DECLARATION

I hereby declare that I have carried out the present research work, entitled **"A Study on Number of Topological Spaces in a Finite Set Based on Neutrosophic Sense"** under the guidance and supervision of Dr. Bhimraj Basumatary, Assistant Professor, Department of Mathematical Sciences, Bodoland University, Kokrajhar, Assam, India. The thesis has been submitted to Bodoland University for the award of the degree of Doctor of Philosophy in the Faculty of Science & Technology.

I further declare that the analyses and results presented in this thesis represent my original work that has not been previously submitted for a degree or diploma at any university or institution of higher education.

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CERTIFICATE

This is to certify that the thesis entitled **"A Study on Number of Topological Spaces in a Finite Set Based on Neutrosophic Sense"** has been submitted by **Ms. Jili Basumatary** for the award of the Degree of Doctor of Philosophy in Mathematics to Bodoland University, Kokrajhar, Assam, India, as a record of bonafide research work carried out by her under my guidance in the Department of Mathematical Sciences, Bodoland University, Kokrajhar.

The thesis satisfies the requirements of the regulation relating to the degree. Also, considerable parts of the thesis have been published in international and national journals. The work reported in the thesis is original and has not been submitted to any other university or institute for the award of any degree or diploma.

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Jili Basumatary
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List of Abbreviations

FS	Fuzzy Set
FT	Fuzzy Topology
FTS	Fuzzy Topological Space
IFS	Intuitionistic Fuzzy Set
IFT	Intuitionistic Fuzzy Topology
IFTS	Intuitionistic Fuzzy Topological Space
NAC	Number of Antichain
NBTS	Neutrosophic Bitopological Space
NC	Number of Chain
NCLBTS	Neutrosophic Clopen Bitopological Space
NCLT	Neutrosophic Clopen Topology
NCLTS	Neutrosophic Clopen Topological Space
NCLTRS	Neutrosophic Clopen Tritopological Space
NCrOS	Neutrosophic Crisp Open Set
NCrS	Neutrosophic Crisp Set
NCrT	Neutrosophic Crisp Topology
NCrTS	Neutrosophic Crisp Topological Space
NNBTS	Number of Neutrosophic Bitopological Space
NNT	Number of Neutrosophic Topology

NNTRS	Number of Neutrosophic Tritopological Space
NNTS	Number of Neutrosophic Topological Space
NOS	Neutrosophic Open Set
NS	Neutrosophic Set
NSub	Neutrosophic Subset
NT	Neutrosophic Topology
NTRS	Neutrosophic Tritopological Space
NTS	Neutrosophic Topological Space

List of Notations

$\mathscr{X}$	A non-empty finite set of cardinality n
$\mathscr{M}$	Ordered set of neutrosophic values
$A^{'}$	Complement of a subset A
$C(A)$	Complement of a NSub A
A^{c}	Complement of a NCrS A
$S(n,k)$	Stirling number of second kind
$C(n,k)$	Number of chain of length k in $\mathscr{X}$
$P(\mathscr{X})$	Power set of $\mathscr{X}$
$\mathscr{P}_{\mathscr{M}}(\mathscr{X})$	Neutrosophic power set of $\mathscr{X}$ whose neutrosophic values lie in $\mathscr{M}$
$\hat{\mathscr{P}}_{\mathscr{M}}(\mathscr{X})$	$\mathscr{P}_{\mathscr{M}}(\mathscr{X}) - \{0^{NT}, 1^{NT}\}$
$\mathscr{P}_{\mathscr{NC}r}(\mathscr{X})$	Neutrosophic crisp power set of $\mathscr{X}$
$\mathscr{N}_{\mathscr{X}}$	Set of all NSubs of $\mathscr{X}$ whose neutrosophic values lie in $\mathscr{M}$
$\mathscr{N}_{\mathscr{X}}^{\mathscr{T}}$	Set of all NCLTs on $\mathscr{X}$ whose neutrosophic values lie in $\mathscr{M}$
k-NCrOSs	k number of NCrOSs, where $k = 2, 3, 4$
k-NOSs	k number of NOSs, where $k = 2, 3, 4$
k-open sets	k number of NOSs in NCLT, where $k = 2, 3, 4, 5$

NCLBTSs having (k,l)-open sets	NCLBTSs having k number of open sets in one topology and l number of open sets in another topology
NCLBTSs having k & l-open sets	NCLBTSs having (k,k), (l,l), and (k,l)-open sets
NCLTRSs having (k,l,m)-open sets	NCLTRSs having k number of open sets in one topology, l number in another topology, and m number in the third topology
NCLTRSs having k & l-open sets	NCLTRSs having (k,k,k), (l,l,l), (k,k,l), (l,l,k)-open sets
NCLTRSs having k,l & m-open sets	NCLTRSs having (k,k,k), (l,l,l), (m,m,m), (k,k,l), (k,k,m), (l,l,k), (l,l,m), (m,m,k), (m,m,l), and (k,l,m)-open sets
$c_k(\mathscr{P})$	Number of chains with k elements in the ordered set $\mathscr{P}$
$\mathscr{C}_N(n,m,k)$	Number of chain of length k in $\mathscr{P}_{\mathscr{M}}(\mathscr{X})$
$\tau_{\mathscr{F}}(n,m,k)$	Number of FTSs having k-open sets on $\mathscr{X}$ whose membership values lie in a totally orderd set of cardinality m.
$\mathscr{T}^{NT}_{\mathscr{X}}(n,m,k)$	Number of NTSs having k-open sets on $\mathscr{X}$ with neutrosophic values in $\mathscr{M}$
$(\mathscr{T}^{NT}_i,\mathscr{T}^{NT}_j)^{NT}_{\mathscr{X}}(n,m,k)$	Number of NBTSs having k-open sets in both NTs on $\mathscr{X}$ with neutrosophic values in $\mathscr{M}$

$(\mathscr{T}_i^{NT}, \mathscr{T}_j^{NT}, \mathscr{T}_k^{NT})_{\mathscr{X}}^{NT}(n, m, k)$	Number of NTRSs having k-open sets in all the three NTs on $\mathscr{X}$ with neutrosophic values in $\mathscr{M}$
η_k	Number of NCLTs having k-open sets
$\eta_B^{CL}(n, m, k)$	Number of NCLBTSs having (k, k)-open sets
$\eta_B^{CL}(n, m, k, l)$	Number of NCLBTSs having (k, l)-open sets with $k \neq l$
$\mathscr{T}_T^{CL}(n, m, k)$	Number of NCLTRSs having (k, k, k)-open sets
$\mathscr{T}_T^{CL}(n, m, k, l)$	Number of NCLTRSs having (k, k, l)-open sets
$\mathscr{T}_T^{CL}(n, m, k, l, \mathfrak{m})$	Number of NCLTRSs having $(k, l, \mathfrak{m})$-open sets
$\mathscr{T}_{\mathscr{C}r}(n, k)$	Number of NCrTs of cardinality k on $\mathscr{X}$

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