mathbf{n}, \mathbf{m}, 3) = \mathbf{m}^{\mathbf{n}} - 2$.*

These NTs necessarily consists of a chain containing $0^{NT}, 1^{NT}$ and any one NSub of \mathbb{X}^{NT} . In this case NTs are in the chain, of the form $0^{NT} \subseteq \mathfrak{U}_1^{NT} \subseteq 1^{NT}$, \mathfrak{U}_1^{NT} is any NSub of \mathbb{X}^{NT} .

Example 3.4. Let $\mathbb{X}^{NT} = \{u, v\}$ and $\mathbb{M}^{NT} = \{(0, 1, 1), (0.6, 0.1, 0.2), (1, 0, 0)\}$. It is seen that, $|\mathbb{X}^{NT}| = n = 2$, $|\mathbb{M}^{NT}| = m = 3$.

Then number of elements in $N_{\mathbb{X}}^{\mathcal{F}}$ i.e., $|N_{\mathbb{X}}^{\mathcal{F}}| = 3^2 = 9$. These are

$$\begin{aligned} 0^{NT}, 1^{NT}, \mathfrak{U}_1^{NT} &= \langle \frac{u}{(0,1,1)}, \frac{v}{(0.6,0.1,0.2)} \rangle, & \mathfrak{U}_2^{NT} &= \langle \frac{u}{(0,1,1)}, \frac{v}{(1,0,0)} \rangle, & \mathfrak{U}_3^{NT} &= \langle \frac{u}{(0.6,0.1,0.2)}, \frac{v}{(0,1,1)} \rangle, \\ \mathfrak{U}_4^{NT} &= \langle \frac{u}{(0.6,0.1,0.2)}, \frac{v}{(0.6,0.1,0.2)} \rangle, & \mathfrak{U}_5^{NT} &= \langle \frac{u}{(0.6,0.1,0.2)}, \frac{v}{(1,0,0)} \rangle, & \mathfrak{U}_6^{NT} &= \langle \frac{u}{(1,0,0)}, \frac{v}{(0,1,1)} \rangle, \\ \mathfrak{U}_7^{NT} &= \langle \frac{u}{((1,0,0)}, \frac{v}{(0.6,0.1,0.2)} \rangle. \end{aligned}$$

So, $\mathcal{F}_{\mathbb{X}}^{NT}(2, 3, 3) = 3^2 - 2 = 7$.

The NTs having 3-open sets are:

$$\begin{aligned} \mathcal{F}_1^{NT} &= \{0^{NT}, 1^{NT}, \mathfrak{U}_1^{NT}\}, & \mathcal{F}_2^{NT} &= \{0^{NT}, 1^{NT}, \mathfrak{U}_2^{NT}\}, & \mathcal{F}_3^{NT} &= \{0^{NT}, 1^{NT}, \mathfrak{U}_3^{NT}\}, \\ \mathcal{F}_4^{NT} &= \{0^{NT}, 1^{NT}, \mathfrak{U}_4^{NT}\}, & \mathcal{F}_5^{NT} &= \{0^{NT}, 1^{NT}, \mathfrak{U}_5^{NT}\}, & \mathcal{F}_6^{NT} &= \{0^{NT}, 1^{NT}, \mathfrak{U}_6^{NT}\}, \\ \mathcal{F}_7^{NT} &= \{0^{NT}, 1^{NT}, \mathfrak{U}_7^{NT}\}. \end{aligned}$$

Result 3.5. An arbitrary NT with 4-NOSs is an NT consisting of $1^{NT}, 0^{NT}$ and other two NSubs. These NSubs are either chain of 2-elements or anti-chain of 2-elements having 1^{NT} and 0^{NT} as union and intersection respectively.

Theorem 3.6. In $\hat{\mathcal{N}}_{\mathbb{X}}^{\mathcal{F}} = \mathcal{N}_{\mathbb{X}}^{\mathcal{F}} - \{0^{NT}, 1^{NT}\}$, the number of chains (NCs) of length 2 is obtained by

$$c_2(\mathcal{N}_{\mathbb{X}}^{\mathcal{F}}) = \binom{m+1}{2}^n - 3m^n + 3.$$

Corollary 3.7. In $\mathcal{N}_{\mathbb{X}}^{\mathcal{F}}$, the NCs of length 4 having both 0^{NT} and 1^{NT} is same as $c_2(\mathcal{N}_{\mathbb{X}}^{\mathcal{F}})$.

Lemma 3.8. In $\mathcal{N}_{\mathbb{X}}^{\mathcal{F}}$, the number of anti-chains (NACs) of size 2 (having 2-elements) with 1^{NT} as union and 0^{NT} as intersection is $2^{n-1} - 1$.

Corollary 3.9. The NAC NTs of $\mathcal{N}_{\mathbb{X}}^{\mathcal{F}}$ consisting of 4-open set is $2^{n-1} - 1$.

Theorem 3.10. The NNTs in $\mathcal{N}_{\mathbb{X}}^{\mathcal{F}}$ with 4-NOSs is

$$\mathcal{F}_{\mathbb{X}}^{NT}(n, m, 4) = \left(\frac{m(m+1)}{2}\right)^n - 3m^n + 2^{n-1} + 2.$$

Follow Cor. 3.7 and Cor. 3.9 for the prove of theorem.

Example 3.11. Let, $\mathbb{X}^{NT} = \{u, v\}$ and $\mathbb{M}^{NT} = \{(0, 1, 1), (0.1, 0.3, 0.8), (1, 0, 0)\}$. Therefore $|\mathcal{N}_{\mathbb{X}}^{\mathcal{F}}| = 3^2 = 9$. These NSubs are

$$\begin{aligned} 0^{NT} &= \langle \frac{u}{(0,1,1)}, \frac{v}{(0,1,1)} \rangle, & 1^{NT} &= \langle \frac{u}{(1,0,0)}, \frac{v}{(1,0,0)} \rangle, & \mathfrak{U}_1^{NT} &= \langle \frac{u}{(0,1,1)}, \frac{v}{(0.1,0.3,0.8)} \rangle, \\ \mathfrak{U}_2^{NT} &= \langle \frac{u}{(0,1,1)}, \frac{v}{(1,0,0)} \rangle, & \mathfrak{U}_3^{NT} &= \langle \frac{u}{(0.1,0.3,0.8)}, \frac{v}{(0,1,1)} \rangle, & \mathfrak{U}_4^{NT} &= \langle \frac{u}{(0.1,0.3,0.8)}, \frac{v}{(0.1,0.3,0.8)} \rangle, \\ \mathfrak{U}_5^{NT} &= \langle \frac{u}{(0.1,0.3,0.8)}, \frac{v}{(1,0,0)} \rangle, & \mathfrak{U}_6^{NT} &= \langle \frac{u}{(1,0,0)}, \frac{v}{(0,1,1)} \rangle, & \mathfrak{U}_7^{NT} &= \langle \frac{u}{(1,0,0)}, \frac{v}{(0.1,0.3,0.8)} \rangle. \end{aligned}$$

In this case, $n = 2$, $m = 3$,

Therefore, $\mathcal{F}_{\mathbb{X}}^{NT}(2, 3, 4) = \left(\frac{3(3+1)}{2}\right)^2 - 3.3^2 + 2^{2-1} + 2 = 6^2 - 23 = 13$.

These NTs with 4-NOSs are

$$\mathcal{F}_1^{NT} = \{0^{NT}, 1^{NT}, \mathfrak{U}_1^{NT}, \mathfrak{U}_2^{NT}\}, \quad \mathcal{F}_2^{NT} = \{0^{NT}, 1^{NT}, \mathfrak{U}_1^{NT}, \mathfrak{U}_4^{NT}\},$$

$$\begin{aligned}
 \mathcal{T}_3^{NT} &= \{0^{NT}, 1^{NT}, \mathfrak{U}_1^{NT}, \mathfrak{U}_5^{NT}\}, & \mathcal{T}_4^{NT} &= \{0^{NT}, 1^{NT}, \mathfrak{U}_1^{NT}, \mathfrak{U}_7^{NT}\}, \\
 \mathcal{T}_5^{NT} &= \{0^{NT}, 1^{NT}, \mathfrak{U}_2^{NT}, \mathfrak{U}_5^{NT}\}, & \mathcal{T}_6^{NT} &= \{0^{NT}, 1^{NT}, \mathfrak{U}_2^{NT}, \mathfrak{U}_6^{NT}\}, \\
 \mathcal{T}_7^{NT} &= \{0^{NT}, 1^{NT}, \mathfrak{U}_3^{NT}, \mathfrak{U}_4^{NT}\}, & \mathcal{T}_8^{NT} &= \{0^{NT}, 1^{NT}, \mathfrak{U}_3^{NT}, \mathfrak{U}_5^{NT}\}, \\
 \mathcal{T}_9^{NT} &= \{0^{NT}, 1^{NT}, \mathfrak{U}_3^{NT}, \mathfrak{U}_6^{NT}\}, & \mathcal{T}_{10}^{NT} &= \{0^{NT}, 1^{NT}, \mathfrak{U}_3^{NT}, \mathfrak{U}_7^{NT}\}, \\
 \mathcal{T}_{11}^{NT} &= \{0^{NT}, 1^{NT}, \mathfrak{U}_4^{NT}, \mathfrak{U}_5^{NT}\}, & \mathcal{T}_{12}^{NT} &= \{0^{NT}, 1^{NT}, \mathfrak{U}_4^{NT}, \mathfrak{U}_7^{NT}\}, \\
 \mathcal{T}_{13}^{NT} &= \{0^{NT}, 1^{NT}, \mathfrak{U}_6^{NT}, \mathfrak{U}_7^{NT}\}.
 \end{aligned}$$

Here, the only anti-chain NTs in $\mathcal{N}_{\mathbb{X}}^{\mathcal{F}}$ is \mathcal{T}_6^{NT} with 0^{NT} and 1^{NT} as intersection and union respectively.

4. Results on NNBTs

In this section, the NBTS having 3-NOSs in both NTs and the NBTS having 3-NOSs in both NTs without repetition means NBTS of the form $(\mathbb{X}^{NT}, \mathcal{T}_i^{NT}, \mathcal{T}_j^{NT})$, where $\mathcal{T}_i^{NT}, \mathcal{T}_j^{NT}$ are identical or non-identical topologies, and non-identical topologies having 3-NOSs respectively. A similar meaning is used for 4-NOSs.

Result 4.1. In $\mathcal{N}_{\mathbb{X}}^{\mathcal{F}}$, the NNBTs with two NOSs in both the NTs is

$$(\mathcal{T}_i^{NT}, \mathcal{T}_j^{NT})_{\mathbb{X}}^{NT}(\mathbf{n}, \mathbf{m}, 2) = 1.$$

From Result 3.2, $\mathcal{F}_{\mathbb{X}}^{\mathcal{F}}(\mathbf{n}, \mathbf{m}, 2) = 1$, which is the indiscrete topology $\mathcal{T}_1^{NT} = \{0^{NT}, 1^{NT}\}$. Hence, NBTS with 2-NOSs is only one i.e., $(\mathbb{X}^{NT}, \mathcal{T}_1^{NT}, \mathcal{T}_1^{NT})$.

Result 4.2. In $\mathcal{N}_{\mathbb{X}}^{\mathcal{F}}$, the NNBTs having 3-NOSs in both NTs is

$$(\mathcal{T}_i^{NT}, \mathcal{T}_j^{NT})_{\mathbb{X}}^{NT}(\mathbf{n}, \mathbf{m}, 3) = \binom{\mathcal{F}_{\mathbb{X}}^{NT}(\mathbf{n}, \mathbf{m}, 3) + 1}{2} = \frac{\mathbf{m}^{2\mathbf{n}} - 3\mathbf{m}^{\mathbf{n} + 2}}{2}.$$

Example 4.3. Example 3.4 gives $\mathcal{F}_{\mathbb{X}}^{NT}(2, 3, 3) = 7$.

Therefore, $(\mathcal{T}_i^{NT}, \mathcal{T}_j^{NT})_{\mathbb{X}}^{NT}(2, 3, 3) = \binom{\mathcal{F}_{\mathbb{X}}^{NT}(2, 3, 3) + 1}{2} = 28$.

Then, these NBTSs are

- $(\mathbb{X}^{NT}, \mathcal{T}_1^{NT}, \mathcal{T}_1^{NT}), (\mathbb{X}^{NT}, \mathcal{T}_1^{NT}, \mathcal{T}_2^{NT}), (\mathbb{X}^{NT}, \mathcal{T}_1^{NT}, \mathcal{T}_3^{NT}), (\mathbb{X}^{NT}, \mathcal{T}_1^{NT}, \mathcal{T}_4^{NT}),$
- $(\mathbb{X}^{NT}, \mathcal{T}_1^{NT}, \mathcal{T}_5^{NT}), (\mathbb{X}^{NT}, \mathcal{T}_1^{NT}, \mathcal{T}_6^{NT}), (\mathbb{X}^{NT}, \mathcal{T}_1^{NT}, \mathcal{T}_7^{NT}),$
- $(\mathbb{X}^{NT}, \mathcal{T}_2^{NT}, \mathcal{T}_2^{NT}), (\mathbb{X}^{NT}, \mathcal{T}_2^{NT}, \mathcal{T}_3^{NT}), (\mathbb{X}^{NT}, \mathcal{T}_2^{NT}, \mathcal{T}_4^{NT}), (\mathbb{X}^{NT}, \mathcal{T}_2^{NT}, \mathcal{T}_5^{NT}),$
- $(\mathbb{X}^{NT}, \mathcal{T}_2^{NT}, \mathcal{T}_6^{NT}), (\mathbb{X}^{NT}, \mathcal{T}_2^{NT}, \mathcal{T}_7^{NT}),$
- $(\mathbb{X}^{NT}, \mathcal{T}_3^{NT}, \mathcal{T}_3^{NT}), (\mathbb{X}^{NT}, \mathcal{T}_3^{NT}, \mathcal{T}_4^{NT}), (\mathbb{X}^{NT}, \mathcal{T}_3^{NT}, \mathcal{T}_5^{NT}), (\mathbb{X}^{NT}, \mathcal{T}_3^{NT}, \mathcal{T}_6^{NT}),$
- $(\mathbb{X}^{NT}, \mathcal{T}_3^{NT}, \mathcal{T}_7^{NT}),$
- $(\mathbb{X}^{NT}, \mathcal{T}_4^{NT}, \mathcal{T}_4^{NT}), (\mathbb{X}^{NT}, \mathcal{T}_4^{NT}, \mathcal{T}_5^{NT}), (\mathbb{X}^{NT}, \mathcal{T}_4^{NT}, \mathcal{T}_6^{NT}), (\mathbb{X}^{NT}, \mathcal{T}_4^{NT}, \mathcal{T}_7^{NT}),$
- $(\mathbb{X}^{NT}, \mathcal{T}_5^{NT}, \mathcal{T}_5^{NT}), (\mathbb{X}^{NT}, \mathcal{T}_5^{NT}, \mathcal{T}_6^{NT}), (\mathbb{X}^{NT}, \mathcal{T}_5^{NT}, \mathcal{T}_7^{NT}),$
- $(\mathbb{X}^{NT}, \mathcal{T}_6^{NT}, \mathcal{T}_6^{NT}), (\mathbb{X}^{NT}, \mathcal{T}_6^{NT}, \mathcal{T}_7^{NT}),$
- $(\mathbb{X}^{NT}, \mathcal{T}_7^{NT}, \mathcal{T}_7^{NT}).$

Result 4.4. In $\mathcal{N}_{\mathbb{X}}^{\mathcal{F}}$, the NNBTs having 3-NOSs in both NTs without repetition is

$$(\mathcal{F}_i^{NT}, \mathcal{F}_j^{NT})_{\mathbb{X}}^{NT}(\mathbf{n}, \mathbf{m}, 3) = \binom{\mathcal{F}_{\mathbb{X}}^{NT}(\mathbf{n}, \mathbf{m}, 3)}{2}.$$

Example 4.5. Following Example 3.4 and Result 4.4., the number of NBTs without repetition is

$$21 = \binom{\mathcal{F}_{\mathbb{X}}^{NT}(2,3,3)}{2} = \binom{7}{2}.$$

Result 4.6. The NNBTs in $\mathcal{N}_{\mathbb{X}}^{\mathcal{F}}$, consisting 4-NOSs in both the NT is

$$(\mathcal{F}_i^{NT}, \mathcal{F}_j^{NT})_{\mathbb{X}}^{NT}(\mathbf{n}, \mathbf{m}, 4) = \binom{\mathcal{F}_{\mathbb{X}}^{NT}(\mathbf{n}, \mathbf{m}, 4)+1}{2}.$$

Example 4.7. Let $\mathbb{X}^{NT} = \{u, v\}$ and $\mathbb{M}^{NT} = \{(0, 1, 1), (0.1, 0.3, 0.8), (1, 0, 0)\}$.

Then, $\mathcal{F}_{\mathbb{X}}^{NT}(2, 3, 4) = 13$.

and the NNBTs is

$$(\mathcal{F}_i^{NT}, \mathcal{F}_j^{NT})_{\mathbb{X}}^{NT}(2, 3, 4) = \binom{\mathcal{F}_{\mathbb{X}}^{NT}(2,3,4)+1}{2} = 91.$$

These NBTs are

- $(\mathbb{X}^{NT}, \mathcal{F}_1^{NT}, \mathcal{F}_1^{NT}), (\mathbb{X}^{NT}, \mathcal{F}_1^{NT}, \mathcal{F}_2^{NT}), (\mathbb{X}^{NT}, \mathcal{F}_1^{NT}, \mathcal{F}_3^{NT}), (\mathbb{X}^{NT}, \mathcal{F}_1^{NT}, \mathcal{F}_4^{NT}),$
- $(\mathbb{X}^{NT}, \mathcal{F}_1^{NT}, \mathcal{F}_5^{NT}), (\mathbb{X}^{NT}, \mathcal{F}_1^{NT}, \mathcal{F}_6^{NT}), (\mathbb{X}^{NT}, \mathcal{F}_1^{NT}, \mathcal{F}_7^{NT}), (\mathbb{X}^{NT}, \mathcal{F}_1^{NT}, \mathcal{F}_8^{NT}),$
- $(\mathbb{X}^{NT}, \mathcal{F}_1^{NT}, \mathcal{F}_9^{NT}), (\mathbb{X}^{NT}, \mathcal{F}_1^{NT}, \mathcal{F}_{10}^{NT}), (\mathbb{X}^{NT}, \mathcal{F}_1^{NT}, \mathcal{F}_{11}^{NT}), (\mathbb{X}^{NT}, \mathcal{F}_1^{NT}, \mathcal{F}_{12}^{NT}),$
- $(\mathbb{X}^{NT}, \mathcal{F}_1^{NT}, \mathcal{F}_{13}^{NT}),$
- $(\mathbb{X}^{NT}, \mathcal{F}_2^{NT}, \mathcal{F}_2^{NT}), (\mathbb{X}^{NT}, \mathcal{F}_2^{NT}, \mathcal{F}_3^{NT}), (\mathbb{X}^{NT}, \mathcal{F}_2^{NT}, \mathcal{F}_4^{NT}), (\mathbb{X}^{NT}, \mathcal{F}_2^{NT}, \mathcal{F}_5^{NT}),$
- $(\mathbb{X}^{NT}, \mathcal{F}_2^{NT}, \mathcal{F}_6^{NT}), (\mathbb{X}^{NT}, \mathcal{F}_2^{NT}, \mathcal{F}_7^{NT}), (\mathbb{X}^{NT}, \mathcal{F}_2^{NT}, \mathcal{F}_8^{NT}), (\mathbb{X}^{NT}, \mathcal{F}_2^{NT}, \mathcal{F}_9^{NT}),$
- $(\mathbb{X}^{NT}, \mathcal{F}_2^{NT}, \mathcal{F}_{10}^{NT}), (\mathbb{X}^{NT}, \mathcal{F}_2^{NT}, \mathcal{F}_{11}^{NT}), (\mathbb{X}^{NT}, \mathcal{F}_2^{NT}, \mathcal{F}_{12}^{NT}), (\mathbb{X}^{NT}, \mathcal{F}_2^{NT}, \mathcal{F}_{13}^{NT}),$
- $(\mathbb{X}^{NT}, \mathcal{F}_3^{NT}, \mathcal{F}_3^{NT}), (\mathbb{X}^{NT}, \mathcal{F}_3^{NT}, \mathcal{F}_4^{NT}), (\mathbb{X}^{NT}, \mathcal{F}_3^{NT}, \mathcal{F}_5^{NT}), (\mathbb{X}^{NT}, \mathcal{F}_3^{NT}, \mathcal{F}_6^{NT}),$
- $(\mathbb{X}^{NT}, \mathcal{F}_3^{NT}, \mathcal{F}_7^{NT}), (\mathbb{X}^{NT}, \mathcal{F}_3^{NT}, \mathcal{F}_8^{NT}), (\mathbb{X}^{NT}, \mathcal{F}_3^{NT}, \mathcal{F}_9^{NT}), (\mathbb{X}^{NT}, \mathcal{F}_3^{NT}, \mathcal{F}_{10}^{NT}),$
- $(\mathbb{X}^{NT}, \mathcal{F}_3^{NT}, \mathcal{F}_{11}^{NT}), (\mathbb{X}^{NT}, \mathcal{F}_3^{NT}, \mathcal{F}_{12}^{NT}), (\mathbb{X}^{NT}, \mathcal{F}_3^{NT}, \mathcal{F}_{13}^{NT}),$
- $(\mathbb{X}^{NT}, \mathcal{F}_4^{NT}, \mathcal{F}_4^{NT}), (\mathbb{X}^{NT}, \mathcal{F}_4^{NT}, \mathcal{F}_5^{NT}), (\mathbb{X}^{NT}, \mathcal{F}_4^{NT}, \mathcal{F}_6^{NT}), (\mathbb{X}^{NT}, \mathcal{F}_4^{NT}, \mathcal{F}_7^{NT}),$
- $(\mathbb{X}^{NT}, \mathcal{F}_4^{NT}, \mathcal{F}_8^{NT}), (\mathbb{X}^{NT}, \mathcal{F}_4^{NT}, \mathcal{F}_9^{NT}), (\mathbb{X}^{NT}, \mathcal{F}_4^{NT}, \mathcal{F}_{10}^{NT}), (\mathbb{X}^{NT}, \mathcal{F}_4^{NT}, \mathcal{F}_{11}^{NT}),$
- $(\mathbb{X}^{NT}, \mathcal{F}_4^{NT}, \mathcal{F}_{12}^{NT}), (\mathbb{X}^{NT}, \mathcal{F}_4^{NT}, \mathcal{F}_{13}^{NT}),$
- $(\mathbb{X}^{NT}, \mathcal{F}_5^{NT}, \mathcal{F}_5^{NT}), (\mathbb{X}^{NT}, \mathcal{F}_5^{NT}, \mathcal{F}_6^{NT}), (\mathbb{X}^{NT}, \mathcal{F}_5^{NT}, \mathcal{F}_7^{NT}), (\mathbb{X}^{NT}, \mathcal{F}_5^{NT}, \mathcal{F}_8^{NT}),$
- $(\mathbb{X}^{NT}, \mathcal{F}_5^{NT}, \mathcal{F}_9^{NT}), (\mathbb{X}^{NT}, \mathcal{F}_5^{NT}, \mathcal{F}_{10}^{NT}), (\mathbb{X}^{NT}, \mathcal{F}_5^{NT}, \mathcal{F}_{11}^{NT}), (\mathbb{X}^{NT}, \mathcal{F}_5^{NT}, \mathcal{F}_{12}^{NT}),$
- $(\mathbb{X}^{NT}, \mathcal{F}_5^{NT}, \mathcal{F}_{13}^{NT}),$
- $(\mathbb{X}^{NT}, \mathcal{F}_6^{NT}, \mathcal{F}_6^{NT}), (\mathbb{X}^{NT}, \mathcal{F}_6^{NT}, \mathcal{F}_7^{NT}), (\mathbb{X}^{NT}, \mathcal{F}_6^{NT}, \mathcal{F}_8^{NT}), (\mathbb{X}^{NT}, \mathcal{F}_6^{NT}, \mathcal{F}_9^{NT}),$
- $(\mathbb{X}^{NT}, \mathcal{F}_6^{NT}, \mathcal{F}_{10}^{NT}), (\mathbb{X}^{NT}, \mathcal{F}_6^{NT}, \mathcal{F}_{11}^{NT}), (\mathbb{X}^{NT}, \mathcal{F}_6^{NT}, \mathcal{F}_{12}^{NT}), (\mathbb{X}^{NT}, \mathcal{F}_6^{NT}, \mathcal{F}_{13}^{NT}),$
- $(\mathbb{X}^{NT}, \mathcal{F}_7^{NT}, \mathcal{F}_7^{NT}), (\mathbb{X}^{NT}, \mathcal{F}_7^{NT}, \mathcal{F}_8^{NT}), (\mathbb{X}^{NT}, \mathcal{F}_7^{NT}, \mathcal{F}_9^{NT}), (\mathbb{X}^{NT}, \mathcal{F}_7^{NT}, \mathcal{F}_{10}^{NT}),$
- $(\mathbb{X}^{NT}, \mathcal{F}_7^{NT}, \mathcal{F}_{11}^{NT}), (\mathbb{X}^{NT}, \mathcal{F}_7^{NT}, \mathcal{F}_{12}^{NT}), (\mathbb{X}^{NT}, \mathcal{F}_7^{NT}, \mathcal{F}_{13}^{NT}),$
- $(\mathbb{X}^{NT}, \mathcal{F}_8^{NT}, \mathcal{F}_8^{NT}), (\mathbb{X}^{NT}, \mathcal{F}_8^{NT}, \mathcal{F}_9^{NT}), (\mathbb{X}^{NT}, \mathcal{F}_8^{NT}, \mathcal{F}_{10}^{NT}), (\mathbb{X}^{NT}, \mathcal{F}_8^{NT}, \mathcal{F}_{11}^{NT}),$
- $(\mathbb{X}^{NT}, \mathcal{F}_8^{NT}, \mathcal{F}_{12}^{NT}), (\mathbb{X}^{NT}, \mathcal{F}_8^{NT}, \mathcal{F}_{13}^{NT}),$
- $(\mathbb{X}^{NT}, \mathcal{F}_9^{NT}, \mathcal{F}_9^{NT}), (\mathbb{X}^{NT}, \mathcal{F}_9^{NT}, \mathcal{F}_{10}^{NT}), (\mathbb{X}^{NT}, \mathcal{F}_9^{NT}, \mathcal{F}_{11}^{NT}), (\mathbb{X}^{NT}, \mathcal{F}_9^{NT}, \mathcal{F}_{12}^{NT}),$
- $(\mathbb{X}^{NT}, \mathcal{F}_9^{NT}, \mathcal{F}_{13}^{NT}),$

$$\begin{aligned}
 &(\mathbb{X}^{NT}, \mathcal{F}_{10}^{NT}, \mathcal{F}_{10}^{NT}), (\mathbb{X}^{NT}, \mathcal{F}_{10}^{NT}, \mathcal{F}_{11}^{NT}), (\mathbb{X}^{NT}, \mathcal{F}_{10}^{NT}, \mathcal{F}_{12}^{NT}), (\mathbb{X}^{NT}, \mathcal{F}_{10}^{NT}, \mathcal{F}_{13}^{NT}), \\
 &(\mathbb{X}^{NT}, \mathcal{F}_{11}^{NT}, \mathcal{F}_{11}^{NT}), (\mathbb{X}^{NT}, \mathcal{F}_{11}^{NT}, \mathcal{F}_{12}^{NT}), (\mathbb{X}^{NT}, \mathcal{F}_{11}^{NT}, \mathcal{F}_{13}^{NT}), \\
 &(\mathbb{X}^{NT}, \mathcal{F}_{12}^{NT}, \mathcal{F}_{12}^{NT}), (\mathbb{X}^{NT}, \mathcal{F}_{12}^{NT}, \mathcal{F}_{13}^{NT}), \\
 &(\mathbb{X}^{NT}, \mathcal{F}_{13}^{NT}, \mathcal{F}_{13}^{NT}).
 \end{aligned}$$

Result 4.8. In $\mathcal{N}_{\mathbb{X}}^{\mathcal{F}}$, the NNBTs having 4-NOSs in both NTs without repetition is

$$(\mathcal{F}_i^{NT}, \mathcal{F}_j^{NT})_{\mathbb{X}}^{NT}(\mathbf{n}, \mathbf{m}, 4) = \binom{\mathcal{F}_{\mathbb{X}}^{NT}(\mathbf{n}, \mathbf{m}, 4)}{2}.$$

Example 4.9. Following Example 3.11 and result 4.8, the number of NBTs without repetition is $78 = \binom{\mathcal{F}_{\mathbb{X}}^{NT}(2,3,4)}{2} = \binom{13}{2}$.

5. Results on NNTRS

In this section, the NTRS having 3-NOS in three NTs and the NTRS having 3-NOS in three NTs without repetition means NTRS of the form $(\mathbb{X}^{NT}, \mathcal{F}_i^{NT}, \mathcal{F}_j^{NT}, \mathcal{F}_k^{NT})$ where $\mathcal{F}_i^{NT}, \mathcal{F}_j^{NT}, \mathcal{F}_k^{NT}$ are identical or non-identical topologies and non-identical topologies having 3-NOS respectively. A similar meaning is used for 4-NOS.

Result 5.1. In $\mathcal{N}_{\mathbb{X}}^{\mathcal{F}}$ the NNTRS consisting 2-NOSs in three NT is

$$(\mathcal{F}_i^{NT}, \mathcal{F}_j^{NT}, \mathcal{F}_k^{NT})_{\mathbb{X}}^{NT}(\mathbf{n}, \mathbf{m}, 2) = 1.$$

In this case NT with 2-NOSs is the indiscrete one i.e., $\mathcal{F}_1^{NT} = \{0^{NT}, 1^{NT}\}$. Therefore, NNTRS with 2-NOSs is exactly one, namely $(\mathbb{X}^{NT}, \mathcal{F}_1^{NT}, \mathcal{F}_1^{NT}, \mathcal{F}_1^{NT})$.

Result 5.2. The NNTRSs consisting 3-NOSs in all three NT in $\mathcal{N}_{\mathbb{X}}^{\mathcal{F}}$ is

$$(\mathcal{F}_i^{NT}, \mathcal{F}_j^{NT}, \mathcal{F}_k^{NT})_{\mathbb{X}}^{NT}(\mathbf{n}, \mathbf{m}, 3) = \binom{\mathcal{F}_{\mathbb{X}}^{NT}(\mathbf{n}, \mathbf{m}, 3)+2}{3}.$$

Example 5.3. Example 3.4 implies $\binom{\mathcal{F}_{\mathbb{X}}^{NT}(2, 3, 3)}{3} = 7$.

Therefore, $(\mathcal{F}_i^{NT}, \mathcal{F}_j^{NT}, \mathcal{F}_k^{NT})_{\mathbb{X}}^{NT}(2, 3, 3) = \binom{\mathcal{F}_{\mathbb{X}}^{NT}(2,3,3)+2}{3} = \frac{9 \times 8 \times 7}{6} = 84$.

Result 5.4. The NNTRSs consisting 3-NOSs in all three NT without repetition in $\mathcal{N}_{\mathbb{X}}^{\mathcal{F}}$ is

$$(\mathcal{F}_i^{NT}, \mathcal{F}_j^{NT}, \mathcal{F}_k^{NT})_{\mathbb{X}}^{NT}(\mathbf{n}, \mathbf{m}, 3) = \binom{\mathcal{F}_{\mathbb{X}}^{NT}(\mathbf{n}, \mathbf{m}, 3)}{3}.$$

Example 5.5. From Example 3.4, $\mathcal{F}_{\mathbb{X}}^{NT}(2, 3, 3) = 7$. In this case, the NTRSs having 3-NOSs in three NTs without repetition are

$$\begin{aligned}
 &(\mathbb{X}^{NT}, \mathcal{F}_1^{NT}, \mathcal{F}_2^{NT}, \mathcal{F}_3^{NT}), (\mathbb{X}^{NT}, \mathcal{F}_1^{NT}, \mathcal{F}_2^{NT}, \mathcal{F}_4^{NT}), (\mathbb{X}^{NT}, \mathcal{F}_1^{NT}, \mathcal{F}_2^{NT}, \mathcal{F}_5^{NT}), \\
 &(\mathbb{X}^{NT}, \mathcal{F}_1^{NT}, \mathcal{F}_2^{NT}, \mathcal{F}_6^{NT}), (\mathbb{X}^{NT}, \mathcal{F}_1^{NT}, \mathcal{F}_2^{NT}, \mathcal{F}_7^{NT}), \\
 &(\mathbb{X}^{NT}, \mathcal{F}_1^{NT}, \mathcal{F}_3^{NT}, \mathcal{F}_4^{NT}), (\mathbb{X}^{NT}, \mathcal{F}_1^{NT}, \mathcal{F}_3^{NT}, \mathcal{F}_5^{NT}), (\mathbb{X}^{NT}, \mathcal{F}_1^{NT}, \mathcal{F}_3^{NT}, \mathcal{F}_6^{NT}), \\
 &(\mathbb{X}^{NT}, \mathcal{F}_1^{NT}, \mathcal{F}_3^{NT}, \mathcal{F}_7^{NT}), \\
 &(\mathbb{X}^{NT}, \mathcal{F}_1^{NT}, \mathcal{F}_4^{NT}, \mathcal{F}_5^{NT}), (\mathbb{X}^{NT}, \mathcal{F}_1^{NT}, \mathcal{F}_4^{NT}, \mathcal{F}_6^{NT}), (\mathbb{X}^{NT}, \mathcal{F}_1^{NT}, \mathcal{F}_4^{NT}, \mathcal{F}_7^{NT}), \\
 &(\mathbb{X}^{NT}, \mathcal{F}_1^{NT}, \mathcal{F}_5^{NT}, \mathcal{F}_6^{NT}), (\mathbb{X}^{NT}, \mathcal{F}_1^{NT}, \mathcal{F}_5^{NT}, \mathcal{F}_7^{NT}), \\
 &(\mathbb{X}^{NT}, \mathcal{F}_1^{NT}, \mathcal{F}_6^{NT}, \mathcal{F}_7^{NT}),
 \end{aligned}$$

$$\begin{aligned}
 & (\mathbb{X}^{NT}, \mathcal{F}_2^{NT}, \mathcal{F}_3^{NT}, \mathcal{F}_4^{NT}), (\mathbb{X}^{NT}, \mathcal{F}_2^{NT}, \mathcal{F}_3^{NT}, \mathcal{F}_5^{NT}), (\mathbb{X}^{NT}, \mathcal{F}_2^{NT}, \mathcal{F}_3^{NT}, \mathcal{F}_6^{NT}), \\
 & \quad (\mathbb{X}^{NT}, \mathcal{F}_2^{NT}, \mathcal{F}_3^{NT}, \mathcal{F}_7^{NT}), \\
 & (\mathbb{X}^{NT}, \mathcal{F}_2^{NT}, \mathcal{F}_4^{NT}, \mathcal{F}_5^{NT}), (\mathbb{X}^{NT}, \mathcal{F}_2^{NT}, \mathcal{F}_4^{NT}, \mathcal{F}_6^{NT}), (\mathbb{X}^{NT}, \mathcal{F}_2^{NT}, \mathcal{F}_4^{NT}, \mathcal{F}_7^{NT}), \\
 & \quad (\mathbb{X}^{NT}, \mathcal{F}_2^{NT}, \mathcal{F}_5^{NT}, \mathcal{F}_6^{NT}), (\mathbb{X}^{NT}, \mathcal{F}_2^{NT}, \mathcal{F}_5^{NT}, \mathcal{F}_7^{NT}), \\
 & \quad (\mathbb{X}^{NT}, \mathcal{F}_2^{NT}, \mathcal{F}_6^{NT}, \mathcal{F}_7^{NT}), \\
 & (\mathbb{X}^{NT}, \mathcal{F}_3^{NT}, \mathcal{F}_4^{NT}, \mathcal{F}_5^{NT}), (\mathbb{X}^{NT}, \mathcal{F}_3^{NT}, \mathcal{F}_4^{NT}, \mathcal{F}_6^{NT}), (\mathbb{X}^{NT}, \mathcal{F}_3^{NT}, \mathcal{F}_4^{NT}, \mathcal{F}_7^{NT}), \\
 & \quad (\mathbb{X}^{NT}, \mathcal{F}_3^{NT}, \mathcal{F}_5^{NT}, \mathcal{F}_6^{NT}), (\mathbb{X}^{NT}, \mathcal{F}_3^{NT}, \mathcal{F}_5^{NT}, \mathcal{F}_7^{NT}), \\
 & \quad (\mathbb{X}^{NT}, \mathcal{F}_3^{NT}, \mathcal{F}_6^{NT}, \mathcal{F}_7^{NT}), \\
 & (\mathbb{X}^{NT}, \mathcal{F}_4^{NT}, \mathcal{F}_5^{NT}, \mathcal{F}_6^{NT}), (\mathbb{X}^{NT}, \mathcal{F}_4^{NT}, \mathcal{F}_5^{NT}, \mathcal{F}_7^{NT}), \\
 & \quad (\mathbb{X}^{NT}, \mathcal{F}_4^{NT}, \mathcal{F}_6^{NT}, \mathcal{F}_7^{NT}). \\
 & (\mathbb{X}^{NT}, \mathcal{F}_5^{NT}, \mathcal{F}_6^{NT}, \mathcal{F}_7^{NT}).
 \end{aligned}$$

Therefore, the NNTRSs consisting 3-NOSs in all three NTs without repetition is

$$(\mathcal{F}_i^{NT}, \mathcal{F}_j^{NT}, \mathcal{F}_k^{NT})_{\mathbb{X}}^{NT}(2, 3, 3) = 35 = \binom{\mathcal{F}_x^{NT}(2,3,3)}{3} = \binom{7}{3}.$$

Result 5.6. $(\mathcal{F}_i^{NT}, \mathcal{F}_j^{NT}, \mathcal{F}_k^{NT})_{\mathbb{X}}^{NT}(\mathbf{n}, \mathbf{m}, 3) = \frac{\mathbf{m}^{\mathbf{n}}}{3} (\mathcal{F}_i^{NT}, \mathcal{F}_j^{NT})_{\mathbb{X}}^{NT}(\mathbf{n}, \mathbf{m}, 3)$.

Example 5.7. From Example 4.3 and 5.3, we have,

$$(\mathcal{F}_i^{NT}, \mathcal{F}_j^{NT})_{\mathbb{X}}^{NT}(2, 3, 3) = 28 \text{ and } (\mathcal{F}_i^{NT}, \mathcal{F}_j^{NT}, \mathcal{F}_k^{NT})_{\mathbb{X}}^{NT}(2, 3, 3) = 84.$$

Therefore $\frac{3^2}{3} \times (\mathcal{F}_i^{NT}, \mathcal{F}_j^{NT})_{\mathbb{X}}^{NT}(2, 3, 3) = \frac{3^2}{3} \times 28 = 84 = (\mathcal{F}_i^{NT}, \mathcal{F}_j^{NT}, \mathcal{F}_k^{NT})_{\mathbb{X}}^{NT}(2, 3, 3)$.

Result 5.8. In $\mathcal{N}_{\mathbb{X}}^{\mathcal{F}}$, the NNTRSs consisting 4-NOSs in three NTs is

$$(\mathcal{F}_i^{NT}, \mathcal{F}_j^{NT}, \mathcal{F}_k^{NT})_{\mathbb{X}}^{NT}(\mathbf{n}, \mathbf{m}, 4) = \binom{\mathcal{F}_x^{NT}(\mathbf{n},\mathbf{m},4)+2}{3}.$$

Example 5.9. Example 3.11 implies,

$$\mathcal{F}_x^{NT}(2, 3, 4) = 13.$$

Then the NNTRS having 4-NOSs is

$$(\mathcal{F}_i^{NT}, \mathcal{F}_j^{NT}, \mathcal{F}_k^{NT})_{\mathbb{X}}^{NT}(2, 3, 4) = \binom{\mathcal{F}_x^{NT}(2,3,4)+2}{3} = \frac{13(13+1)(13+2)}{6} = 455.$$

Result 5.10. The NNTRSs consisting 4-NOSs in all three NT without repetition in $\mathcal{N}_{\mathbb{X}}^{\mathcal{F}}$ is

$$(\mathcal{F}_i^{NT}, \mathcal{F}_j^{NT}, \mathcal{F}_k^{NT})_{\mathbb{X}}^{NT}(\mathbf{n}, \mathbf{m}, 4) = \binom{\mathcal{F}_x^{NT}(\mathbf{n},\mathbf{m},4)}{3}.$$

Example 5.11. From Example 3.11, $\mathcal{F}_x^{NT}(2, 3, 4) = 13$. Following Example 5.5 and result 5.10, the NNTRSs consisting 4-NOSs in all three NT without repetition is $(\mathcal{F}_i^{NT}, \mathcal{F}_j^{NT}, \mathcal{F}_k^{NT})_{\mathbb{X}}^{NT}(2, 3, 4) = 286$.

Result 5.12. $(\mathcal{F}_i^{NT}, \mathcal{F}_j^{NT}, \mathcal{F}_k^{NT})_{\mathbb{X}}^{NT}(\mathbf{n}, \mathbf{m}, 4) = \frac{(\mathcal{F}_x^{NT}(\mathbf{n},\mathbf{m},4)+2)}{3} (\mathcal{F}_i^{NT}, \mathcal{F}_j^{NT})_{\mathbb{X}}^{NT}(\mathbf{n}, \mathbf{m}, 4)$.

Example 5.13. From Examples 3.11, 4.7 and 5.9, we have

$$\mathcal{F}_x^{NT}(2, 3, 4) = 13, (\mathcal{F}_i^{NT}, \mathcal{F}_j^{NT})_{\mathbb{X}}^{NT}(2, 3, 4) = 91 \text{ and } (\mathcal{F}_i^{NT}, \mathcal{F}_j^{NT}, \mathcal{F}_k^{NT})_{\mathbb{X}}^{NT}(2, 3, 4) = 455.$$

Therefore,

$$\frac{(\mathcal{T}_{\mathbb{X}}^{NT}(2,3,4)+2)}{3}(\mathcal{T}_i^{NT}, \mathcal{T}_j^{NT})_{\mathbb{X}}^{NT}(2, 3, 4) = \frac{13+2}{3} \times 91 = 455 = (\mathcal{T}_i^{NT}, \mathcal{T}_j^{NT}, \mathcal{T}_k^{NT})_{\mathbb{X}}^{NT}(2, 3, 4).$$

6. Effective of the proposed method

The formula for giving the number of topologies $T(\mathbf{n})$ is still not obtained for a finite set \mathbb{X} having \mathbf{n} elements. If \mathbf{n} is small, then we can compute it by hand. But the difficulty increases when \mathbf{n} becomes large. Studying this particular area is also a highly valued part of the topology, and this is one of the fascinating and challenging research areas. Note that the explicit formula for finding the number of topologies is undetermined till now. This paper is towards the formulae for finding the number of neutrosophic topological spaces having 2, 3, 4-open sets, the number of neutrosophic bitopological spaces, and tritopological spaces having the same number of open sets in topologies.

7. Conclusions

In this paper, the NNTSs consisting of small NOSs i.e., 2, 3, and 4-open sets are computed. Moreover, the NNBTSSs and NNTRSs are computed. It is also observed that formulae for finding NNTSs, NNBTSSs, and NNTRSs are interrelated. Hope this work will help in further study of NNTSs with greater open sets. In the future, the NNBTSSs having k, l -open sets and the NNTRSs having k, l, m -open sets can be found where $k \neq l \neq m$. Moreover, we aim to extend our work to study the existence of NNTSs in the topological group.

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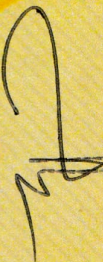


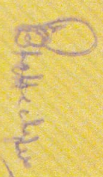
Ms. Jli Basumatary


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