A STUDY ON NUMBER OF TOPOLOGICAL SPACES IN A FINITE SET BASED ON NEUTROSOPHIC SENSE

A THESIS

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CHAPTER 6

6.1 Summary and Conclusion

This thesis is about the study of the number of topological spaces. Many researchers have studied the number of topological spaces from different angles, such as in the sense of general and fuzzy sets. The formula for determining the number of topologies $T(n)$ is still not obtained. So, each calculation is precious in itself. If the number of elements n of the finite set $\mathscr{X}$ is small (i.e., $n = 1, 2, 3$), we can calculate it by hand. But when $n \geq 4$, the difficulty in finding $T(n)$ increases. This thesis provides some formulae to determine the number of NTSs having $k \leq 4$ neutrosophic open sets, NCLTSs having $k \leq 5$ open sets based on $\mathscr{M}$, and NCrTSs having $k \leq 4$ neutrosophic crisp open sets on a finite set $\mathscr{X}$.

In Chapter 2, the cardinalities of the power set of the neutrosophic set for a non-empty finite set $\mathscr{X}$ with neutrosophic values in $\mathscr{M}$, $|\mathscr{M}| = m \geq 2$ and the cardinalities of the neutrosophic crisp set have been studied. Additionally, it has been enumerated how many chains of cardinality less than 4 are in the power sets of the neutrosophic sets. In addition, some interesting propositions have been studied.

In Chapter 3, the number of neutrosophic topological spaces having two, three, and four neutrosophic open sets have been computed for a finite set $\mathscr{X}$ with neutrosophic values in $\mathscr{M}$. Also, the number of neutrosophic bitopological spaces and neutrosophic tritopological spaces having two, three, and four neutrosophic open sets on finite sets have been computed. Additionally, it has been observed that the formulae for NNTSs, NNBTSs, and NNTRSs are interrelated.

Chapter 4 contains the formulae for determining how many neutrosophic clopen topological spaces are there with 2-open sets, 3-open sets, 4-open sets, and 5-open sets depending on different $\mathscr{M}$. This chapter also includes the formulae for counting the number of neutrosophic clopen bitopological spaces with (k, k)-open sets, (k, l)-open sets, and k & l-open sets. Additionally, this chapter provides methods for determining the number of neutrosophic clopen tritopological spaces consisting of, respectively, (k, k, k)-open sets, (k, k, l)-open sets, (k, l, m)-open sets, k & l-open sets, and k, l & m-open sets.

The beauty of chapter 5 is its fascinating formulae. In this chapter, the formulae for finding the number of neutrosophic crisp topological spaces of cardinalities $2, 3$, and 4 have been presented.

The results of this work are expected to inspire readers to undertake more research on the counting number of NTSs, NCLTSs, and NCrTSs for large open sets.

6.2 Limitation of the Study

In this thesis, the number of neutrosophic topologies with 2-neutrosophic open sets, 3-neutrosophic open sets, and 4-neutrosophic open sets for a finite set $\mathscr{X}$ with neutrosophic values in $\mathscr{M}$ have been computed. Despite the finite set $\mathscr{X}$, this thesis does not address the infinite cardinality problem. Also, it is observed that finding the number of NTSs and NCLTSs whose neutrosophic values lie in $\mathscr{M}$ ($|\mathscr{M}| = \infty$) is complicated.

6.3 Future Scope

One of the challenging and fascinating research areas is the computation of the number of topological spaces. There will be a lot of work to be done in the future on the number of topologies. Here is a list of some of them:

- To find the number of T_0-topological spaces having $k \geq n + 6$ open sets.
- To compute the number of fuzzy topological spaces having $k > 5$ open sets.
- To study the number of neutrosophic subgroups with respect to topological spaces.
- To study convergence and divergence of the number of topological spaces.