CHAPTER 4

On the Structure of Number of Neutrosophic Clopen Topological Spaces on a Finite Set

In the previous chapter, the NNTS, NNBTS, and NNTRS having k $(k = 2, 3, 4)$ *neutrosophic open sets on finite sets have been computed. This chapter presents the computation of the formulae to determine the number of NCLTSs on* $\mathscr{X}$ *with neutrosophic values in* $\mathscr{M}$ *containing* 2*-open sets,* 3*-open sets,* 4*-open sets, and* 5*-open sets. The formulae for determining the number of NCLBTS having* (k, k)*-open sets,* (k, l)*-open sets, and* k *&* l*-open sets are also presented in this chapter. This chapter further provides formulae for computing the number of NCLTRSs having* (k, k, k)*-open sets,* (k, k, l)*-open sets,* (k, l, m)*-open sets,* k *&* l*-open sets, and* $k, l,$ *&* m*-open sets, respectively.*

Before going to the number of NCLTs, let us discuss some basic definition and propositions on NCLTS.

Some of the results discussed in this chapter have been published in the journal, Basumatary, J., Basumatary, B., and Broumi, S.(2022). On the Structure of Number of Neutrosophic Clopen Topological Space. International Journal of Neutrosophic Science, 18(4):192-203

Definition 4.0.1

A NT τ on a non-empty set $\mathscr{X}$ is said to be a NCLT if it consists of neutrosophic clopen sets, i.e. if every one of its NOSs are closed too.

The pair $(\mathscr{X}, \tau)$ is called a neutrosophic clopen topological space (NCLTS), and if τ contains k-open sets then $(\mathscr{X}, \tau)$ is called a NCLTS having k-open sets.

Example 4.0.1

Let $\mathscr{X} = \{u_1, v_1, w_1\}$ and consider the family

$\tau = \{0^{NT}, 1^{NT}, A_1, A_2\}$, *where*

$0^{NT} = \langle \frac{u_1}{(0,1,1)}, \frac{v_1}{(0,1,1)}, \frac{w_1}{(0,1,1)} \rangle, 1^{NT} = \langle \frac{u_1}{(1,0,0)}, \frac{v_1}{(1,0,0)}, \frac{w_1}{(1,0,0)} \rangle,$

$A_1 = \langle \frac{u_1}{(0.2,0.5,0.3)}, \frac{v_1}{(0.3,0.6,0.5)}, \frac{w_1}{(0,0.7,0.4)} \rangle,$

$A_2 = \langle \frac{u_1}{(0.3,0.5,0.2)}, \frac{v_1}{(0.5,0.4,0.3)}, \frac{w_1}{(0.4,0.3,0)} \rangle.$

Then τ is a NCLT on $\mathscr{X}$ and so $(\mathscr{X}, \tau)$ is a NCLTS on $\mathscr{X}$.

Proposition 4.0.1

Arbitrary intersection of the NCLTs on $\mathscr{X}$ is a NCLT.

Proof:

Let

$$\mathscr{N}_\tau = \bigcap_{i \in \lambda} \tau_i,$$

where λ is an index set and $\tau_i \in \mathscr{N}_{\mathscr{X}}^{\mathscr{T}}$, the collection of all NCLTs on $\mathscr{X}$.

Clearly $\mathscr{N}_\tau \neq \emptyset$ as $0^{NT}, 1^{NT} \in \mathscr{N}_\tau$.

Let A, B be any two members of $\mathscr{N}_\tau$. Then

$A, B \in \mathscr{N}_\tau$

$\Longrightarrow A, B \in \bigcap_{i \in \lambda} \tau_i$

$\Longrightarrow A, B \in \tau_i \quad ; \forall i$

$\Longrightarrow A \cap B \in \tau_i$ and $A \cup B \in \tau_i \quad ; \forall i$

$\Longrightarrow A \cap B \in \bigcap_{i \in \lambda} \tau_i$ and $A \cup B \in \bigcap_{i \in \lambda} \tau_i \quad ; \forall i$

$\Longrightarrow A \cap B \in \mathscr{N}_\tau$ and $A \cup B \in \mathscr{N}_\tau$

Therefore, $\mathscr{N}_\tau$ is a NT on $\mathscr{X}$.

Let A be any element of $\mathscr{N}_\tau$, then

$$\begin{aligned} A \in \mathscr{N}_\tau &\Longrightarrow A \in \bigcap_{i\in\ \lambda} \tau_i \\ &\Longrightarrow A \in \tau_i \qquad ;\forall i \\ &\Longrightarrow C(A) \in \tau_i \qquad ;\forall i \\ &\Longrightarrow C(A) \in \bigcap_{i\in\ \lambda} \tau_i \\ &\Longrightarrow C(A) \in \mathscr{N}_\tau \end{aligned}$$

Since, A is an arbitrary element of $\mathscr{N}_\tau$ and $C(A) \in \mathscr{N}_\tau$ implies that every element of $\mathscr{N}_\tau$ is a neutrosophic clopen set. Hence, arbitrary intersection of the NCLTs is a NCLT.

Remark 4.0.1

The union of the NCLTs on $\mathscr{X}$ is not a NCLT.

Illustration:

Let $\mathscr{X} = \{u_1, v_1\}$ and $\mathscr{M} = \{(0,1,1), (0.5,0.5,0.5), (1,0,0)\}$. Then the number of elements in $\mathscr{N}_{\mathscr{X}}$, i.e., $|\mathscr{N}_{\mathscr{X}}| = 3^2 = 9$. These are

$0^{NT} = \langle \frac{u_1}{(0,1,1)}, \frac{v_1}{(0,1,1)} \rangle$, $1^{NT} = \langle \frac{u_1}{(1,0,0)}, \frac{v_1}{(1,0,0)} \rangle$,
$A_1 = \langle \frac{u_1}{(0,1,1)}, \frac{v_1}{(0.5,0.5,0.5)} \rangle$, $A_2 = \langle \frac{u_1}{(0,1,1)}, \frac{v_1}{(1,0,0)} \rangle$,
$A_3 = \langle \frac{u_1}{(0.5,0.5,0.5)}, \frac{v_1}{(0,1,1)} \rangle$, $A_4 = \langle \frac{u_1}{(0.5,0.5,0.5)}, \frac{v_1}{(0.5,0.5,0.5)} \rangle$,
$A_5 = \langle \frac{u_1}{(0.5,0.5,0.5)}, \frac{v_1}{(1,0,0)} \rangle$, $A_6 = \langle \frac{u_1}{(1,0,0)}, \frac{v_1}{(0,1,1)} \rangle$,
$A_7 = \langle \frac{u_1}{(1,0,0)}, \frac{v_1}{(0.5,0.5,0.5)} \rangle$.

In this case,

$\tau_1 = \{0^{NT}, 1^{NT}, A_4\}$ *and*
$\tau_2 = \{0^{NT}, 1^{NT}, A_1, A_2, A_6, A_7\}$ *are NCLTs on $\mathscr{X}$.*

But $\tau_1 \cup \tau_2 = \{0^{NT}, 1^{NT}, A_1, A_2, A_4, A_6, A_7\}$ is not a NCLT on $\mathscr{X}$ as $A_4 \cap\ A_6 = A_3 \notin \tau_1 \cup \tau_2$.

Proposition 4.0.2

The union of two NCLTs is again a NCLT if one is contained in the other.

Proof:

Let τ_1 and τ_2 be two NCLTs on $\mathscr{X}$. Let $\tau_1 \subseteq \tau_2$, then $\tau_1 \cup \tau_2 = \tau_2$, which is a NCLT on $\mathscr{X}$. Similarly, if $\tau_2 \subseteq \tau_1$, then $\tau_1 \cup \tau_2 = \tau_1$, which is a

NCLT on $\mathscr{X}$. This shows that the union of two NCLTs is again a NCLT if one is contained in the other.

4.1 Number of Neutrosophic Clopen Topologies having 2-Open Sets

This section discusses and computes the number of NCLTs with 2-open sets.

1. If $\mathscr{X} = \{p\}$ or $|\mathscr{X}| = 1$ whose neutrosophic values lie in $\mathscr{M}$.

Case I: If $\mathscr{M} = \{(0,1,1),(1,0,0)\}$

Then, $|\mathscr{N}_{\mathscr{X}}| = 2^1 = 2$. These are

$$0^{NT} = \langle \frac{p}{(0,1,1)} \rangle, 1^{NT} = \langle \frac{p}{(1,0,0)} \rangle.$$

In this case, we have obtained only one NT, i.e., $\tau_1 = \{0^{NT}, 1^{NT}\}$. This NT is also a NCLT.

Case II: If $\mathscr{M} = \{(0,1,1),(0.5,0.5,0.5),(1,0,0)\}$

Then, $|\mathscr{N}_{\mathscr{X}}| = 3^1 = 3$. These are

$$0^{NT}, 1^{NT}, A_1 = \langle \frac{p}{(0.5,0.5,0.5)} \rangle.$$

In this case, we have only one NT having 2-open sets, i.e., $\tau_1 = \{0^{NT}, 1^{NT}\}$, and this NT is also a NCLT.

Case III: If $\mathscr{M} = \{(0,1,1),(T,I,F),(F,1-I,T),(1,0,0)\}; T,I,F \in [0,1]$

In this case, $|\mathscr{N}_{\mathscr{X}}| = 4^1 = 4$. These are

$$0^{NT}, 1^{NT}, A_1 = \langle \frac{p}{(T,I,F)} \rangle, A_2 = \langle \frac{p}{(F,1-I,T)} \rangle.$$

In this case, we have only one NT having 2-open sets, i.e., $\tau_1 = \{0^{NT}, 1^{NT}\}$, and this NT is also a NCLT.

Figure 4.1: NCLT having 2-open sets

2. If $\mathscr{X} = \{p, q\}$ or $|\mathscr{X}| = 2$ whose neutrosophic values lie in $\mathscr{M}$.

Case I: If $\mathscr{M} = \{(0,1,1),(1,0,0)\}$

Then, $|\mathscr{N}_{\mathscr{X}}| = 2^2 = 4$. These are

$$0^{NT} = \langle \tfrac{p}{(0,1,1)}, \tfrac{q}{(0,1,1)} \rangle, 1^{NT} = \langle \tfrac{p}{(1,0,0)}, \tfrac{q}{(1,0,0)} \rangle,$$
$$A_1 = \langle \tfrac{p}{(0,1,1)}, \tfrac{q}{(1,0,0)} \rangle, A_2 = \langle \tfrac{p}{(1,0,0)}, \tfrac{q}{(0,1,1)} \rangle.$$

In this case, the NCLT having 2-open sets is one, i.e., $\tau_1 = \{0^{NT}, 1^{NT}\}$.

Case II: If $\mathscr{M} = \{(0,1,1),(0.5,0.5,0.5),(1,0,0)\}$

Then, $|\mathscr{N}_{\mathscr{X}}| = 3^2 = 9$. These are

$$0^{NT}, 1^{NT}, A_1 = \langle \tfrac{p}{(0,1,1)}, \tfrac{q}{(0.5,0.5,0.5)} \rangle, A_2 = \langle \tfrac{p}{(0,1,1)}, \tfrac{q}{(1,0,0)} \rangle,$$
$$A_3 = \langle \tfrac{p}{(0.5,0.5,0.5)}, \tfrac{q}{(0,1,1)} \rangle, A_4 = \langle \tfrac{p}{(0.5,0.5,0.5)}, \tfrac{q}{(0.5,0.5,0.5)} \rangle,$$
$$A_5 = \langle \tfrac{p}{(0.5,0.5,0.5)}, \tfrac{q}{(1,0,0)} \rangle, A_6 = \langle \tfrac{p}{(1,0,0)}, \tfrac{q}{(0,1,1)} \rangle,$$
$$A_7 = \langle \tfrac{p}{(1,0,0)}, \tfrac{q}{(0.5,0.5,0.5)} \rangle.$$

In this case, the NCLT having 2-open sets is one, i.e., $\tau_1 = \{0^{NT}, 1^{NT}\}$.

Case III: If $\mathscr{M} = \{(0,1,1),(T,I,F),(F,1-I,T),(1,0,0)\}; T, I, F \in [0,1]$

Then, $|\mathscr{N}_{\mathscr{X}}| = 4^2 = 16$. These are

$$0^{NT}, 1^{NT}, A_1 = \langle \tfrac{p}{(0,1,1)}, \tfrac{q}{(T,I,F)} \rangle, A_2 = \langle \tfrac{p}{(0,1,1)}, \tfrac{q}{(1,0,0)} \rangle,$$
$$A_3 = \langle \tfrac{p}{(T,I,F)}, \tfrac{q}{(0,1,1)} \rangle, \qquad A_4 = \langle \tfrac{p}{(T,I,F)}, \tfrac{q}{(T,I,F)} \rangle,$$
$$A_5 = \langle \tfrac{p}{(T,I,F)}, \tfrac{q}{(1,0,0)} \rangle, \qquad A_6 = \langle \tfrac{p}{(1,0,0)}, \tfrac{q}{(0,1,1)} \rangle,$$
$$A_7 = \langle \tfrac{p}{(1,0,0)}, \tfrac{q}{(T,I,F)} \rangle, \qquad A_8 = \langle \tfrac{p}{(0,1,1)}, \tfrac{q}{(F,1-I,T)} \rangle,$$
$$A_9 = \langle \tfrac{p}{(T,I,F)}, \tfrac{q}{(F,1-I,T)} \rangle, \qquad A_{10} = \langle \tfrac{p}{(F,1-I,T)}, \tfrac{q}{(0,1,1)} \rangle,$$
$$A_{11} = \langle \tfrac{p}{(F,1-I,T)}, \tfrac{q}{(F,1-I,T)} \rangle, \qquad A_{12} = \langle \tfrac{p}{(F,1-I,T)}, \tfrac{q}{(1,0,0)} \rangle,$$
$$A_{13} = \langle \tfrac{p}{(F,1-I,T)}, \tfrac{q}{(T,I,F)} \rangle, \qquad A_{14} = \langle \tfrac{p}{(1,0,0)}, \tfrac{q}{(F,1-I,T)} \rangle.$$

In this case, the NCLT having 2-open sets is $\tau_1 = \{0^{NT}, 1^{NT}\}$.

Proposition 4.1.1

For $|\mathscr{X}| = n$, $|\mathscr{M}| = m$, $\mathscr{M}$ is the set of neutrosophic values containing $(1,0,0)$ and $(0,1,1)$, then the number of NCLT having 2-open sets is one.

Proof:

The NT having 2-open sets is indiscrete NT only i.e., $\tau = \{0^{NT}, 1^{NT}\}$. This NT is also a NCLT as 0^{NT} and 1^{NT} are complements of each other.

Therefore, the number of NCLTS having 2-open sets is one.

4.2 Number of Neutrosophic Clopen Topologies having 3-Open Sets

This section analyses and determines the formulae for the number of NCLTs with 3-open sets.

1. If $\mathscr{X} = \{p\}$ or $|\mathscr{X}| = 1$ whose neutrosophic values lie in $\mathscr{M}$.

Case I: If $\mathscr{M} = \{(0,1,1),(1,0,0)\}$

In this case, $|\mathscr{N}_{\mathscr{X}}| = 2^1 = 2$. We have obtained only one NCLT, i.e., $\tau_1 = \{0^{NT}, 1^{NT}\}$ and so there exists no NCLT having 3-open sets. Therefore, the number of NCLT having 3-open sets is zero.

Case II: If $\mathscr{M} = \{(0,1,1),(0.5,0.5,0.5),(1,0,0)\}$

In this case, $|\mathscr{N}_{\mathscr{X}}| = 3^1 = 3$ and we have obtained one NCLT having 3-open sets, i.e., $\tau_1 = \{0^{NT}, A_1, 1^{NT}\}$ as complement of A_1 is A_1 i.e., $C(A_1) = A_1$. Therefore, the number of NCLT having 3-open sets is one.

Case III: If $\mathscr{M} = \{(0,1,1),(T,I,F),(F,1-I,T),(1,0,0)\}; T,I,F \in [0,1]$

In this case, $|\mathscr{N}_{\mathscr{X}}| = 4^1 = 4$ and we have obtained no NT having 3-open sets. Therefore, the number of NCLT having 3-open sets is zero.

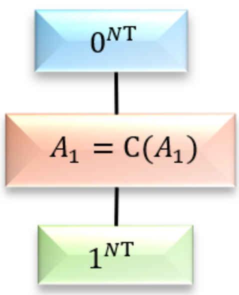

Figure 4.2: NCLTs having 3-open sets

2. If $\mathscr{X} = \{p, q\}$ or $|\mathscr{X}| = 2$ whose neutrosophic values lie in $\mathscr{M}$.

Case I: If $\mathscr{M} = \{(0,1,1), (1,0,0)\}$

In this case, $|\mathscr{N}_{\mathscr{X}}| = 2^2 = 4$ and we have obtained no NCLT having 3-open sets. Therefore, the number of NCLT having 3-open sets is zero.

Case II: If $\mathscr{M} = \{(0,1,1), (0.5,0.5,0.5), (1,0,0)\}$

Then, $|\mathscr{N}_{\mathscr{X}}| = 3^2 = 9$ and we have obtained one NCLT having 3-open sets, i.e., $\tau_1 = \{0^{NT}, A_1, 1^{NT}\}$. Therefore, the number of NCLT having 3-open sets is one.

Case III: If $\mathscr{M} = \{(0,1,1), (T,I,F), (F,1-I,T), (1,0,0)\}$, where $T, I, F \in [0,1]$

In this case, $|\mathscr{N}_{\mathscr{X}}| = 4^2 = 16$ and we have obtained no NCLT having 3-open sets. Therefore, the number of NCLT having 3-open sets is zero.

Proposition 4.2.1

For $|\mathscr{X}| = n$, the number of NCLT having 3-open sets is always one for $\mathscr{M}$ containing $(0,1,1), (1,0,0)$ and $(T, 0.5, F)$, where $T = F; T, F \in [0,1]$, and zero for $\mathscr{M}$ that does not include neutrosophic values which are complement to each other.

Proof:

Let $\mathscr{M}$ be the set containing $(0,1,1), (1,0,0), (T,0.5,F), T = F; T, F \in [0,1]$.

Case 1: Let $\mathscr{X} = \{a\}$ and $|\mathscr{X}| = 1$

In this case, the NCLT having 3-open sets is $\{0^{NT}, 1^{NT}, A_1\}$, where $A_1 = \langle \frac{a}{(T,0.5,F)} \rangle$.

Case II: Let $\mathscr{X} = \{a, b\}$ and $|\mathscr{X}| = 2$

In this case, the NCLT having 3-open sets is $\{0^{NT}, 1^{NT}, A_2\}$, where $A_2 = \langle \frac{a}{(T,0.5,F)}, \frac{b}{(T,0.5,F)} \rangle$.

Case III: Let $\mathscr{X} = \{a, b, c\}$ and $|\mathscr{X}| = 3$

In this case, the NCLT having 3-open sets is $\{0^{NT}, 1^{NT}, A_3\}$, where $A_3 = \langle \frac{a}{(T,0.5,F)}, \frac{b}{(T,0.5,F)}, \frac{c}{(T,0.5,F)} \rangle$.

Case IV: Let $\mathscr{X} = \{a_1, a_2, a_3, \ldots, a_n\}$ and $|\mathscr{X}| = n$ (finite)

In this case, the NCLT having 3-open sets is $\{0^{NT}, 1^{NT}, A_n\}$, where $A_n = \langle \frac{a_1}{(T,0.5,F)}, \frac{a_2}{(T,0.5,F)}, \frac{a_3}{(T,0.5,F)}, \ldots, \frac{a_n}{(T,0.5,F)} \rangle$.

It is seen that there exists only one NCLT having 3-open sets. This NCLT contains $0^{NT}, 1^{NT}$ and A_n. Note that, in A_n every member of $\mathscr{X}$ has neutrosophic value as $(T, 0.5, F), T = F; T, F \in [0, 1]$, whose complement is itself i.e., $A_n = C(A_n)$.

On the other hand, if $\mathscr{M}$ does not contain $(T, 0.5, F), T = F; T, F \in [0, 1]$, then there exists no neutrosophic subset of $\mathscr{X}$ of the form A_n, i.e., $A_n \neq C(A_n)$. Hence, there exists no NCLT having 3-open sets.

4.3 Number of Neutrosophic Clopen Topologies having 4-Open Sets

This section analyses and determines the formulae for the number of NCLTs with 4-open sets.

1. If $\mathscr{X} = \{p\}$ or $|\mathscr{X}| = 1$ whose neutrosophic values lie in $\mathscr{M}$.

Case I: If $\mathscr{M} = \{(0, 1, 1), (1, 0, 0)\}$

In this case, $|\mathscr{N}_{\mathscr{X}}| = 2^1 = 2$ and so, there exists no NCLT having 4-open sets. Therefore, the number of NCLT having 4-open sets is zero.

Case II: If $\mathscr{M} = \{(0, 1, 1), (0.5, 0.5, 0.5), (1, 0, 0)\}$

In this case, $|\mathscr{N}_{\mathscr{X}}| = 3^1 = 3$ and we have obtained no NT having 4-open sets. Therefore, the number of NCLT having 4-open sets is zero.

Case III: If $\mathscr{M} = \{(0, 1, 1), (T, I, F), (F, 1 - I, T), (1, 0, 0)\}; T, I, F \in [0, 1]$

In this case, $|\mathscr{N}_{\mathscr{X}}| = 4^1 = 4$ and we have obtained one NCLT having 4-open sets. Therefore, the number of NCLT having 4-open sets is one.

2. If $\mathscr{X} = \{p, q\}$ or $|\mathscr{X}| = 2$ whose neutrosophic values lie in $\mathscr{M}$.

Case I: If $\mathscr{M} = \{(0, 1, 1), (1, 0, 0)\}$

In this case, $|\mathscr{N}_{\mathscr{X}}| = 2^2 = 4$ and we have obtained one NCLT having

4-open sets. Therefore, the number of NCLT having 4-open sets is one i.e., $\tau_1 = \{0^{NT}, A_1, A_2, 1^{NT}\}$.

Case II: If $\mathscr{M} = \{(0,1,1), (0.5,0.5,0.5), (1,0,0)\}$

In this case, $|\mathscr{N}_{\mathscr{X}}| = 3^2 = 9$ and we have obtained three NCLTs having 4-open sets. These are

$\tau_1 = \{0^{NT}, A_1, A_7, 1^{NT}\}$,

$\tau_2 = \{0^{NT}, A_2, A_6, 1^{NT}\}$,

$\tau_3 = \{0^{NT}, A_3, A_5, 1^{NT}\}$.

Therefore, the number of NCLTs having 4-open sets is three.

Case III: If $\mathscr{M} = \{(0,1,1), (T,I,F), (F,1-I,T), (1,0,0)\}; T,I,F \in [0,1]$

In this case, $|\mathscr{N}_{\mathscr{X}}| = 4^2 = 16$ and we have obtained four NCLTs having 4-open sets. These are

$\tau_1 = \{0^{NT}, A_1, A_{14}, 1^{NT}\}$,

$\tau_2 = \{0^{NT}, A_2, A_6, 1^{NT}\}$,

$\tau_3 = \{0^{NT}, A_3, A_{12}, 1^{NT}\}$,

$\tau_4 = \{0^{NT}, A_4, A_{11}, 1^{NT}\}$.

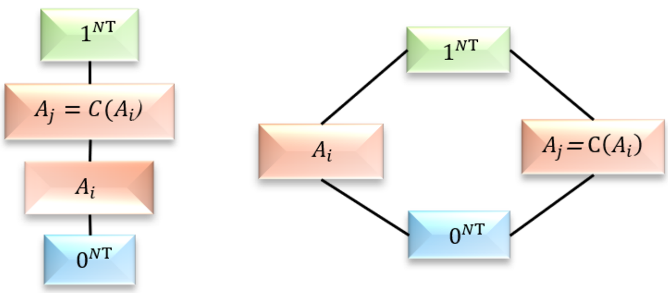

Figure 4.3: NCLTs having 4-open sets

Proposition 4.3.1

Let $\mathscr{X}$ be a finite set with $|\mathscr{X}| = n$ and $\mathscr{M} = \{(0,1,1), (1,0,0), (0.5,0.5,0.5)\}$. Then the number of NCLTSs having 4-open sets is obtained by

$$t_n = 0 + 3.2^0 + 3.2^1 + 3.2^2 + 3.2^3 + \cdots + 3.2^{n-2} = 6.2^{n-2} - 3,$$

where t_n is the sum of first n^{th} term and $t_1 = 0$.

Proof:

For $n = 1$, we have $t_1 = 0 = 6.2^{1-2} - 3$.

Let $\mathscr{X} = \{u_1\}$ then $|\mathscr{N}_{\mathscr{X}}| = 3^1 = 3$. So, there is no NCLT having 4-open sets.

Therefore, the result is true for $n = 1$.

For $n = 2$, we have $t_2 = 0 + 3.2^0 = 3 = 6.2^{2-2} - 3$.

Let $\mathscr{X} = \{u_1, v_1\}$ then $|\mathscr{N}_{\mathscr{X}}| = 3^2 = 9$. In this case, NCLTs having 4-open sets are

$\tau_1 = \{0^{NT}, 1^{NT}, \langle\frac{u_1}{(0,1,1)}, \frac{v_1}{(0.5,0.5,0.5)}\rangle, \langle\frac{u_1}{(1,0,0)}, \frac{v_1}{(0.5,0.5,0.5)}\rangle\}$,

$\tau_2 = \{0^{NT}, 1^{NT}, \langle\frac{u_1}{(0.5,0.5,0.5)}, \frac{v_1}{(0,1,1)}\rangle, \langle\frac{u_1}{(0.5,0.5,0.5)}, \frac{v_1}{(1,0,0)}\rangle\}$,

$\tau_3 = \{0^{NT}, 1^{NT}, \langle\frac{u_1}{(0,1,1)}, \frac{v_1}{(1,0,0)}\rangle, \langle\frac{u_1}{(1,0,0)}, \frac{v_1}{(0,1,1)}\rangle\}$.

Let us consider the result is true for $n = k$ i.e., $t_k = 6.2^{k-2} - 3$.

We now try to prove the result for $n = k + 1$.

Therefore, $t_n = t_{k+1} = t_k + 3.2^{(k+1)-2}$

$= 6.2^{k-2} - 3 + 3.2^{k-1}$

$= 3.2^{k-2}(2 + 2) - 3$

$= 3.4.2^{k-2} - 3$

$= 6.2^{k-1} - 3$

$= 6.2^{(k+1)-2} - 3.$

Thus, the result is true for $n = k + 1$. Hence, for all the natural numbers the result is true.

The following table 4.1 shows the number of NCLTSs having 4-open sets on $\mathscr{X}$ whose neutrosophic values lie in

$$\mathscr{M} = \{(0, 1, 1), (0.5, 0.5, 0.5), (1, 0, 0)\}.$$

Table 4.1: Number of NCLTSs having 4-open sets on $\mathscr{X}$ with neutrosophic values in $\mathscr{M} = \{(0, 1, 1), (0.5, 0.5, 0.5), (1, 0, 0)\}$

$\lvert\mathscr{X}\rvert$	1	2	3	4	5	$\cdots$	n
Number of NCLTs having 4-open sets	0	3	9	21	45	$\cdots$	$6.2^{n-2} - 3$

From the Table 4.1, the following figure 4.4 is constructed. In the figure

4.4, every small sphere represents a NCLT having 4-open sets on $\mathscr{X}$ whose neutrosophic values lie in $\mathscr{M}$.

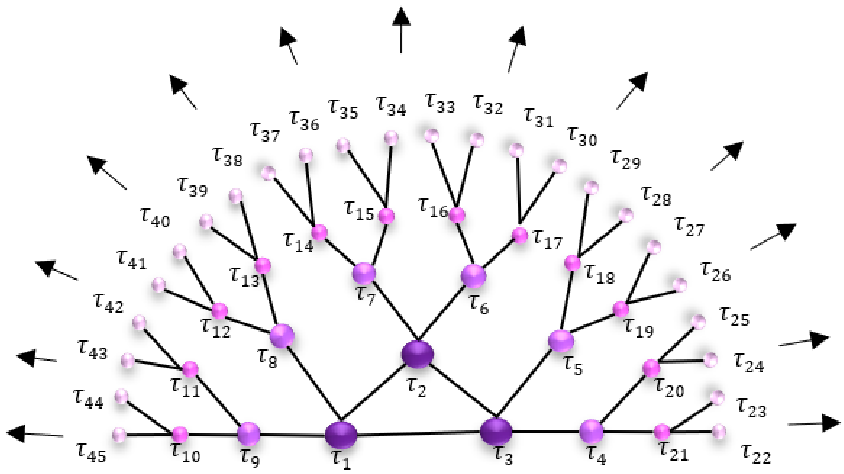

Figure 4.4: Representation of NCLTSs having 4-open sets on $\mathscr{X}$ with $|\mathscr{X}| \geq 2$ whose neutrosophic values lie in $\mathscr{M}$

Proposition 4.3.2

Let $|\mathscr{X}| = n$ (finite) and $\mathscr{M} = \{(0,1,1),(1,0,0),(T,I,F),(F,1-I,T)\}$. Then the number of NCLTSs having 4-open sets is obtained by

$$t_n = 1 + 3.2^0 + 3.2^1 + 3.2^2 + \cdots + 3.2^{n-2} = 3.2^{n-1} - 2,$$

where t_n is the sum of first n^{th} term and $t_1 = 1$.

Proof:

For $n = 1$, we have, $t_1 = 1 = 3.2^{1-1} - 2$.

Let $\mathscr{X} = \{u_1\}$ then $|\mathscr{N}_{\mathscr{X}}| = 4^1 = 4$. So, there is one NCLT having 4-open sets which is

$$\{\langle\tfrac{u_1}{(0,1,1)}\rangle, \langle\tfrac{u_1}{(1,0,0)}\rangle, \langle\tfrac{u_1}{(T,I,F)}\rangle, \langle\tfrac{u_1}{(F,1-I,T)}\rangle\}.$$

Therefore, the result is true for $n = 1$.

For $n = 2$, we have $t_2 = 1 + 3.2^0 = 4 = 3.2^1 - 2$.

Let $\mathscr{X} = \{u_1, v_1\}$ and $\mathscr{M} = \{(0,1,1),(1,0,0),(T,I,F),(F,1-I,T)\}$.

Then $|\mathscr{N}_{\mathscr{X}}| = 4^2 = 16$. In this case, NCLTs having 4-open sets are

$$\tau_1 = \{0^{NT}, 1^{NT}, \langle\tfrac{u_1}{(0,1,1)}, \tfrac{v_1}{(T,I,F)}\rangle, \langle\tfrac{u_1}{(1,0,0)}, \tfrac{v_1}{(F,1-I,T)}\rangle\},$$

$$\tau_2 = \{0^{NT}, 1^{NT}, \langle\tfrac{u_1}{(T,I,F)}, \tfrac{v_1}{(T,I,F)}\rangle, \langle\tfrac{u_1}{(F,1-I,T)}, \tfrac{v_1}{(F,1-I,T)}\rangle\},$$

$\tau_3 = \{0^{NT}, 1^{NT}, \langle \frac{v_1}{(0,1,1)}, \frac{v_1}{(1,0,0)} \rangle, \langle \frac{u_1}{(1,0,0)}, \frac{v_1}{(0,1,1)} \rangle\}$,
$\tau_4 = \{0^{NT}, 1^{NT}, \langle \frac{u_1}{(T,I,F)}, \frac{v_1}{(0,1,1)} \rangle, \langle \frac{u_1}{(F,1-I,T)}, \frac{v_1}{(1,0,0)} \rangle\}$.

Let us consider, the result is true for $n = k$ i.e., $t_k = 3.2^{k-1} - 2$.

We now try to prove the result for $n = k + 1$.

Therefore,

$$\begin{aligned} t_n &= t_{k+1} \\ &= t_k + 3.2^{(k+1)-2} \\ &= 3.2^{k-1} - 2 + 3.2^{k-1} \\ &= 2.3.2^{k-1} - 2 \\ &= 3.2^k - 2 \\ &= 3.2^{(k+1)-1} - 2 \end{aligned}$$

Hence, the result is true for $n = k + 1$. So, for all the natural numbers the result is true.

The following table 4.2 shows the number of NCLTSs having 4-open sets on $\mathscr{X}$ whose neutrosophic values lie in

$\mathscr{M} = \{(0,1,1),(1,0,0),(T,I,F),(F,1-I,T)\}$.

Table 4.2: Number of NCLTSs having 4-open sets on $\mathscr{X}$ with neutrosophic values in $\mathscr{M} = \{(0,1,1),(1,0,0),(T,I,F),(F,1-I,T)\}$

$\lvert\mathscr{X}\rvert$	1	2	3	4	5	$\cdots$	n
Number of NCLTs having 4-open sets	1	4	10	22	46	$\cdots$	$3.2^{n-1} - 2$

From the Table 4.2, the following figure 4.5 is constructed. In the figure 4.5, every small sphere represents a NCLT having 4-open sets on $\mathscr{X}$ whose neutrosophic values lie in $\mathscr{M}$.

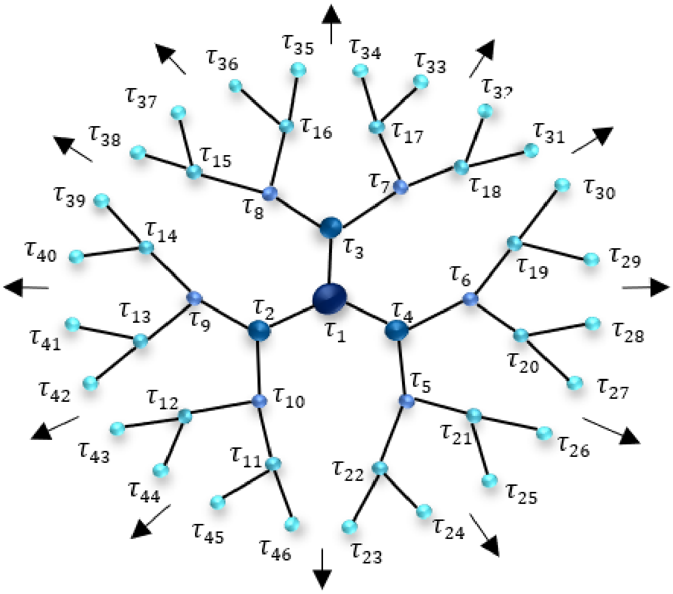

Figure 4.5: Representation of NCLTSs having 4-open sets on $\mathscr{X}$ with $|\mathscr{X}| \geq 1$ whose neutrosophic values lie in $\mathscr{M}$

4.4 Number of Neutrosophic Clopen Topologies having 5-Open Sets

This section analyses and determines the formulae for the number of NCLTs with 5-open sets.

1. If $\mathscr{X} = \{p\}$ or $|\mathscr{X}| = 1$ whose neutrosophic values lie in $\mathscr{M}$.

Case I: If $\mathscr{M} = \{(0,1,1),(1,0,0)\}$

In this case, $|\mathscr{N}_{\mathscr{X}}| = 2^1 = 2$ and we have obtained only one NT which is $\tau_1 = \{0^{NT}, 1^{NT}\}$ and so, there exists no NCLT having 5-open sets. Therefore, the number of NCLT having 5-open sets is zero.

Case II: If $\mathscr{M} = \{(0,1,1),(0.5,0.5,0.5),(1,0,0)\}$

In this case, $|\mathscr{N}_{\mathscr{X}}| = 3^1 = 3$ and we have obtained only two NCLTs having 2 and 3-open sets. Therefore, the number of NCLT having 5-open sets is zero.

Case III: If $\mathscr{M} = \{(0,1,1),(T,I,F),(F,1-I,T),(1,0,0)\}; T,I,F \in [0,1]$

In this case, $|\mathscr{N}_{\mathscr{X}}| = 4^1 = 4$ and we have obtained no NT having 5-open sets. Therefore, the number of NCLT having 5-open sets is zero.

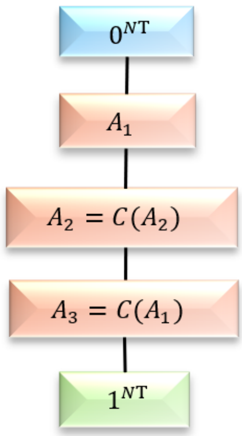

Figure 4.6: NCLTs having 5-open sets

2. If $\mathscr{X} = \{p, q\}$ or $|\mathscr{X}| = 2$ whose neutrosophic values lie in $\mathscr{M}$.

Case I: If $\mathscr{M} = \{(0, 1, 1), (1, 0, 0)\}$

In this case, $|\mathscr{N}_{\mathscr{X}}| = 2^2 = 4$ and we have obtained no NCLT having 5-open sets. Therefore, the number of NCLT having 5-open sets is zero.

Case II: If $\mathscr{M} = \{(0, 1, 1), (0.5, 0.5, 0.5), (1, 0, 0)\}$

In this case, $|\mathscr{N}_{\mathscr{X}}| = 3^2 = 9$ and we have obtained two NCLTs having 5-open sets. These are

$\tau_1 = \{0^{NT}, A_1, A_4, A_7, 1^{NT}\}$,

$\tau_2 = \{0^{NT}, A_3, A_4, A_5, 1^{NT}\}$.

Therefore, the number of NCLTs having 5-open sets is two.

Case III: If $\mathscr{M} = \{(0, 1, 1), (T, I, F), (F, 1 - I, T), (1, 0, 0)\}; T, I, F \in [0, 1]$

In this case, $|\mathscr{N}_{\mathscr{X}}| = 4^2 = 16$ and we have obtained no NCLT having 5-open sets. Therefore, the number of NCLT having 5-open sets is zero.

Proposition 4.4.1

Let $\mathscr{X}$ be a non-empty finite set, $|\mathscr{X}| = n$ and $\mathscr{M} = \{(0, 1, 1), (1, 0, 0), (0.5, 0.5, 0.5)\}$. Then the number of NCLTs having 5-open sets is obtained by

$t_n = 0 + 2^1 + 2^2 + 2^3 + \cdots + 2^{n-1} = 2^n - 2$, *where* t_n *is the sum of first* n^{th} *term and* $t_1 = 0$.

Proof:

For $n = 1$, we have $t_1 = 0 = 2^1 - 2$.

Let $\mathscr{X} = \{u_1\}$ then $|\mathscr{N}_{\mathscr{X}}| = 3^1 = 3$. So, there is no NCLT having 5-open sets.

Therefore, the result is true for $n = 1$.

For $n = 2$, we have $t_2 = 0 + 2^1 = 2 = 2^2 - 2$.

Let $\mathscr{X} = \{u_1, v_1\}$ then $|\mathscr{N}_{\mathscr{X}}| = 3^2 = 9$. In this case, NCLTs having 5-open sets are

$$\tau_1 = \left\{ \begin{array}{c} \left\langle \frac{u_1}{(0,1,1)}, \frac{v_1}{(0,1,1)} \right\rangle, \left\langle \frac{u_1}{(1,0,0)}, \frac{v_1}{(1,0,0)} \right\rangle, \left\langle \frac{u_1}{(0,1,1)}, \frac{v_1}{(0.5,0.5,0.5)} \right\rangle, \\ \left\langle \frac{u_1}{(1,0,0)}, \frac{v_1}{(0.5,0.5,0.5)} \right\rangle, \left\langle \frac{u_1}{(0.5,0.5,0.5)}, \frac{v_1}{(0.5,0.5,0.5)} \right\rangle \end{array} \right\},$$

$$\tau_2 = \left\{ \begin{array}{c} \left\langle \frac{u_1}{(0,1,1)}, \frac{v_1}{(0,1,1)} \right\rangle, \left\langle \frac{u_1}{(1,0,0)}, \frac{v_1}{(1,0,0)} \right\rangle, \left\langle \frac{u_1}{(0.5,0.5,0.5)}, \frac{v_1}{(0,1,1)} \right\rangle, \\ \left\langle \frac{u_1}{(0.5,0.5,0.5)}, \frac{v_1}{(1,0,0)} \right\rangle, \left\langle \frac{u_1}{(0.5,0.5,0.5)}, \frac{v_1}{(0.5,0.5,0.5)} \right\rangle \end{array} \right\}.$$

Let us consider, the result is true for $n = k$, i.e., $t_k = 2^k - 2$.

We now try to prove the result for $n = k + 1$.

Therefore, $t_n = t_{k+1} = t_k + 2^{(k+1)-1} = 2^k - 2 + 2^k = 2.2^k - 2 = 2^{k+1} - 2$.

Hence, for $n = k + 1$ the result is true. So, for all the natural numbers the result is true.

Table 4.3: NCLTSs having 5-open sets on $\mathscr{X}$ with neutrosophic values in $\mathscr{M} = \{(0, 1, 1), (1, 0, 0), (0.5, 0.5, 0.5)\}$

$\lvert\mathscr{X}\rvert$	1	2	3	4	5	$\cdots$	n
Number of NCLTs having 5-open sets	0	2	6	14	30	$\cdots$	$2^n - 2$

From the Table 4.3, the following figure 4.7 is constructed. In the figure 4.7, every small sphere represents a NCLT having 4-open sets on $\mathscr{X}$ whose neutrosophic values lie in $\mathscr{M}$.

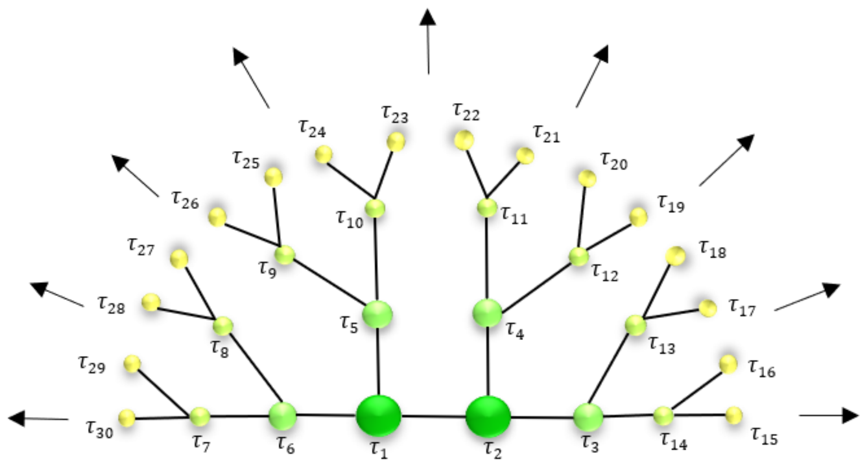

Figure 4.7: Representation of NCLTSs having 5-open sets on $\mathscr{X}$ with $|\mathscr{X}| \geq 2$ whose neutrosophic values lie in $\mathscr{M}$

Proposition 4.4.2 *Let $\mathscr{X}$ be a non-empty finite set, $|\mathscr{X}| = n$ and $\mathscr{M} = \{(0,1,1),(T,I,F),(1,0,0)\}, T = F; I = 0.5$. Then the number of NCLTs having*

(i) 4-open sets is obtained by

$$t_n = 0 + 3.2^0 + 3.2^1 + 3.2^2 + 3.2^3 + \cdots + 3.2^{n-2} = 6.2^{n-2} - 3.$$

(ii) 5-open sets is obtained by

$$t_n = 0 + 2^1 + 2^2 + 2^3 + \cdots + 2^{n-1} = 2^n - 2.$$

Proof:

The proof is straightforward.

Remark 4.4.1

Result obtained for $k = 4$ is true for all ordered set

$$\mathscr{M} = \{(0,1,1),(1,0,0),(T,I,F),(F,1-I,T)\}$$

provided $I = 0.5, T = F$, and $T < F$, $I < 0.5$.

Moreover, in this case, the results obtained for $I = 0.5$, $T = F$ coincides with results obtained for $\mathscr{M} = \{(0,1,1),(1,0,0),(0.5,0.5,0.5)\}$ and $\mathscr{M} = \{(0,1,1),(1,0,0),(T,0.5,F)\}$, $T = F$. Also, the results obtained for $T < F$ and $I < 0.5$ coincides with results obtained for $\mathscr{M} = \{(0,1,1),(1,0,0)\}$.

Proposition 4.4.3

Let $|\mathscr{X}| = n$, $|\mathscr{M}| = m$, $\mathscr{M}$ be any finite set of neutrosophic values containing $(1,0,0),(0,1,1)$ and other neutrosophic values which are complement to each other, then the number of NCLTs having m^n-open sets is one and for any other $\mathscr{M}$ it is zero.

Proof:

In this case, the NT having m^n-open sets is discrete NT. This NCLT is also clopen as it contains all neutrosophic subsets of $\mathscr{X}$ and complement of each neutrosophic subset is also in that NT. Therefore, the number of NCLTS having m^n-open sets is one.

Further, for NCLT, it is found that if $\mathscr{M}$ consists of $(1,0,0),(0,1,1)$, and other neutrosophic values which are not complement to each other, then $\mathscr{M}$, $|\mathscr{M}| = m > 2$, is equivalent to $\mathscr{M} = \{(1,0,0),(0,1,1)\}$. But for $\mathscr{M} = \{(1,0,0),(0,1,1)\}$, the maximum number of open sets in NCLTs is 2^n. Since $m^n \geq 2^n$, the number of NCLTS having m^n-open sets is zero.

Corollary 4.4.1

The minimum number of open sets in a NCLT is 2 and the maximum number of open sets in a NCLT is m^n, where n is the number of elements in $\mathscr{X}$ and m is the number of elements in $\mathscr{M}$.

Proof:

This result is obtained by using Proposition 4.4.3.

Proposition 4.4.4

Let $|\mathscr{X}| = n$, $\mathscr{M} = \{(1,0,0),(0,1,1)\}$ and $\mathscr{N}_{\mathscr{X}}^{\mathscr{T}}$ be the set of all NCLTSs on $\mathscr{X}$ whose neutrosophic values lie in $\mathscr{M}$ then

(i) $\mathscr{N}_{\mathscr{X}}^{\mathscr{T}}$ contains only NCLTs having 2^k-open sets where $k = 1,2,3,\ldots,n$.

(ii) Number of NCLTs having 2^k-open sets is $S(n,k), k = 1,2,\ldots,n$.

Proof:

Let $|\mathscr{X}| = n$, $\mathscr{M} = \{(1,0,0),(0,1,1)\}$ and η_k be the number of NCLTs having k-open sets.

Case I: If $\mathscr{X} = \{u_1\}$ and $|\mathscr{X}| = 1$

Then $|\mathscr{N}_{\mathscr{X}}| = 2^1 = 2$. These are

$$0^{NT} = \langle \tfrac{u_1}{(0,1,1)} \rangle, 1^{NT} = \langle \tfrac{u_1}{(1,0,0)} \rangle.$$

In this case, we have obtained only one NCLT which is the indiscrete NT i.e., $\tau_1 = \{0^{NT}, 1^{NT}\}$. This shows that for $|\mathscr{X}| = 1$,

(a) there exists only NCLT having $2^n, n = 1$-open sets.

(b) there exists one NCLT having 2^1-open sets i.e., $\eta_{2^1} = S(1, 1)$.

Case II: If $\mathscr{X} = \{u_1, v_1\}$ and $|\mathscr{X}| = 2$

Then $|\mathscr{N}_{\mathscr{X}}| = 2^2 = 4$. These are

$$0^{NT} = \langle \tfrac{u_1}{(0,1,1)}, \tfrac{v_1}{(0,1,1)} \rangle, 1^{NT} = \langle \tfrac{u_1}{(1,0,0)}, \tfrac{v_1}{(1,0,0)} \rangle,$$
$$A_1 = \langle \tfrac{u_1}{(0,1,1)}, \tfrac{v_1}{(1,0,0)} \rangle, A_2 = \langle \tfrac{u_1}{(1,0,0)}, \tfrac{v_1}{(0,1,1)} \rangle.$$

In this case, the NCLTs are

$$\tau_1 = \{0^{NT}, 1^{NT}\}, \tau_2 = \{0^{NT}, A_1, A_2, 1^{NT}\}.$$

This shows that there are 2-open sets in τ_1 and $4 = 2^2$-open sets in τ_2. Therefore, for $|\mathscr{X}| = 2$,

(a) there exists only NCLTs having $2^n, n = 1, 2$-open sets i.e., τ_1 and τ_2.

(b) there exists one NCLT having 2^1-open sets and one NCLT having 2^2-open sets i.e., $\eta_{2^1} = 1 = S(2, 1)$ and $\eta_{2^2} = 1 = S(2, 2)$ respectively.

Case III: If $\mathscr{X} = \{u_1, v_1, w_1\}$ and $|\mathscr{X}| = 3$

Then $|\mathscr{N}_{\mathscr{X}}| = 2^3 = 8$. These are

$$0^{NT} = \langle \tfrac{u_1}{(0,1,1)}, \tfrac{v_1}{(0,1,1)}, \tfrac{w_1}{(0,1,1)} \rangle, 1^{NT} = \langle \tfrac{u_1}{(1,0,0)}, \tfrac{v_1}{(1,0,0)}, \tfrac{w_1}{(1,0,0)} \rangle,$$
$$A_1 = \langle \tfrac{u_1}{(1,0,0)}, \tfrac{v_1}{(0,1,1)}, \tfrac{w_1}{(0,1,1)} \rangle, A_2 = \langle \tfrac{u_1}{(0,1,1)}, \tfrac{v_1}{(1,0,0)}, \tfrac{w_1}{(0,1,1)} \rangle,$$
$$A_3 = \langle \tfrac{u_1}{(0,1,1)}, \tfrac{v_1}{(0,1,1)}, \tfrac{w_1}{(1,0,0)} \rangle, A_4 = \langle \tfrac{u_1}{(1,0,0)}, \tfrac{v_1}{(1,0,0)}, \tfrac{w_1}{(0,1,1)} \rangle,$$
$$A_5 = \langle \tfrac{u_1}{(1,0,0)}, \tfrac{v_1}{(0,1,1)}, \tfrac{w_1}{(1,0,0)} \rangle\ A_6 = \langle \tfrac{u_1}{(0,1,1)}, \tfrac{v_1}{(1,0,0)}, \tfrac{w_1}{(1,0,0)} \rangle.$$

In this case, the NCLTs are

$$\tau_1 = \{0^{NT}, 1^{NT}\},$$
$$\tau_2 = \{0^{NT}, A_1, A_6, 1^{NT}\},$$
$$\tau_3 = \{0^{NT}, A_2, A_5, 1^{NT}\},$$
$$\tau_4 = \{0^{NT}, A_3, A_4, 1^{NT}\},$$

$\tau_5 = \{0^{NT}, A_1, A_2, A_3, A_4, A_5, A_6, 1^{NT}\}$.

This shows that there are 2-open sets in τ_1, $4 = 2^2$-open sets in each of τ_2, τ_3, and τ_4, and $8 = 2^3$-open sets in τ_6. Therefore, for $|\mathscr{X}| = 3$,

(a) there exists only NCLTs having $2^n, n = 1, 2, 3$-open sets.

(b) there exists one NCLT having 2^1-open sets, three NCLTs having 2^2-open sets, and one NCLT having 2^3-open sets i.e., $\eta_{2^1} = 1 = S(3, 1), \eta_{2^2} = 3 = S(3, 2)$, and $\eta_{2^3} = 1 = S(3, 3)$ respectively.

Continuing in this way, it is seen that for $|\mathscr{X}| = n$ (finite) and $\mathscr{M} = \{(0, 1, 1), (1, 0, 0)\}$,

(i) there exists only NCLTs having $2^n, n = 1, 2, 3, \ldots, n$-open sets.

(ii) $\eta_{2^1} = 1 = S(n, 1)$, $\eta_{2^2} = S(n, 2)$, $\eta_{2^3} = S(n, 3)$,$\ldots, \eta_{2^n} = S(n, n)$.

Hence, $\mathscr{N}_{\mathscr{X}}$ contains only NCLTs having 2^k-open sets and the number of NCLTs having 2^k-open sets is $S(n, k), k = 1, 2, \ldots, n$.

Table 4.4: Number of NCLTSs on $\mathscr{X}$ with neutrosophic values in $\mathscr{M} = \{(0, 1, 1), (1, 0, 0)\}$

$\mathscr{M} = \{(0,1,1),(1,0,0)\}$	Number of NCLTSs having k-open sets					
$\lvert\mathscr{X}\rvert$	$k = 2^1$	$k = 2^2$	$k = 2^3$	$k = 2^4$	$\ldots$	$k = 2^n$
1	$S(1,1)$	-	-	-	-	-
2	$S(2,1)$	$S(2,2)$	-	-	-	
3	$S(3,1)$	$S(3,2)$	$S(3,3)$	-	-	-
4	$S(4,1)$	$S(4,2)$	$S(4,3)$	$S(4,4)$	-	-
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
n	$S(n,1)$	$S(n,2)$	$S(n,3)$	$S(n,4)$	$\cdots$	$S(n,n)$

Proposition 4.4.5

Let $|\mathscr{X}| = n$, $\mathscr{M} = \{(0, 1, 1), (T, I, F), (F, 1 - I, T), (1, 0, 0)\}$, where $T \neq F; T, I, F \in [0, 1]$, then the number of NCLTS having odd number of open sets is always zero.

Proof:

For NCLT, it is found that if $\mathscr{M}$ consists of $(1, 0, 0), (0, 1, 1)$, and other neutrosophic values which are not complement to each other, then $\mathscr{M}$, $|\mathscr{M}| = m > 2$, is equivalent to $\mathscr{M} = \{(1, 0, 0), (0, 1, 1)\}$. Here, $\mathscr{M} =$

$\{(0,1,1),(T,I,F),(F,1-I,T),(1,0,0)\}$, where $T \neq F$, and so, following Proposition 4.4.4, it is clear that the number of NCLTS having odd number of open sets is zero.

Remark 4.4.2

There is no clopen topologies having odd number of open sets in general topology but in case of NT we may have NCLTs consisting of odd number of open sets.

Example 4.4.1

Let $\mathscr{X} = \{p,q\}$ *and* $\mathscr{M} = \{(0,1,1),(0.5,0.5,0.5),(1,0,0)\}$. *Then* $\rho_1 = \{0^{NT}, 1^{NT}, A_1\}$, *where* $A_1 = \langle \frac{p}{(0.5,0.5,0.5)}, \frac{q}{(0.5,0.5,0.5)} \rangle$ *is a NCLT on* $\mathscr{X}$ *having 3-open sets, i.e., odd number of open set. On the other hand, in case of general topology, for* $\mathscr{X} = \{p,q\}$, *if* $\tau = \{\phi, \mathscr{X}, A\}$, *where* $A = \{p\}$, *then* τ *is a topology on* $\mathscr{X}$, *but it is not a clopen topology as* $A^{'} = \{q\} \notin \tau$. *Here, the only clopen topologies on* $\mathscr{X}$ *are* $\tau_1 = \{\phi, \mathscr{X}\}$ *and* $\tau_2 = \{\phi, \mathscr{X}, \{p\}, \{q\}\}$.

4.5 Number of Neutrosophic Clopen Bitopological Spaces

Definition 4.5.1

A neutrosophic clopen bitopological space (NCLBTS) is a triplet $(\mathscr{X}, \tau_i, \tau_j)$, *where* τ_i *and* τ_j *are any two NCLTs on* $\mathscr{X}$.

Remark 4.5.1

In this section, the NCLBTS with repetition refers to the NCLBTS of the form $(\mathscr{X}, \tau_i, \tau_j)$, *where* τ_i *and* τ_j *are identical or non-identical NCLTs, and the NCLBTS without repetition refers to the NCLBTS of the form* $(\mathscr{X}, \tau_i, \tau_j)$, *where* τ_i *and* τ_j *are non-identical NCLTs.*

Definition 4.5.2

Let τ_i *and* τ_j *be two NCLTs having* i *and* j*-open sets respectively. Then* $(\mathscr{X}, \tau_i, \tau_j)$ *is called a NCLBTS having* (i,j)*-open sets.*

Example 4.5.1

If $\mathscr{X} = \{p, q, r\}$ *and if we consider the family* $\tau_1 = \{0^{NT}, 1^{NT}\}$ *and* $\tau_2 = \{0^{NT}, 1^{NT}, A_1\}$,

where $A_1 = \langle \frac{p}{(0.7,0.1,0.5)}, \frac{q}{(0.5,0.2,0.3)}, \frac{r}{(0.3,0.4,0.4)} \rangle$,

$A_2 = \langle \frac{p}{(0.5,0.5,0.5)}, \frac{q}{(0.5,0.5,0.5)}, \frac{r}{(0.5,0.5,0.5)} \rangle$.

Then $(\mathscr{X}, \tau_1)$ *and* $(\mathscr{X}, \tau_2)$ *form NCLTSs. Therefore,* $(\mathscr{X}, \tau_1, \tau_2)$ *is a NCLBTS on* $\mathscr{X}$.

Proposition 4.5.1

For $k = l, \eta_B^{CL}(n, m, k)$*, the number of NCLBTSs having* (k, l)*-open sets on* $\mathscr{X}$, $|\mathscr{X}| = n$ *(finite), whose neutrosophic values lie in* $\mathscr{M}$ *with* $|\mathscr{M}| = m$ *is obtained by the formulae*

(i) with repetition is $\eta_B^{CL}(n, m, k) = \frac{1}{2}({\eta_k}^2 + \eta_k)$ *or* $\frac{1}{2}({\eta_l}^2 + \eta_l)$.

(ii) without repetition is $\eta_B^{CL}(n, m, k) = \frac{1}{2}({\eta_k}^2 - \eta_k)$ *or* $\frac{1}{2}({\eta_l}^2 - \eta_l)$.

Proof:

(i) Let $\mathscr{X}$ be a finite set having n elements and $\eta_k = m_1 = \eta_l$ as $k = l$. Let $\tau_1, \tau_2, \ldots, \tau_{m_1}$ be such NCLTs on $\mathscr{X}$.
Let $\mathfrak{B}_i$ be the collection of NCLBTSs on $\mathscr{X}$ having (k, k)-open sets or (l, l)-open sets with repetition, where

$$\mathfrak{B}_i = \{(\mathscr{X}, \tau_i, \tau_j) : 1 \leq j \leq i\}, i = 1, 2, \ldots, m_1.$$

Then,

$$\mathfrak{B}_1 = \{(\mathscr{X}, \tau_1, \tau_1)\},$$
$$\mathfrak{B}_2 = \{(\mathscr{X}, \tau_2, \tau_1), (\mathscr{X}, \tau_2, \tau_2)\},$$
$$\mathfrak{B}_3 = \{(\mathscr{X}, \tau_3, \tau_1), (\mathscr{X}, \tau_3, \tau_2), (\mathscr{X}, \tau_3, \tau_3)\},$$
$$\vdots$$
$$\mathfrak{B}_{m_1} = \{(\mathscr{X}, \tau_{m_1}, \tau_1), (\mathscr{X}, \tau_{m_1}, \tau_2), \ldots, (\mathscr{X}, \tau_{m_1}, \tau_{m_1})\}.$$

This shows that $|\mathfrak{B}_1| = 1$, $|\mathfrak{B}_2| = 2$, …, $|\mathfrak{B}_{m_1}| = m_1$ and $\bigcup_{i=1}^{m_1} \mathfrak{B}_i$ contains all the NCLBTSs having (k, k)-open sets on $\mathscr{X}$ with repetition.

Therefore, $\eta_B^{CL}(n,m,k) = \text{cardinality of } \bigcup_{i=1}^{m_1} \mathfrak{B}_i$

$= \sum_{i=1}^{m_1} |\mathfrak{B}_i|$, as $\mathfrak{B}_i$ are distinct

$= |\mathfrak{B}_1| + |\mathfrak{B}_2| + \ldots + |\mathfrak{B}_{m_1}|$

$= 1 + 2 + 3 + \ldots + m_1$

$= \frac{m_1(m_1+1)}{2}$

$= \frac{1}{2}({\eta_k}^2 + \eta_k)$.

Hence, the number of NCLBTSs having (k,k)-open sets on $\mathscr{X}$ is

$$\eta_B^{CL}(n,m,k) = \frac{1}{2}({\eta_k}^2 + \eta_k).$$

(ii) Let $\mathscr{X}$ be a finite set having n elements and $\eta_k = \eta_l$ as $k = l$.

Let $\tau_1, \tau_2, \ldots, \tau_{\eta_k}$ be such NCLTs on $\mathscr{X}$.

Let $\mathfrak{B}_i$ be the collection of NCLBTSs on $\mathscr{X}$ having (k,k)-open sets without repetition, where

$$\mathfrak{B}_i = \{(\mathscr{X}, \tau_i, \tau_j) : i < j \leq \eta_k\}, i = 1, 2, \ldots, \eta_k - 1.$$

Then,

$$\mathfrak{B}_1 = \Big\{(\mathscr{X}, \tau_1, \tau_2), (\mathscr{X}, \tau_1, \tau_3), (\mathscr{X}, \tau_1, \tau_4), \ldots, (\mathscr{X}, \tau_1, \tau_{\eta_k})\Big\},$$
$$\mathfrak{B}_2 = \Big\{(\mathscr{X}, \tau_2, \tau_3), (\mathscr{X}, \tau_2, \tau_4), (\mathscr{X}, \tau_2, \tau_5), \ldots, (\mathscr{X}, \tau_2, \tau_{\eta_k})\Big\},$$
$$\mathfrak{B}_3 = \Big\{(\mathscr{X}, \tau_3, \tau_4), (\mathscr{X}, \tau_3, \tau_5), (\mathscr{X}, \tau_3, \tau_6), \ldots, (\mathscr{X}, \tau_3, \tau_{\eta_k})\Big\},$$
$$\vdots$$
$$\mathfrak{B}_{\eta_k-2} = \{(\mathscr{X}, \tau_{\eta_k-2}, \tau_{\eta_k-1}), (\mathscr{X}, \tau_{\eta_k-2}, \tau_{\eta_k})\},$$
$$\mathfrak{B}_{\eta_k-1} = \{(\mathscr{X}, \tau_{\eta_{k-1}}, \tau_{\eta_k})\}.$$

This shows that

$|\mathfrak{B}_1| = \eta_k - 1, |\mathfrak{B}_2| = \eta_k - 2, \ldots, |\mathfrak{B}_{\eta_k-2}| = 2, |\mathfrak{B}_{\eta_k-1}| = 1$ and $\bigcup_{i=1}^{\eta_k-1} \mathfrak{B}_i$ contains all the NCLBTSs having (k,k)-open sets without repetition on $\mathscr{X}$. Therefore,

$\eta_B^{CL}(n,m,k) = \text{cardinality of } \bigcup_{i=1}^{\eta_k-1} \mathfrak{B}_i$

$= \sum_{i=1}^{\eta_k-1} |\mathfrak{B}_i|$, as A_i are distinct

$= |\mathfrak{B}_1| + |\mathfrak{B}_2| + \ldots + |\mathfrak{B}_{\eta_k-1}|$

$= (\eta_k - 1) + (\eta_k - 2) + \ldots + 2 + 1$

$= 1 + 2 + \ldots + (\eta_k - 2) + (\eta_k - 1)$

$= \frac{\eta_k(\eta_k-1)}{2}$
$= \frac{1}{2}({\eta_k}^2 - \eta_k).$

Hence, the number of NCLBTSs having (k, k)-open sets without repetition on $\mathscr{X}$ is $\eta_B^{CL}(n, m, k) = \frac{1}{2}({\eta_k}^2 - \eta_k)$.

Example 4.5.2

For $k = l$

(i) Let $k = 2 = l$, $|\mathscr{X}| = n(finite)$, *and* $\mathscr{M} = \{(0, 1, 1), (1, 0, 0), (T, I, F)\}$, $T, I, F \in [0, 1]$.

In this case, we have only one NCLT having 2*-open sets which is the indiscrete topology, i.e.,* $\tau = \{0^{NT}, 1^{NT}\}$. *So, the number of NCLTS having* $k = 2$*-open sets* $\eta_k = \eta_2 = 1$.

Therefore, the NCLBTSs having $(k, k) = (2, 2)$*-open sets*

(a) *with repetition:* $(\mathscr{X}, \tau, \tau)$ *and hence* $\eta_B^{CL}(n, 3, 2) = 1 = \frac{1}{2}({\eta_2}^2 + \eta_2)$.

(b) *without repetition: None, and hence* $\eta_B^{CL}(n, 3, 2) = 0 = \frac{1}{2}({\eta_2}^2 - \eta_2)$.

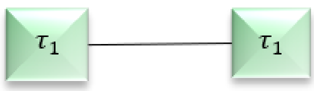

Figure 4.8: NCLBTSs for Example 4.5.2 (i)

(ii) Let $k = 4 = l$, $|\mathscr{X}| = 2$, *and* $\mathscr{M} = \{(0, 1, 1), (1, 0, 0), (0.5, 0.5, 0.5)\}$.

In this case, the NCLTs having $k = 4$*-open sets is* $\eta_k = \eta_2 = 3$ *say* τ_1, τ_2, τ_3.

Therefore, the NCLBTSs having $(k, k) = (4, 4)$*-open sets*

(a) *with repetition:*

$(\mathscr{X}, \tau_1, \tau_1), (\mathscr{X}, \tau_1, \tau_2), (\mathscr{X}, \tau_1, \tau_3),$
$(\mathscr{X}, \tau_2, \tau_2), (\mathscr{X}, \tau_2, \tau_3),$
$(\mathscr{X}, \tau_3, \tau_3).$

Here, the number of NCLBTSs having $(k, k) = (4, 4)$*-open sets with repetition* $= 6 = \frac{1}{2}(3^2 + 3)$.

(b) without repetition:

$$(\mathscr{X}, \tau_1, \tau_2), (\mathscr{X}, \tau_1, \tau_3),$$
$$(\mathscr{X}, \tau_2, \tau_3).$$

Here, the number of NCLBTSs having $(k, k) = (4, 4)$*-open sets without repetition* $= 3 = \frac{1}{2}(3^2 - 3)$.

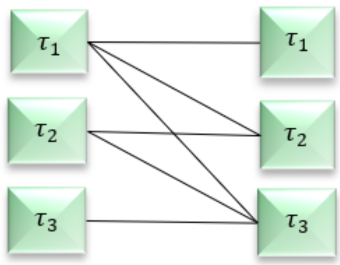

Figure 4.9: NCLBTSs for Example 4.5.2 (ii)

Proposition 4.5.2

If $|\mathscr{X}| = n$, $|\mathscr{M}| = m$, *and* $k \neq l$, *then the number of NCLBTSs whose neutrosophic values lie in* $\mathscr{M}$ *having* (k, l)*-open sets is* $\eta_B^{CL}(n, m, k, l) = \eta_k.\eta_l$.

Proof:

Let $\eta_k = m_1$ and τ_1,$\tau_2, \ldots, \tau_{m_1}$ be such NCLTs on $\mathscr{X}$.

Also, let $\eta_l = m_2$ and $\rho_1, \rho_2, \rho_3, \ldots, \rho_{m_2}$ be such NCLTs on $\mathscr{X}$.

Let $\mathfrak{B}_i = \{(\mathscr{X}, \tau_i, \rho_j) : j = 1, 2, 3, \ldots m_2\}, i = 1, 2, 3, \ldots m_1$.

Then,

$$\mathfrak{B}_1 = \Big\{(\mathscr{X}, \tau_1, \rho_1), (\mathscr{X}, \tau_1, \rho_2), (\mathscr{X}, \tau_1, \rho_3), \ldots, (\mathscr{X}, \tau_1, \rho_{m_2})\Big\},$$
$$\mathfrak{B}_2 = \Big\{(\mathscr{X}, \tau_2, \rho_1), (\mathscr{X}, \tau_2, \rho_2), (\mathscr{X}, \tau_2, \rho_3), \ldots, (\mathscr{X}, \tau_2, \rho_{m_2})\Big\},$$
$$\mathfrak{B}_3 = \Big\{(\mathscr{X}, \tau_3, \rho_1), (\mathscr{X}, \tau_3, \rho_2), (\mathscr{X}, \tau_3, \rho_3), \ldots, (\mathscr{X}, \tau_3, \rho_{m_2})\Big\},$$
$$\vdots$$
$$\mathfrak{B}_{m_1-1} = \left\{\begin{array}{c} (\mathscr{X}, \tau_{m_1-1}, \rho_1), (\mathscr{X}, \tau_{m_1-1}, \rho_2), \\ (\mathscr{X}, \tau_{m_1-1}, \rho_3), \ldots, (\mathscr{X}, \tau_{m_1-1}, \rho_{m_2}) \end{array}\right\},$$
$$\mathfrak{B}_{m_1} = \left\{\begin{array}{c} (\mathscr{X}, \tau_{m_1}, \rho_1), (\mathscr{X}, \tau_{m_1}, \rho_2), (\mathscr{X}, \tau_{m_1}, \rho_3), \\ \ldots, (\mathscr{X}, \tau_{m_1}, \rho_{m_2}) \end{array}\right\}.$$

Here, $|\mathfrak{B}_1| = m_2, |\mathfrak{B}_2| = m_2, \ldots, |\mathfrak{B}_{m_1-1}| = m_2, |\mathfrak{B}_{m_1}| = m_2$.
We see that $\bigcup_{i=1}^{m_1} \mathfrak{B}_i$ contains all the NCLBTSs having (k,l)-open sets on $\mathscr{X}$. So,

The required number of NCLBTSs in $\mathscr{X}$ $=$ cardinality of $\bigcup_{i=1}^{m_1} \mathfrak{B}_i$
$= \sum_{i=1}^{m_1} |\mathfrak{B}_i|$, as $\mathfrak{B}_i$ are distinct
$= |\mathfrak{B}_1| + |\mathfrak{B}_2| + \ldots + |\mathfrak{B}_{m_1}|$
$= m_2 + m_2 + \ldots + m_2$
$= m_1.m_2$
$= \eta_k.\eta_l.$

Hence, the required number of NCLBTSs on $\mathscr{X}$ is $\eta_{\mathscr{B}}^{CL}(n,m,k,l) = \eta_k.\eta_l$.

Corollary 4.5.1

If $k \neq l, \eta_k = \eta_l$, then the number of NCLBTSs having (k,l)-open sets is η_k^2 or η_l^2.

Proof:

Putting $\eta_k = \eta_l$ in the above Proposition 4.5.2 we get the required result of corollary.

Example 4.5.3

Let $k = 4, l = 5$, $\mathscr{X} = \{a,b,c\}$, and $\mathscr{M} = \{(0,1,1),(1,0,0),(0.5,0.5,0.5)\}$.
In this case, $\eta_k = \eta_4 = 9$ and $\eta_l = \eta_5 = 6$.
Let τ_1, τ_2, τ_3, τ_4, τ_5, τ_6, τ_7, τ_8, τ_9 be the NCLTs having 4-open sets and ρ_1, ρ_2, ρ_3, ρ_4, ρ_5, ρ_6 be the NCLTs having 5-open sets.
Then, the NCLBTSs having $(4,5)$-open sets are
$(\mathscr{X},\tau_1,\rho_1),(\mathscr{X},\tau_1,\rho_2),(\mathscr{X},\tau_1,\rho_3),(\mathscr{X},\tau_1,\rho_4),(\mathscr{X},\tau_1,\rho_5),(\mathscr{X},\tau_1,\rho_6),$
$(\mathscr{X},\tau_2,\rho_1),(\mathscr{X},\tau_2,\rho_2),(\mathscr{X},\tau_2,\rho_3),(\mathscr{X},\tau_2,\rho_4),(\mathscr{X},\tau_2,\rho_5),(\mathscr{X},\tau_2,\rho_6),$
$(\mathscr{X},\tau_3,\rho_1),(\mathscr{X},\tau_3,\rho_2),(\mathscr{X},\tau_3,\rho_3),(\mathscr{X},\tau_3,\rho_4),(\mathscr{X},\tau_3,\rho_5),(\mathscr{X},\tau_3,\rho_6),$
$(\mathscr{X},\tau_4,\rho_1),(\mathscr{X},\tau_4,\rho_2),(\mathscr{X},\tau_4,\rho_3),(\mathscr{X},\tau_4,\rho_4),(\mathscr{X},\tau_4,\rho_5),(\mathscr{X},\tau_4,\rho_6),$
$(\mathscr{X},\tau_5,\rho_1),(\mathscr{X},\tau_5,\rho_2),(\mathscr{X},\tau_5,\rho_3),(\mathscr{X},\tau_5,\rho_4),(\mathscr{X},\tau_5,\rho_5),(\mathscr{X},\tau_5,\rho_6),$
$(\mathscr{X},\tau_6,\rho_1),(\mathscr{X},\tau_6,\rho_2),(\mathscr{X},\tau_6,\rho_3),(\mathscr{X},\tau_6,\rho_4),(\mathscr{X},\tau_6,\rho_5),(\mathscr{X},\tau_6,\rho_6),$

$(\mathscr{X}, \tau_7, \rho_1), (\mathscr{X}, \tau_7, \rho_2), (\mathscr{X}, \tau_7, \rho_3), (\mathscr{X}, \tau_7, \rho_4), (\mathscr{X}, \tau_7, \rho_5), (\mathscr{X}, \tau_7, \rho_6),$
$(\mathscr{X}, \tau_8, \rho_1), (\mathscr{X}, \tau_8, \rho_2), (\mathscr{X}, \tau_8, \rho_3), (\mathscr{X}, \tau_8, \rho_4), (\mathscr{X}, \tau_8, \rho_5), (X, \tau_8, \rho_6),$
$(\mathscr{X}, \tau_9, \rho_1), (\mathscr{X}, \tau_9, \rho_2), (\mathscr{X}, \tau_9, \rho_3), (\mathscr{X}, \tau_9, \rho_4), (\mathscr{X}, \tau_9, \rho_5), (\mathscr{X}, \tau_9, \rho_6).$
Therefore, the number of NCLBTSs having $(4, 5)$-open sets $= 54 = 9.6 = \eta_4.\eta_5 = \eta_k.\eta_l$.

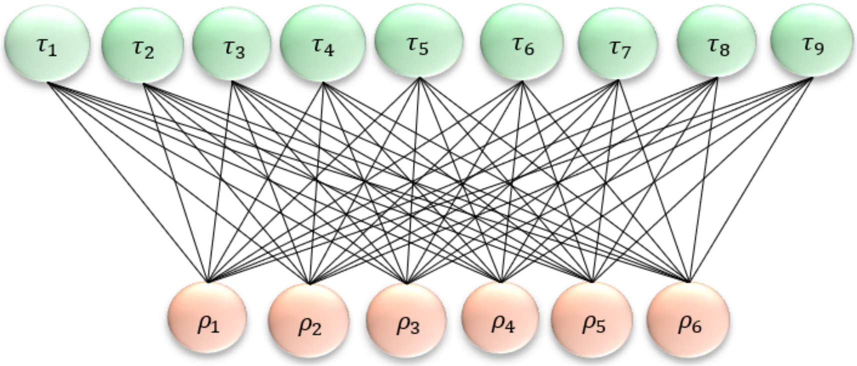

Figure 4.10: NCLBTSs having $(4, 5)$-open sets for Example 4.5.3

Example 4.5.4

For $k \neq l, \eta_k = \eta_l$, let the number of NCLTs having k-open sets and l-open sets on a finite set $\mathscr{X}$ be three i.e., $\eta_k = \eta_l = 3$.
Let the NCLTs having k-open sets be τ_1, τ_2, τ_3 and the NCLTs having l-open sets be ρ_1, ρ_2, ρ_3. Then, the NCLBTS having (k, l)-open sets are

$(\mathscr{X}, \tau_1, \rho_1), (\mathscr{X}, \tau_1, \rho_2), (\mathscr{X}, \tau_1, \rho_3),$
$(\mathscr{X}, \tau_2, \rho_1), (\mathscr{X}, \tau_2, \rho_2), (\mathscr{X}, \tau_2, \rho_3),$
$(\mathscr{X}, \tau_3, \rho_1), (\mathscr{X}, \tau_3, \rho_2), (\mathscr{X}, \tau_3, \rho_3).$

Therefore, the number of NCLBTSs having (k, l)-open sets $= 9 = 3.3 = \eta_k.\eta_k = \eta_l.\eta_l = {\eta_k}^2 = {\eta_l}^2$.

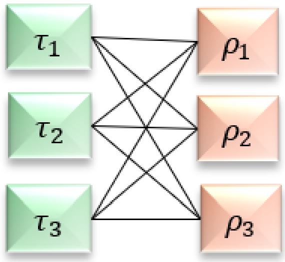

Figure 4.11: NCLBTSs having (k, l)-open sets for Example 4.5.4

Proposition 4.5.3

If $|\mathscr{X}| = n$ and $|\mathscr{M}| = m$, then the total number of NCLBTSs whose neutrosophic values lie in $\mathscr{M}$ having k & l-open sets is

(i) with repetition $\frac{1}{2}[(n_k + n_l)^2 + (n_k + n_l)]$.

(ii) without repetition $\frac{1}{2}[(n_k + n_l)^2 - (n_k + n_l)]$.

Proof:

(i) The total number of NCLBTSs having k & l-open sets with repetition includes:

(k, k)-open sets with repetition: Following Proposition 4.5.1 (i),

$$\eta_B^{CL}(n, m, k) = \frac{1}{2}({\eta_k}^2 + \eta_k).$$

(l, l)-open sets with repetition: Following Proposition 4.5.1 (i),

$$\eta_B^{CL}(n, m, l) = \frac{1}{2}({\eta_l}^2 + \eta_l).$$

(k, l)-open sets: Following Proposition 4.5.2,

$$\eta_B^{CL}(n, m, k, l) = \eta_k.\eta_l.$$

Therefore, the total number of NCLBTSs is obtained by

$$\frac{1}{2}({\eta_k}^2 + \eta_k) + \frac{1}{2}({\eta_l}^2 + \eta_l) + \eta_k.\eta_l = \frac{1}{2}\left[(n_k + n_l)^2 + (n_k + n_l)\right].$$

(ii) The total number of NCLBTSs having k & l-open sets without repetition includes:

(k, k)-open sets without repetition: Following Proposition 4.5.1 (ii),

$$\eta_B^{CL}(n, m, k) = \frac{1}{2}({\eta_k}^2 - \eta_k).$$

(l,l)-open sets without repetition: Following Proposition 4.5.1 (ii),

$$\eta_B^{CL}(n,m,l) = \frac{1}{2}({\eta_l}^2 - \eta_l).$$

(k,l)-open sets: Following Proposition 4.5.2,

$$\eta_B^{CL}(n,m,k,l) = \eta_k.\eta_l.$$

Therefore, the required total number of NCLBTSs is obtained by

$$\frac{1}{2}({\eta_k}^2-\eta_k)+\frac{1}{2}({\eta_l}^2-\eta_l)+\eta_k.\eta_l = \frac{1}{2}\left[(n_k+n_m)^2-(n_k+n_m)\right].$$

Example 4.5.5

For $k \neq l, \eta_k \neq \eta_l$

Let $\mathscr{X} = \{a,b\}, \mathscr{M} = \{(0,1,1),(1,0,0),(0.5,0.5,0.5)\}$ *and* $k=3, l=4$.

In this case, $\eta_k = \eta_3 = 1$ *and* $\eta_k = \eta_4 = 3$.

Let the NCLTs having 3*-open sets be* τ_1 *and the NCLTs having* 4*-open sets be* ρ_1,ρ_2,ρ_3.

The NCLBTS having $(3,3)$*-open sets with repetition is* $(\mathscr{X},\tau_1,\tau_1)$.

The NCLBTSs having $(4,4)$*-open sets with repetition are*

$(\mathscr{X},\rho_1,\rho_1),(\mathscr{X},\rho_1,\rho_2),(\mathscr{X},\rho_1,\rho_3),$
$(\mathscr{X},\rho_2,\rho_2),(\mathscr{X},\rho_2,\rho_3),$
$(\mathscr{X},\rho_3,\rho_3).$

Then, the NCLBTSs having $(3,4)$*-open sets are*

$(\mathscr{X},\tau_1,\rho_1),(\mathscr{X},\tau_1,\rho_2),(\mathscr{X},\tau_1,\rho_3).$

Therefore, the number of NCLBTSs having 3 *&* 4*-open sets*

(i) with repetition $= 10 = \frac{1}{2}[(1+3)^2+(1+3)] = \frac{1}{2}[(n_k+n_l)^2+(n_k+n_l)]$.

(ii) without repetition $= 6 = \frac{1}{2}[(1+3)^2-(1+3)] = \frac{1}{2}[(n_k+n_l)^2-(n_k+n_l)]$.

4.6 Number of Neutrosophic Clopen Tritopological Spaces

Definition 4.6.1

A neutrosophic clopen tritopological space (NCLTRS) is a quadruple $(\mathscr{X}, \tau_i, \tau_j, \tau_k)$, where τ_i, τ_j, and τ_k are any three NCLTs on $\mathscr{X}$.

Remark 4.6.1

In this section, the NCLTRS with repetition refers to the NCLTRS of the form $(\mathscr{X}, \tau_i, \tau_j, \tau_k)$, where τ_i, τ_j, and τ_k are identical or non-identical NCLTs, and the NCLTRS without repetition refers to the NCLTRS of the form $(\mathscr{X}, \tau_i, \tau_j, \tau_k)$, where τ_i, τ_j, and τ_k are non-identical NCLTs.

Definition 4.6.2

Let τ_i, τ_j, and τ_k be the NCLTs having i, j, and k-open sets respectively. Then $(\mathscr{X}, \tau_i, \tau_j, \tau_k)$ is called a NCLTRS having (i, j, k)-open sets.

Example 4.6.1

Let $\mathscr{X} = \{p, q, r\}$ and if we consider the family $\tau_1 = \{0^{NT}, 1^{NT}\}$, $\tau_2 = \{0^{NT}, 1^{NT}, A_1\}$ and $\tau_3 = \{0^{NT}, 1^{NT}, A_3, A_4\}$, where

$$A_1 = \langle \frac{p}{(0.7,0.1,0.5)}, \frac{q}{(0.5,0.2,0.3)}, \frac{r}{(0.3,0.4,0.4)} \rangle,$$
$$A_2 = \langle \frac{p}{(0.5,0.5,0.5)}, \frac{q}{(0.5,0.5,0.5)}, \frac{r}{(0.5,0.5,0.5)} \rangle,$$
$$A_3 = \langle \frac{p}{(0.1,0.5,0.5)}, \frac{q}{(0.2,0.5,0.6)}, \frac{r}{(0.5,0.7,0.5)} \rangle,$$
$$A_4 = \langle \frac{p}{(0.5,0.5,0.1)}, \frac{q}{(0.6,0.5,0.2)}, \frac{r}{(0.5,0.3,0.5)} \rangle.$$

Then, $(\mathscr{X}, \tau_1), (\mathscr{X}, \tau_2)$, and $(\mathscr{X}, \tau_3)$ form a NCLTSs.

Therefore, $(\mathscr{X}, \tau_1, \tau_2, \tau_3)$ is a NCLTRS on $\mathscr{X}$.

Proposition 4.6.1

Let $|\mathscr{X}| = n$ and $|\mathscr{M}| = m$, then the number of NCLTRSs having (k, k, k)-open sets whose neutrosophic values lie in $\mathscr{M}$ is obtained by

(i) with repetition is $\mathscr{T}_T^{CL}(n, m, k) = \frac{1}{6}({\eta_k}^3 + 3{\eta_k}^2 + 2\eta_k)$.

(ii) without repetition is $\mathscr{T}_T^{CL}(n, m, k) = \frac{1}{6}({\eta_k}^3 - 3{\eta_k}^2 + 2\eta_k)$.

Proof:

(i) Let $\mathscr{X}$ be a finite set having n elements and the number of NCLTs having k-open sets be η_k.
Let $\tau_1, \tau_2, \ldots, \tau_{\eta_k}$ be such NCLTs on $\mathscr{X}$.
Let $\mathfrak{T}_i$ be the collection of NCLTRSs on $\mathscr{X}$ having (k,k,k)-open sets with repetition, where
$\mathfrak{T}_i = \{(\mathscr{X}, \tau_p, \tau_q, \tau_r) : p = i, 1 \leq q \leq p,\ q \leq r \leq p\},\ i = 1, 2, \ldots, \eta_k$.
Then,

$$\mathfrak{T}_1 = \{(\mathscr{X}, \tau_1, \tau_1, \tau_1)\},$$

$$\mathfrak{T}_2 = \begin{Bmatrix} (\mathscr{X}, \tau_2, \tau_1, \tau_1), (\mathscr{X}, \tau_2, \tau_1, \tau_2), \\ (\mathscr{X}, \tau_2, \tau_2, \tau_2) \end{Bmatrix},$$

$$\mathfrak{T}_3 = \begin{Bmatrix} (\mathscr{X}, \tau_3, \tau_1, \tau_1), (\mathscr{X}, \tau_3, \tau_1, \tau_2), (\mathscr{X}, \tau_3, \tau_1, \tau_3), \\ (\mathscr{X}, \tau_3, \tau_2, \tau_2), (\mathscr{X}, \tau_3, \tau_2, \tau_3), \\ (\mathscr{X}, \tau_3, \tau_3, \tau_3) \end{Bmatrix},$$

$$\vdots$$

$$\mathfrak{T}_{\eta_k} = \begin{Bmatrix} (\mathscr{X}, \tau_{\eta_k}, \tau_1, \tau_1), (\mathscr{X}, \tau_{\eta_k}, \tau_1, \tau_2), \ldots, (\mathscr{X}, \tau_{\eta_k}, \tau_1, \tau_{\eta_k}), \\ (\mathscr{X}, \tau_{\eta_k}, \tau_2, \tau_2), (\mathscr{X}, \tau_{\eta_k}, \tau_2, \tau_3), \ldots, (\mathscr{X}, \tau_{\eta_k}, \tau_2, \tau_{\eta_k}), \\ \vdots \\ (\mathscr{X}, \tau_{\eta_k}, \tau_{\eta_k}, \tau_{\eta_k}) \end{Bmatrix}.$$

Here, $|\mathfrak{T}_1| = 1, |\mathfrak{T}_2| = 3 = 1+2, |\mathfrak{T}_3| = 6 = 1+2+3, \ldots, |\mathfrak{T}_{\eta_k}| = 1+2+3+\ldots+\eta_k$. Also, $\bigcup_{i=1}^{\eta_k} \mathfrak{T}_i$ contains all the NCLTRSs having (k,k,k)-open sets with repetition on $\mathscr{X}$.
Therefore,

$$\begin{aligned}
\mathscr{T}_T^{CL}(n,m,k) &= \text{cardinality of } \textstyle\bigcup_{i=1}^{\eta_k} \mathfrak{T}_i \\
&= \textstyle\sum_{i=1}^{\eta_k} |\mathfrak{T}_i|, \text{ as } \mathfrak{T}_i \text{ are distinct} \\
&= |\mathfrak{T}_1| + |\mathfrak{T}_2| + \ldots + |\mathfrak{T}_{\eta_k}| \\
&= 1+(1+2)+(1+2+3)+\ldots(1+2+\ldots+\eta_k) \\
&= \tfrac{\eta_k(\eta_k+1)(\eta_k+2)}{6} \\
&= \tfrac{1}{6}({\eta_k}^3 + 3{\eta_k}^2 + 2\eta_k).
\end{aligned}$$

Hence, the number of NCLTRSs having (k, k, k)-open sets on $\mathscr{X}$ with repetition is $\mathscr{T}_T^{CL}(n, m, k) = \frac{1}{6}({\eta_k}^3 + 3{\eta_k}^2 + 2\eta_k)$.

(ii) Let $\mathscr{X}$ be a finite set having n elements and the number of NCLTs having k-open sets be η_k.
Let $\tau_1, \tau_2, \ldots, \tau_{\eta_k}$ be such NCLTs on $\mathscr{X}$.
Let $\mathfrak{T}_i$ be the collection of NCLTRSs on $\mathscr{X}$ having (k, k, k)-open sets without repetition, where
$\mathfrak{T}_i = \{(\mathscr{X}, \tau_p, \tau_q, \tau_r) : p = i, p < q < r \leq \eta_k\}, i = 1, 2, \ldots, \eta_k - 2.$
Then,

$$\mathfrak{T}_1 = \left\{\begin{array}{c} (\mathscr{X}, \tau_1, \tau_2, \tau_3), (\mathscr{X}, \tau_1, \tau_2, \tau_4), \ldots, (\mathscr{X}, \tau_1, \tau_2, \tau_{\eta_k}), \\ (\mathscr{X}, \tau_1, \tau_3, \tau_4), \ldots, (\mathscr{X}, \tau_1, \tau_3, \tau_{\eta_k}), \\ \vdots \\ (\mathscr{X}, \tau_1, \tau_{\eta_k-1}, \tau_{\eta_k}) \end{array}\right\},$$

$$\mathfrak{T}_2 = \left\{\begin{array}{c} (\mathscr{X}, \tau_2, \tau_3, \tau_4), (\mathscr{X}, \tau_2, \tau_3, \tau_5), \ldots, (\mathscr{X}, \tau_2, \tau_3, \tau_{\eta_k}), \\ (\mathscr{X}, \tau_2, \tau_4, \tau_5), \ldots, (\mathscr{X}, \tau_2, \tau_4, \tau_{\eta_k}), \\ \vdots \\ (\mathscr{X}, \tau_2, \tau_{\eta_k-1}, \tau_{\eta_k}) \end{array}\right\},$$

$$\vdots$$

$$\mathfrak{T}_{\eta_k-3} = \left\{\begin{array}{c} (\mathscr{X}, \tau_{\eta_k-3}, \tau_{\eta_k-2}, \tau_{\eta_k-1}), (\mathscr{X}, \tau_{\eta_k-3}, \tau_{\eta_k-2}, \tau_{\eta_k}), \\ (\mathscr{X}, \tau_{\eta_k-3}, \tau_{\eta_k-1}, \tau_{\eta_k}) \end{array}\right\},$$

$$\mathfrak{T}_{\eta_k-2} = \left\{(\mathscr{X}, \tau_{\eta_k-2}, \tau_{\eta_k-1}, \tau_{\eta_k})\right\}.$$

Here, $|\mathfrak{T}_1| = 1+2+\ldots+(\eta_k-2), |\mathfrak{T}_2| = 1+2+\ldots+(\eta_k-3), \ldots,$
$|\mathfrak{T}_{\eta_k-3}| = 1 + 2, |\mathfrak{T}_{\eta_k-2}| = 1.$
Also, $\bigcup_{i=1}^{\eta_k-2} \mathfrak{T}_i$ contains all the NCLTRSs having (k, k, k)-open sets without repetition on $\mathscr{X}$.
Therefore, in this case,

$$\begin{aligned} \mathscr{T}_T^{CL}(n, m, k) &= \text{cardinality of } \textstyle\bigcup_{i=1}^{\eta_k-2} \mathfrak{T}_i \\ &= \textstyle\sum_{i=1}^{\eta_k-2} |\mathfrak{T}_i|, \text{ as } \mathfrak{T}_i \text{ are distinct} \\ &= |\mathfrak{T}_1| + |\mathfrak{T}_2| + \ldots + |\mathfrak{T}_{\eta_k-2}| \\ &= 1+(1+2)+(1+2+3)+\ldots+\{1+2+\ldots+(\eta_k-2)\} \end{aligned}$$

$= \frac{\eta_k(\eta_k-1)(\eta_k-2)}{6}$
$= \frac{1}{6}({\eta_k}^3 - 3{\eta_k}^2 + 2\eta_k)$.

Hence, the number of NCLTRSs having (k, k, k)-open sets on $\mathscr{X}$ without repetition is $\mathscr{T}_T^{CL}(n, m, k) = \frac{1}{6}({\eta_k}^3 - 3{\eta_k}^2 + 2\eta_k)$.

Example 4.6.2

(i) For $k = 2$*, let* $\mathscr{X} = \{a, b\}$ *and* $\mathscr{M} = \{(0, 1, 1), (1, 0, 0), (0.5, 0.5, 0.5)\}$*. In this case, we have only one NCLT having two open sets which is indiscrete topology say* $\tau = \{0^{NT}, 1^{NT}\}$*. So, the number of NCLTs having* $k = 2$*-open sets* $\eta_k = \eta_2 = 1$*. Therefore, the NCLTRS having* $(2, 2, 2)$*-open sets is* $(\mathscr{X}, \tau, \tau, \tau)$*. Hence, the number of NCLTRS is one.*

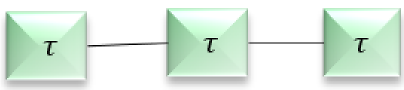

Figure 4.12: NCLTRS having $(2, 2, 2)$-open sets for Example 4.6.2

(ii) For $k = 4$*, let* $|\mathscr{X}| = 2$ *and* $\mathscr{M} = \{(0, 1, 1), (1, 0, 0), (0.5, 0.5, 0.5)\}$*. In this case, the number of NCLTs having* $k = 4$*-open sets is* $\eta_k = \eta_4 = 3$ *say* τ_1, τ_2, τ_3*. Therefore, the NCLTRSs having* $(4, 4, 4)$*-open sets*

(a) *with repetition:*
$(\mathscr{X}, \tau_1, \tau_1, \tau_1), (\mathscr{X}, \tau_1, \tau_1, \tau_2), (\mathscr{X}, \tau_1, \tau_1, \tau_3),$
$(\mathscr{X}, \tau_1, \tau_2, \tau_2), (\mathscr{X}, \tau_1, \tau_2, \tau_3),$
$(\mathscr{X}, \tau_1, \tau_3, \tau_3),$
$(\mathscr{X}, \tau_2, \tau_2, \tau_2), (\mathscr{X}, \tau_2, \tau_2, \tau_3),$
$(\mathscr{X}, \tau_2, \tau_3, \tau_3),$
$(\mathscr{X}, \tau_3, \tau_3, \tau_3).$
Here, the number of NCLTRSs having (4,4,4)-open sets with repetition $= 10 = 1 + (1 + 2) + (1 + 2 + 3) = \frac{1}{6}(3^3 + 3.3^2 + 2.3) = \frac{1}{6}({\eta_k}^3 + 3{\eta_k}^2 + 2\eta_k)$.

(b) without repetition: $(\mathscr{X}, \tau_1, \tau_2, \tau_3)$.

Here, the number of NCLTRSs having (4,4,4)-open sets without repetition $= 1 = \frac{1}{6}(3^3 - 3.3^2 + 2.3) = \frac{1}{6}({\eta_k}^3 - 3{\eta_k}^2 + 2\eta_k)$.

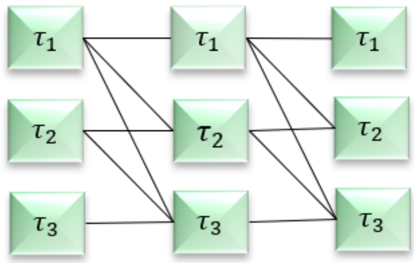

Figure 4.13: NCLTRSs having $(4,4,4)$-open sets for example 4.6.2

Proposition 4.6.2

For $k \neq l, \eta_k \neq \eta_l$*, the number of NCLTRSs having* (k,k,l)*-open sets on* $\mathscr{X}$*,* $|\mathscr{X}| = n$ *(finite), whose neutrosophic values lie in* $\mathscr{M}$ *with* $|\mathscr{M}| = m$ *for with and without repetition are obtained respectively by the formulae*

(i) $\mathscr{T}_T^{CL}(n,m,k,l) = \frac{1}{2}[\eta_k\eta_l(\eta_k + 1)]$.

(ii) $\mathscr{T}_T^{CL}(n,m,k,l) = \frac{1}{2}[\eta_k\eta_l(\eta_k - 1)]$.

Proof:

(i) Let $\mathscr{X}$ be a finite set having n elements and the number of NCLTs having k-open sets be η_k and the number of NCLTs having l-open sets be η_l.

Let $\tau_1, \tau_2, \ldots, \tau_{\eta_k}$ be the NCLTs on $\mathscr{X}$ having k-open sets and ρ_1, $\rho_2, \ldots, \rho_{\eta_l}$ be the NCLTs on $\mathscr{X}$ having l-open sets.

Let A_i be the collection of NCLTRSs having (k,k,l)-open sets with repetition, where

$A_i = \{(\mathscr{X}, \tau_i, \tau_j, \rho_r) : 1 \leq j \leq i, 1 \leq r \leq \eta_l\}, i = 1,2,\ldots,\eta_k$.

Then,

$$A_1 = \Big\{(\mathscr{X}, \tau_1, \tau_1, \rho_1), (\mathscr{X}, \tau_1, \tau_1, \rho_2), \ldots, (\mathscr{X}, \tau_1, \tau_1, \rho_{\eta_l})\Big\},$$

$$A_2 = \begin{Bmatrix} (\mathscr{X}, \tau_2, \tau_1, \rho_1), (\mathscr{X}, \tau_2, \tau_1, \rho_2), \ldots, (\mathscr{X}, \tau_2, \tau_1, \rho_{\eta_l}), \\ (\mathscr{X}, \tau_2, \tau_2, \rho_1), (\mathscr{X}, \tau_2, \tau_2, \rho_2), \ldots, (\mathscr{X}, \tau_2, \tau_2, \rho_{\eta_l}) \end{Bmatrix},$$

$$\vdots$$

$$A_{\eta_k} = \begin{Bmatrix} (\mathscr{X}, \tau_{\eta_k}, \tau_1, \rho_1), (\mathscr{X}, \tau_{\eta_k}, \tau_1, \rho_2), \ldots, (\mathscr{X}, \tau_{\eta_k}, \tau_1, \rho_{\eta_l}), \\ (\mathscr{X}, \tau_{\eta_k}, \tau_2, \rho_1), (\mathscr{X}, \tau_{\eta_k}, \tau_2, \rho_2), \ldots, (\mathscr{X}, \tau_{\eta_k}, \tau_2, \rho_{\eta_l}), \\ \ldots \\ (\mathscr{X}, \tau_{\eta_k}, \tau_{\eta_k}, \rho_1), (\mathscr{X}, \tau_{\eta_k}, \tau_{\eta_k}, \rho_2), \ldots, (\mathscr{X}, \tau_{\eta_k}, \tau_{\eta_k}, \rho_{\eta_l}) \end{Bmatrix}.$$

Here, $|A_1| = \eta_l, |A_2| = 2\eta_l, |A_3| = 3\eta_l, \ldots, |A_{\eta_k}| = \eta_k\eta_l$.

We see that $\bigcup_{i=1}^{\eta_k} A_i$ contains all the NCLTRSs with repetition on $\mathscr{X}$.

So, the required number of NCLTRSs on $\mathscr{X}$ = cardinality of $\bigcup_{i=1}^{\eta_k} A_i$

$= \sum_{i=1}^{\eta_k} |A_i|$, as A_i are distinct

$= |A_1| + |A_2| + |A_3| + \ldots + |A_{\eta_k}|$

$= \eta_l + 2\eta_l + 3\eta_l + \ldots + \eta_k\eta_l$

$= (1 + 2 + 3 + \ldots + \eta_k)\eta_l$

$= \frac{\eta_k(\eta_k+1)}{2}.\eta_l$

$= \frac{\eta_k\eta_l(\eta_k+1)}{2}$

Hence, the number of NCLTRSs having (k, k, l)-open sets with repetition is $\mathscr{T}_T^{CL}(n, m, k, l) = \frac{\eta_k\eta_l(\eta_k+1)}{2}$.

(ii) Let $\mathscr{X}$ be a finite set having n elements and the number of NCLTs having k-open sets be η_k and the number of NCLTs having l-open sets be η_l.

Let $\tau_1, \tau_2, \ldots, \tau_{\eta_k}$ be the NCLTs on $\mathscr{X}$ having k-open sets and ρ_1, $\rho_2, \ldots, \rho_{\eta_l}$ be the NCLTs on $\mathscr{X}$ having l-open sets.

Let A_i be the collection of NCLTRSs having (k, k, l)-open sets without repetition, where

$A_i = \{(\mathscr{X}, \tau_i, \tau_j, \tau_r) : i < j \leq \eta_k, 1 \leq r \leq \eta_l\}, i = 1, 2, \ldots, \eta_k - 1.$

Then,

$$A_1 = \left\{ \begin{array}{c} (\mathscr{X}, \tau_1, \tau_2, \rho_1), (\mathscr{X}, \tau_1, \tau_3, \rho_1), \ldots, (\mathscr{X}, \tau_1, \tau_{\eta_k}, \rho_1), \\ (\mathscr{X}, \tau_1, \tau_2, \rho_2), (\mathscr{X}, \tau_1, \tau_3, \rho_2), \ldots, (\mathscr{X}, \tau_1, \tau_{\eta_k}, \rho_2), \\ \vdots \\ (\mathscr{X}, \tau_1, \tau_2, \rho_{\eta_l}), (\mathscr{X}, \tau_1, \tau_3, \rho_{\eta_l}), \ldots, (\mathscr{X}, \tau_1, \tau_{\eta_k}, \rho_{\eta_l}) \end{array} \right\},$$

$$A_2 = \left\{ \begin{array}{c} (\mathscr{X}, \tau_2, \tau_3, \rho_1), (\mathscr{X}, \tau_2, \tau_4, \rho_1), \ldots, (\mathscr{X}, \tau_2, \tau_{\eta_k}, \rho_1), \\ (\mathscr{X}, \tau_2, \tau_3, \rho_2), (\mathscr{X}, \tau_2, \tau_4, \rho_2), \ldots, (\mathscr{X}, \tau_2, \tau_{\eta_k}, \rho_2), \\ \vdots \\ (\mathscr{X}, \tau_2, \tau_3, \rho_{\eta_l}), (\mathscr{X}, \tau_2, \tau_4, \rho_{\eta_l}), \ldots, (\mathscr{X}, \tau_2, \tau_{\eta_k}, \rho_{\eta_l}) \end{array} \right\},$$

$$\vdots$$

$$A_{\eta_k-2} = \left\{ \begin{array}{c} (\mathscr{X}, \tau_{\eta_{k-2}}, \tau_{\eta_{k-1}}, \rho_1), (\mathscr{X}, \tau_{\eta_{k-2}}, \tau_{\eta_k}, \rho_1), \\ (\mathscr{X}, \tau_{\eta_{k-2}}, \tau_{\eta_{k-1}}, \rho_2), (\mathscr{X}, \tau_{\eta_{k-2}}, \tau_{\eta_k}, \rho_2), \\ \vdots \\ (\mathscr{X}, \tau_{\eta_{k-2}}, \tau_{\eta_{k-1}}, \rho_{\eta_1}), (\mathscr{X}, \tau_{\eta_{k-2}}, \tau_{\eta_k}, \rho_{\eta_1}) \end{array} \right\},$$

$$A_{\eta_k-1} = \left\{ \begin{array}{c} (\mathscr{X}, \tau_{\eta_{k-1}}, \tau_{\eta_k}, \rho_1), \\ (\mathscr{X}, \tau_{\eta_{k-1}}, \tau_{\eta_k}, \rho_2), \\ \vdots \\ (\mathscr{X}, \tau_{\eta_{k-1}}, \tau_{\eta_k}, \rho_{\eta_l}) \end{array} \right\}.$$

Here, $|A_1| = (\eta_k - 1)\eta_l$, $|A_2| = (\eta_k - 2)\eta_l$, $\ldots$, $|A_{\eta_{k-2}}| = 2\eta_l$, $|A_{\eta_{k-1}}| = \eta_l$.

We see that $\bigcup_{i=1}^{\eta_k-1} A_i$ contains all the NCLTRSs without repetition on $\mathscr{X}$.

So, the required number of NCLTRSs on $\mathscr{X}$ = cardinality of $\bigcup_{i=1}^{\eta_k-1} A_i$

$= \sum_{i=1}^{\eta_k-1} |A_i|$, as A_i are distinct

$= |A_1| + |A_2| + \ldots + |A_{\eta_k-2}| + |A_{\eta_k-1}|$

$= (\eta_k - 1)\eta_l + (\eta_k - 2)\eta_l + \ldots + 2\eta_l + \eta_l$

$= [1 + 2 + 3 + \ldots + (\eta_k - 1)]\eta_l$

$= \frac{\eta_k(\eta_k-1)}{2}.\eta_l$

$= \frac{\eta_k\eta_l(\eta_k-1)}{2}.$

Hence, the number of NCLTRSs having (k, k, l)-open sets without repetition is $\mathscr{T}_T^{CL}(n, m, k, l) = \frac{\eta_k\eta_l(\eta_k-1)}{2}$.

Proposition 4.6.3

For $k \neq l, \eta_k \neq \eta_l$, the total number of NCLTRSs on $\mathscr{X}$ having k & l-open sets with repetition is obtained by the formula

$$\frac{1}{6}[(\eta_k + \eta_l)^3 + 3(\eta_k + \eta_l)^2 + 2(\eta_k + \eta_l)].$$

Proof:

The total number of NCLTRSs having k & l-open sets with repetition includes:

(i) (k, k, k)-open sets with repetition: Following Proposition 4.6.1(i), the number of NCLTRSs is computed by

$$\mathscr{T}_T^{CL}(n, m, k) = \frac{\eta_k(\eta_k+1)(\eta_k+2)}{6}.$$

(ii) (l, l, l)-open sets with repetition: Following Proposition 4.6.1(i),

$$\mathscr{T}_T^{CL}(n, m, l) = \frac{\eta_l(\eta_l+1)(\eta_l+2)}{6}.$$

(iii) (k, k, l)-open sets with repetition: Following Proposition 4.6.2(i),

$$\mathscr{T}_T^{CL}(n, m, k, l) = \frac{\eta_k\eta_l(\eta_k+1)}{2}.$$

(iv) (l, l, k)-open sets with repetition: Following Proposition 4.6.2(i),

$$\mathscr{T}_T^{CL}(n, m, l, k) = \frac{\eta_k\eta_l(\eta_l+1)}{2}.$$

Therefore, adding results from (i) to (iv), the required total number of NCLTRSs having k & l-open sets with repetition is obtained as

$$\frac{1}{6}[(\eta_k + \eta_l)^3 + 3(\eta_k + \eta_l)^2 + 2(\eta_k + \eta_l)].$$

Proposition 4.6.4 *For $k \neq l, \eta_k \neq \eta_l$, the number of NCLTRSs having k & l-open sets without repetition is obtained by the formula*

$$\frac{1}{6}[(\eta_k + \eta_l)^3 - 3(\eta_k + \eta_l)^2 + 2(\eta_k + \eta_l)].$$

Proof:

The total number of NCLTRSs having k & l-open sets without repetition includes:

(i) (k, k, k)-open sets without repetition: Following Proposition 4.6.1(ii),

$$\mathscr{T}_T^{CL}(n, m, k) = \frac{\eta_k(\eta_k-1)(\eta_k-2)}{6}.$$

(ii) (l, l, l)-open sets without repetition: Following Proposition 4.6.1(ii),

$$\mathscr{T}_T^{CL}(n, m, l) = \frac{\eta_l(\eta_l-1)(\eta_l-2)}{6}.$$

(iii) (k, k, l)-open sets without repetition: Following Proposition 4.6.2(ii),

$$\mathscr{T}_T^{CL}(n, m, k, l) = \frac{\eta_k\eta_l(\eta_k-1)}{2}.$$

(iv) (l, l, k)-open sets without repetition: Following Proposition 4.6.2(ii),

$$\mathscr{T}_T^{CL}(n, m, l, k) = \frac{\eta_k\eta_l(\eta_l-1)}{2}.$$

Therefore, adding results from (i) to (iv), the required total number of NCLTRSs having k & l-open sets without repetition is obtained as

$$\frac{1}{6}[(\eta_k + \eta_l)^3 - 3(\eta_k + \eta_l)^2 + 2(\eta_k + \eta_l)].$$

Example 4.6.3

For $k \neq l, \eta_k \neq \eta_l$

(i) Let $k = 3, l = 4$, $\mathscr{X} = \{a, b\}$, *and* $\mathscr{M} = \{(0, 1, 1), (1, 0, 0), (0.5, 0.5, 0.5)\}$.

In this case, $\eta_k = \eta_3 = 1$ *and* $\eta_k = \eta_4 = 3$.

Let the NCLT having 3-open sets be τ_1 *and the NCLTs having* 4*-open sets be* ρ_1, ρ_2, ρ_3.

Then, the NCLTRS having $(3, 3, 3)$*-open sets is* $(\mathscr{X}, \tau_1, \tau_1, \tau_1)$.

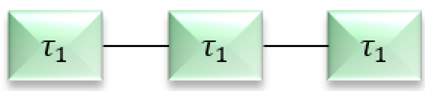

Figure 4.14: NCLTRS having $(3, 3, 3)$-open sets for Example 4.6.3

The NCLTRSs having $(4, 4, 4)$*-open sets are*

$$(\mathscr{X}, \rho_1, \rho_1, \rho_1), (\mathscr{X}, \rho_1, \rho_1, \rho_2), (\mathscr{X}, \rho_1, \rho_1, \rho_3),$$
$$(\mathscr{X}, \rho_1, \rho_2, \rho_2), (\mathscr{X}, \rho_1, \rho_2, \rho_3), (\mathscr{X}, \rho_1, \rho_3, \rho_3),$$
$$(\mathscr{X}, \rho_2, \rho_2, \rho_2), (\mathscr{X}, \rho_2, \rho_2, \rho_3), (\mathscr{X}, \rho_2, \rho_3, \rho_3),$$
$$(\mathscr{X}, \rho_3, \rho_3, \rho_3).$$

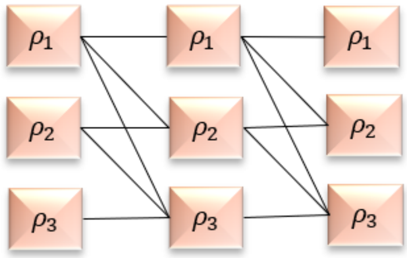

Figure 4.15: NCLTRSs having $(4, 4, 4)$-open sets for Example 4.6.3

The NCLTRSs having $(3, 3, 4)$-*open sets are*

$$(\mathscr{X}, \tau_1, \tau_1, \rho_1), (\mathscr{X}, \tau_1, \tau_1, \rho_2), (\mathscr{X}, \tau_1, \tau_1, \rho_3).$$

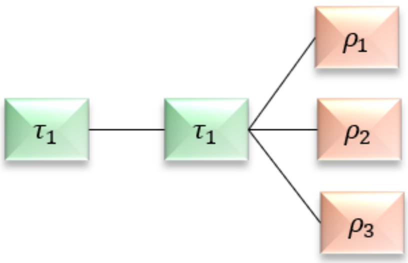

Figure 4.16: NCLTRSs having $(3, 3, 4)$-open sets for Example 4.6.3

The NCLTRSs having $(4, 4, 3)$-*open sets are*

$$(\mathscr{X}, \rho_1, \rho_1, \tau_1), (\mathscr{X}, \rho_1, \rho_2, \tau_1), (\mathscr{X}, \rho_1, \rho_3, \tau_1),$$
$$(\mathscr{X}, \rho_2, \rho_2, \tau_1), (\mathscr{X}, \rho_2, \rho_3, \tau_1), (\mathscr{X}, \rho_3, \rho_3, \tau_1).$$

Here, the total number of NCLTRSs having 3 *&* 4-*open sets*

$$= 20 = 1 + 10 + 3 + 6 = \frac{1}{6}[(1+3)^3 + 3.(1+3)^2 + 2.(1+3)].$$

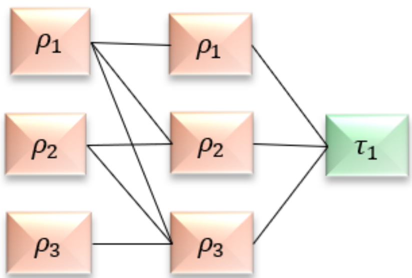

Figure 4.17: NCLTRSs having $(4, 4, 3)$-open sets for Example 4.6.3

(ii) Let $k = 4, l = 5$*,* $\eta_k = \eta_4 = 2$ *and* $\eta_l = \eta_5 = 3$*.*
Let the NCLTs having 4*-open sets be* τ_1, τ_2 *and the NCLTs having* 5*-open sets are* ρ_1, ρ_2, ρ_3*.*
Then, the NCLTRSs having $(4, 4, 4)$*-open sets are*

$$(\mathscr{X}, \tau_1, \tau_1, \tau_1), (\mathscr{X}, \tau_1, \tau_1, \tau_2), (\mathscr{X}, \tau_1, \tau_2, \tau_2),$$
$$(\mathscr{X}, \tau_2, \tau_2, \tau_2).$$

The NCLTRSs having $(5, 5, 5)$*-open sets are*

$$(\mathscr{X}, \rho_1, \rho_1, \rho_1), (\mathscr{X}, \rho_1, \rho_1, \rho_2), (\mathscr{X}, \rho_1, \rho_1, \rho_3),$$
$$(\mathscr{X}, \rho_1, \rho_2, \rho_2), (\mathscr{X}, \rho_1, \rho_2, \rho_3), (\mathscr{X}, \rho_1, \rho_3, \rho_3),$$
$$(\mathscr{X}, \rho_2, \rho_2, \rho_2), (\mathscr{X}, \rho_2, \rho_2, \rho_3), (\mathscr{X}, \rho_2, \rho_3, \rho_3),$$
$$(\mathscr{X}, \rho_3, \rho_3, \rho_3).$$

The NCLTRSs having $(4, 4, 5)$*-open sets are*

$$(\mathscr{X}, \tau_1, \tau_1, \rho_1), (\mathscr{X}, \tau_1, \tau_1, \rho_2), (\mathscr{X}, \tau_1, \tau_1, \rho_3),$$
$$(\mathscr{X}, \tau_1, \tau_2, \rho_1), (\mathscr{X}, \tau_1, \tau_2, \rho_2), (\mathscr{X}, \tau_1, \tau_2, \rho_3),$$
$$(\mathscr{X}, \tau_2, \tau_2, \rho_1), (\mathscr{X}, \tau_2, \tau_2, \rho_2), (\mathscr{X}, \tau_2, \tau_2, \rho_3).$$

The NCLTRSs having (5,5,4)-open sets are

$$(\mathscr{X}, \rho_1, \rho_1, \tau_1), (\mathscr{X}, \rho_1, \rho_1, \tau_2), (\mathscr{X}, \rho_1, \rho_2, \tau_1),$$
$$(\mathscr{X}, \rho_1, \rho_2, \tau_2), (\mathscr{X}, \rho_1, \rho_3, \tau_1), (\mathscr{X}, \rho_1, \rho_3, \tau_2),$$
$$(\mathscr{X}, \rho_2, \rho_2, \tau_1), (\mathscr{X}, \rho_2, \rho_2, \tau_2), (\mathscr{X}, \rho_2, \rho_3, \tau_1),$$
$$(\mathscr{X}, \rho_2, \rho_3, \tau_2), (\mathscr{X}, \rho_3, \rho_3, \tau_1), (\mathscr{X}, \rho_3, \rho_3, \tau_2).$$

Here, the total number of NCLTRSs having 4 *and* 5*-open sets*

$$= 35 = 4 + 10 + 9 + 12 = \frac{1}{6}[(2+3)^3 + 3.(2+3)^2 + 2.(2+3)].$$

Proposition 4.6.5

For $k \neq l \neq \mathfrak{m}$ *, the number of NCLTRSs on* $\mathscr{X}$ *having* $(k, l, \mathfrak{m})$*-open sets is*

$$\mathscr{T}_T^{CL}(n, m, k, l, \mathfrak{m}) = \eta_k \eta_l \eta_{\mathfrak{m}}.$$

Proof:

Let the number of NCLTs having k-open sets be η_k, the number of NCLTs having l-open sets be η_l, and the number of NCLTs having $\mathfrak{m}$-open sets

be $\eta_{\mathfrak{m}}$.
Let $\tau_1, \tau_2, \ldots, \tau_{\eta_k}$ be the NCLTs on $\mathscr{X}$ having k-open sets, $\rho_1, \rho_2, \ldots, \rho_{\eta_l}$ be the NCLTs on $\mathscr{X}$ having l-open sets, and $\mu_1, \mu_2, \ldots, \mu_{\eta_{\mathfrak{m}}}$ be the NCLTs on $\mathscr{X}$ having $\mathfrak{m}$-open sets.
Let A_i be the collections of NCLTRSs having $(k, l, \mathfrak{m})$-open sets, where $A_i = \{(\mathscr{X}, \tau_i, \rho_j, \mu_r) : 1 \leq j \leq \eta_l, 1 \leq r \leq \eta_{\mathfrak{m}}\}, i = 1, 2, \ldots, \eta_k$.
Then,

$$A_1 = \left\{ \begin{array}{c} (\mathscr{X}, \tau_1, \rho_1, \mu_1), (\mathscr{X}, \tau_1, \rho_1, \mu_2), \ldots, (\mathscr{X}, \tau_1, \rho_1, \mu_{\eta_{\mathfrak{m}}}), \\ (\mathscr{X}, \tau_1, \rho_2, \mu_1), (\mathscr{X}, \tau_1, \rho_2, \mu_2), \ldots, (\mathscr{X}, \tau_1, \rho_2, \mu_{\eta_{\mathfrak{m}}}) \\ \vdots \\ (\mathscr{X}, \tau_1, \rho_{\eta_l}, \mu_1), (\mathscr{X}, \tau_1, \rho_{\eta_l}, \mu_2), \ldots, (\mathscr{X}, \tau_1, \rho_{\eta_l}, \mu_{\eta_{\mathfrak{m}}}) \end{array} \right\},$$

$$A_2 = \left\{ \begin{array}{c} (\mathscr{X}, \tau_2, \rho_1, \mu_1), (\mathscr{X}, \tau_2, \rho_1, \mu_2), \ldots, (\mathscr{X}, \tau_2, \rho_1, \mu_{\eta_{\mathfrak{m}}}), \\ (\mathscr{X}, \tau_2, \rho_2, \mu_1), (\mathscr{X}, \tau_2, \rho_2, \mu_2), \ldots, (\mathscr{X}, \tau_2, \rho_2, \mu_{\eta_{\mathfrak{m}}}), \\ \vdots \\ (\mathscr{X}, \tau_2, \rho_{\eta_l}, \mu_1), (\mathscr{X}, \tau_2, \rho_{\eta_l}, \mu_2), \ldots, (\mathscr{X}, \tau_2, \rho_{\eta_l}, \mu_{\eta_{\mathfrak{m}}}) \end{array} \right\},$$

$$\vdots$$

$$A_{\eta_k} = \left\{ \begin{array}{c} (\mathscr{X}, \tau_{\eta_k}, \rho_1, \mu_1), (\mathscr{X}, \tau_{\eta_k}, \rho_1, \mu_2), \ldots, (\mathscr{X}, \tau_{\eta_k}, \rho_1, \mu_{\eta_{\mathfrak{m}}}), \\ (\mathscr{X}, \tau_{\eta_k}, \rho_2, \mu_1), (\mathscr{X}, \tau_{\eta_k}, \rho_2, \mu_2), \ldots, (\mathscr{X}, \tau_{\eta_k}, \rho_2, \mu_{\eta_{\mathfrak{m}}}), \\ \vdots \\ (\mathscr{X}, \tau_{\eta_k}, \rho_{\eta_l}, \mu_1), (\mathscr{X}, \tau_{\eta_k}, \rho_{\eta_l}, \mu_2), \ldots, (\mathscr{X}, \tau_{\eta_k}, \rho_{\eta_l}, \mu_{\eta_{\mathfrak{m}}}) \end{array} \right\}.$$

Here, $|A_1| = \eta_l \eta_{\mathfrak{m}}, |A_2| = \eta_l \eta_{\mathfrak{m}}, \ldots, |A_{\eta_k}| = \eta_l \eta_{\mathfrak{m}}$.
We see that $\bigcup_{i=1}^{\eta_k} A_i$ contains all the NCLTRSs having $(k, l, \mathfrak{m})$-open sets on $\mathscr{X}$.
So, $\mathscr{T}_T^{CL}(n, m, k, l, \mathfrak{m}) =$ cardinality of $\bigcup_{i=1}^{\eta_k} A_i$

$= \sum_{i=1}^{\eta_k} |A_i|$, as A_i are distinct

$= |A_1| + |A_2| + \ldots + |A_{\eta_k}|$

$= \eta_l \eta_{\mathfrak{m}} + \eta_l \eta_{\mathfrak{m}} + \ldots + \eta_l \eta_{\mathfrak{m}}$

$= \eta_k \eta_l \eta_{\mathfrak{m}}$.

Corollary 4.6.1

For $k \neq l \neq \mathfrak{m}$ *, if* $\eta_k = \eta_l = \eta_{\mathfrak{m}}$*, the number of NCLTRSs having* $(k, l, \mathfrak{m})$*-open sets is*

$$\mathscr{T}_T^{CL}(n, m, k, l, \mathfrak{m}) = \eta_k^3 \text{ or } \eta_l^3 \text{ or } \eta_{\mathfrak{m}}^3.$$

Proof:

Putting $\eta_k = \eta_l = \eta_{\mathfrak{m}}$ in the Proposition 4.6.5, the required result is obtained.

Example 4.6.4

For $k \neq l \neq \mathfrak{m}, \eta_k = \eta_l = \eta_{\mathfrak{m}}$

Let $\mathscr{X} = \{a, b\}$*,* $\mathscr{M} = \{(0, 1, 1), (1, 0, 0), (0.5, 0.5, 0.5)\}$ *and* $k = 2, l = 3$*, and* $\mathfrak{m} = 9$.

Then, $\eta_k = \eta_l = \eta_{\mathfrak{m}} = 1$*, say* τ_1, ρ_1, μ_1 *are the NCLTSs having* k, l*,* $\mathfrak{m}$*-open sets respectively.*

The NCLTRSs having

(a) (k, k, k)*-open sets is* $(\mathscr{X}, \tau_1, \tau_1, \tau_1)$.

(b) (l, l, l)*-open sets is* $(\mathscr{X}, \rho_1, \rho_1, \rho_1)$.

(c) $(\mathfrak{m}, \mathfrak{m}, \mathfrak{m})$*-open sets is* $(\mathscr{X}, \mu_1, \mu_1, \mu_1)$.

(d) (k, k, l)*-open sets is* $(\mathscr{X}, \tau_1, \tau_1, \rho_1)$.

(e) $(k, k, \mathfrak{m})$ *open sets is* $(\mathscr{X}, \tau_1, \tau_1, \mu_1)$.

(f) (l, l, k)*-open sets is* $(\mathscr{X}, \rho_1, \rho_1, \tau_1)$.

(g) $(l, l, \mathfrak{m})$*-open sets is* $(\mathscr{X}, \rho_1, \rho_1, \mu_1)$.

(h) $(\mathfrak{m}, \mathfrak{m}, k)$*-open sets is* $(\mathscr{X}, \mu_1, \mu_1, \tau_1)$.

(i) $(\mathfrak{m}, \mathfrak{m}, l)$*-open sets is* $(\mathscr{X}, \mu_1, \mu_1, \rho_1)$.

(j) $(k, l, \mathfrak{m})$*-open sets is* $(\mathscr{X}, \tau_1, \rho_1, \mu_1)$.

Therefore, the total number of NCLTRSs is

$= 10 = \frac{1}{6}[(1+1+1)^3 + 3(1+1+1)^2 + 2(1+1+1)]$.

Proposition 4.6.6

For $k \neq l \neq \mathfrak{m}, \eta_k \neq \eta_l \neq \eta_\mathfrak{m}$, the number of NCLTRSs having k, l & $\mathfrak{m}$-open sets with repetition is obtained by the formula

$\frac{1}{6}[(\eta_k + \eta_l + \eta_\mathfrak{m})^3 + 3(\eta_k + \eta_l + \eta_\mathfrak{m})^2 + 2(\eta_k + \eta_l + \eta_\mathfrak{m})]$.

Proof:

The total number of NCLTRSs having k, l & $\mathfrak{m}$-open sets with repetition includes:

(i) (k, k, k)-open sets with repetition: Following Proposition 4.6.1(i),
$\mathscr{T}_T^{CL}(n, m, k) = \frac{\eta_k(\eta_k+1)(\eta_k+2)}{6}$.

(ii) (l, l, l)-open sets with repetition: Following Proposition 4.6.1(i),
$\mathscr{T}_T^{CL}(n, m, l) = \frac{\eta_l(\eta_l+1)(\eta_l+2)}{6}$.

(iii) $(\mathfrak{m}, \mathfrak{m}, \mathfrak{m})$-open sets with repetition: Following Proposition 4.6.1(i),
$\mathfrak{J}_T^{CL}(n, m, \mathfrak{m}) = \frac{\eta_\mathfrak{m}(\eta_\mathfrak{m}+1)(\eta_\mathfrak{m}+2)}{6}$

(iv) (k, k, l)-open sets and (k, k, m)-open sets with repetition: Following Proposition 4.6.2(i),
$\mathscr{T}_T^{CL}(n, m, k, l) = \frac{\eta_k\eta_l(\eta_k+1)}{2}$ and $\mathscr{T}_T^{CL}(n, m, k, \mathfrak{m}) = \frac{\eta_k\eta_\mathfrak{m}(\eta_k+1)}{2}$
respectively.

(v) (l, l, k)-open sets and $(l, l, \mathfrak{m})$-open sets with repetition: Following Proposition 4.6.2(i),
$\mathscr{T}_T^{CL}(n, m, l, k) = \frac{\eta_k\eta_l(\eta_l+1)}{2}$ and $\mathscr{T}_T^{CL}(n, m, l, \mathfrak{m}) = \frac{\eta_l\eta_\mathfrak{m}(\eta_l+1)}{2}$
respectively.

(vi) $(\mathfrak{m}, \mathfrak{m}, k)$-open sets and $(\mathfrak{m}, \mathfrak{m}, l)$-open sets with repetition: Following Proposition 4.6.2(i),
$\mathscr{T}_T^{CL}(n, m, \mathfrak{m}, k) = \frac{\eta_k\eta_\mathfrak{m}(\eta_\mathfrak{m}+1)}{2}$ and $\mathscr{T}_T^{CL}(n, m, \mathfrak{m}, l) = \frac{\eta_l\eta_\mathfrak{m}(\eta_\mathfrak{m}+1)}{2}$
respectively.

(vii) $(k, l, \mathfrak{m})$-open sets: By Proposition 4.6.5,

$$\mathscr{T}_T^{CL}(n, m, k, l, \mathfrak{m}) = \eta_k \eta_l \eta_{\mathfrak{m}}.$$

Therefore, adding results from (i) to (vii), the required total number of NCLTRSs is obtained as

$$\frac{1}{6}[(\eta_k + \eta_l + \eta_{\mathfrak{m}})^3 + 3(\eta_k + \eta_l + \eta_{\mathfrak{m}})^2 + 2(\eta_k + \eta_l + \eta_{\mathfrak{m}})].$$

Proposition 4.6.7

For $k \neq l \neq \mathfrak{m}, \eta_k \neq \eta_l \neq \eta_{\mathfrak{m}}$, the number of NCLTRSs having k, l & $\mathfrak{m}$-open sets without repetition is obtained by the formula

$$\frac{1}{6}[(\eta_k + \eta_l + \eta_{\mathfrak{m}})^3 - 3(\eta_k + \eta_l + \eta_{\mathfrak{m}})^2 + 2(\eta_k + \eta_l + \eta_{\mathfrak{m}})].$$

Proof:

The total number of NCLTRSs having k, l & $\mathfrak{m}$-open sets without repetition includes:

(i) (k, k, k)-open sets without repetition: Following Proposition 4.6.1(ii),

$$\mathscr{T}_T^{CL}(n, m, k) = \frac{\eta_k(\eta_k-1)(\eta_k-2)}{6}.$$

(ii) (l, l, l)-open sets without repetition: Following Proposition 4.6.1(ii),

$$\mathscr{T}_T^{CL}(n, m, l) = \frac{\eta_l(\eta_l-1)(\eta_l-2)}{6}$$

(iii) $(\mathfrak{m}, \mathfrak{m}, \mathfrak{m})$-open sets without repetition: Following Proposition 4.6.1(ii),

$$\mathscr{T}_T^{CL}(n, m, \mathfrak{m}) = \frac{\eta_{\mathfrak{m}}(\eta_{\mathfrak{m}}-1)(\eta_{\mathfrak{m}}-2)}{6}$$

(iv) (k, k, l)-open sets and $(k, k, \mathfrak{m})$-open sets without repetition: Following Proposition 4.6.2(ii),

$\mathscr{T}_T^{CL}(n, m, k, l) = \frac{\eta_k\eta_l(\eta_k-1)}{2}$ and $\mathscr{T}_T^{CL}(n, m, k, \mathfrak{m}) = \frac{\eta_k\eta_{\mathfrak{m}}(\eta_k-1)}{2}$ respectively.

(v) (l, l, k)-open sets and $(l, l, \mathfrak{m})$-open sets without repetition: Following Proposition 4.6.2(ii),

$\mathscr{T}_T^{CL}(n, m, l, k) = \frac{\eta_k\eta_l(\eta_l-1)}{2}$ and $\mathscr{T}_T^{CL}(n, m, l, \mathfrak{m}) = \frac{\eta_l\eta_{\mathfrak{m}}(\eta_l-1)}{2}$ respectively.

(vi) $(\mathfrak{m},\mathfrak{m},k)$-open sets and $(\mathfrak{m},\mathfrak{m},l)$-open sets: Following Proposition 4.6.2(ii),
$\mathscr{T}_T^{CL}(n,m,\mathfrak{m},k) = \frac{\eta_k\eta_{\mathfrak{m}}(\eta_{\mathfrak{m}}-1)}{2}$ and $\mathscr{T}_T^{CL}(n,m,\mathfrak{m},l) = \frac{\eta_l\eta_{\mathfrak{m}}(\eta_{\mathfrak{m}}-1)}{2}$ respectively.

(vii) $(k,l,\mathfrak{m})$-open sets: Following Proposition 4.6.5,

$$\mathscr{T}_T^{CL}(n,m,k,l,\mathfrak{m}) = \eta_k\eta_l\eta_{\mathfrak{m}}.$$

Therefore, adding results from (i) to (vii), the required total number of NCLTRSs is obtained as

$$\frac{1}{6}[(\eta_k+\eta_l+\eta_{\mathfrak{m}})^3 - 3(\eta_k+\eta_l+\eta_{\mathfrak{m}})^2 + 2(\eta_k+\eta_l+\eta_{\mathfrak{m}})].$$

Corollary 4.6.2

For $k \neq l \neq \mathfrak{m}, \eta_k = \eta_l = \eta_{\mathfrak{m}}$, the total number of NCLTRSs is obtained by

(i) with repetition: $\frac{1}{2}[9{\eta_k}^3 + 9{\eta_k}^2 + 2\eta_k]$,

(ii) without repetition: $\frac{1}{2}[9{\eta_k}^3 - 9{\eta_k}^2 + 2\eta_k]$.

Proof:

These results are obtained by substituting $\eta_k = \eta_l = \eta_{\mathfrak{m}}$ in Proposition 4.6.6 and Proposition 4.6.7 respectively.

Example 4.6.5

For $k \neq l \neq \mathfrak{m}, \eta_k \neq \eta_l \neq \eta_{\mathfrak{m}}$

Let us assume that, $\eta_k = 1, \eta_l = 2, \eta_{\mathfrak{m}} = 3$ and let τ_1 be the NCLTs having k-open sets, ρ_1,ρ_2 be the NCLTs having l-open sets and μ_1,μ_2,μ_3 be the NCLTs having $\mathfrak{m}$-open sets.

The NCLTRS having (k,k,k)-open sets is $(\mathscr{X},\tau_1,\tau_1,\tau_1)$.

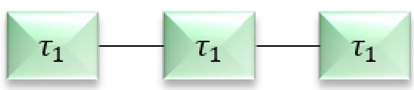

Figure 4.18: NCLTRSs having (k,k,k)-open sets for example 4.6.5

The NCLTRSs having (l, l, l)-open sets are

$(\mathscr{X}, \rho_1, \rho_1, \rho_1), (\mathscr{X}, \rho_1, \rho_1, \rho_2), (\mathscr{X}, \rho_1, \rho_2, \rho_2),$
$(\mathscr{X}, \rho_2, \rho_2, \rho_2).$

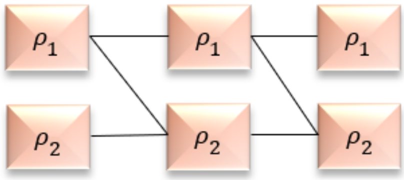

Figure 4.19: NCLTRSs having (l, l, l)-open sets for example 4.6.5

The NCLTRSs having $(\mathfrak{m}, \mathfrak{m}, \mathfrak{m})$-open sets are

$(\mathscr{X}, \mu_1, \mu_1, \mu_1), (\mathscr{X}, \mu_1, \mu_1, \mu_2), (\mathscr{X}, \mu_1, \mu_1, \mu_3),$
$(\mathscr{X}, \mu_1, \mu_2, \mu_2), (\mathscr{X}, \mu_1, \mu_2, \mu_3), (\mathscr{X}, \mu_1, \mu_3, \mu_3),$
$(\mathscr{X}, \mu_2, \mu_2, \mu_2), (\mathscr{X}, \mu_2, \mu_2, \mu_3), (\mathscr{X}, \mu_2, \mu_3, \mu_3),$
$(\mathscr{X}, \mu_3, \mu_3, \mu_3).$

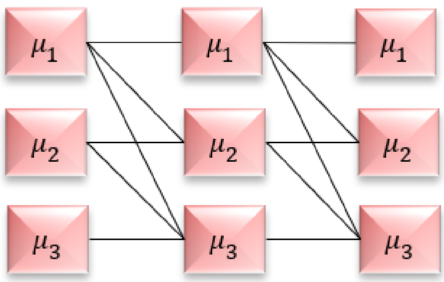

Figure 4.20: NCLTRSs having $(\mathfrak{m}, \mathfrak{m}, \mathfrak{m})$-open sets for example 4.6.5

The NCLTRSs having

(i) (k, k, l)*-open sets are* $(\mathscr{X}, \tau_1, \tau_1, \rho_1), (\mathscr{X}, \tau_1, \tau_1, \rho_2).$

(ii) $(k, k, \mathfrak{m})$*-open sets are*

$(\mathscr{X}, \tau_1, \tau_1, \mu_1), (\mathscr{X}, \tau_1, \tau_1, \mu_2), (\mathscr{X}, \tau_1, \tau_1, \mu_3).$

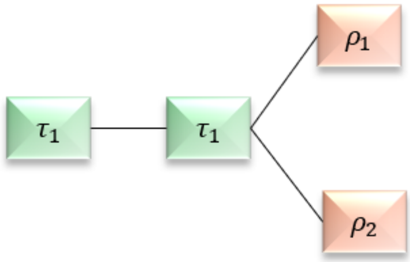

Figure 4.21: NCLTRSs having (k, k, l)-open sets for example 4.6.5

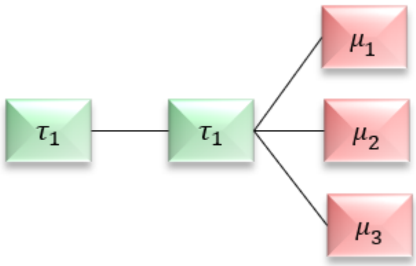

Figure 4.22: NCLTRSs having $(k, k, \mathfrak{m})$-open sets for example 4.6.5

The NCLTRSs having

(a) (l, l, k)*-open sets are*

$(\mathscr{X}, \rho_1, \rho_1, \tau_1), (\mathscr{X}, \rho_1, \rho_2, \tau_1), (\mathscr{X}, \rho_2, \rho_2, \tau_1)$*, and*

(b) $(l, l, \mathfrak{m})$*-open sets are*

$(\mathscr{X}, \rho_1, \rho_1, \mu_1), (\mathscr{X}, \rho_1, \rho_1, \mu_2), (\mathscr{X}, \rho_1, \rho_1, \mu_3),$
$(\mathscr{X}, \rho_1, \rho_2, \mu_1), (\mathscr{X}, \rho_1, \rho_2, \mu_2), (\mathscr{X}, \rho_1, \rho_2, \mu_3),$
$(\mathscr{X}, \rho_2, \rho_2, \mu_1), (\mathscr{X}, \rho_2, \rho_2, \mu_2), (\mathscr{X}, \rho_2, \rho_2, \mu_3).$

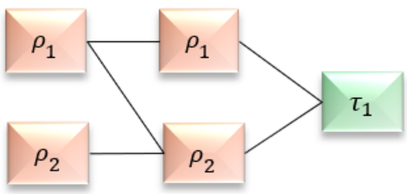

Figure 4.23: NCLTRSs having (l, l, k)-open sets for example 4.6.5

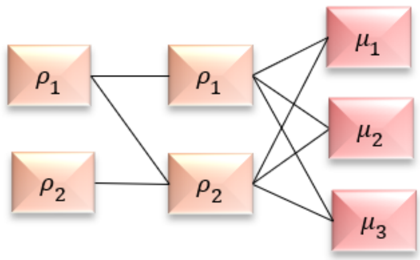

Figure 4.24: NCLTRSs having $(l, l, \mathfrak{m})$-open sets for example 4.6.5

The NCLTRSs having

(a) $(\mathfrak{m}, \mathfrak{m}, k)$*-open sets are*

$(\mathscr{X}, \mu_1, \mu_1, \tau_1), (\mathscr{X}, \mu_1, \mu_2, \tau_1), (\mathscr{X}, \mu_1, \mu_3, \tau_1),$
$(\mathscr{X}, \mu_2, \mu_2, \tau_1), (\mathscr{X}, \mu_2, \mu_3, \tau_1), (\mathscr{X}, \mu_3, \mu_3, \tau_1).$

(b) $(\mathfrak{m}, \mathfrak{m}, l)$*-open sets are*

$(\mathscr{X}, \mu_1, \mu_1, \rho_1), (\mathscr{X}, \mu_1, \mu_1, \rho_2), (\mathscr{X}, \mu_1, \mu_2, \rho_1), (\mathscr{X}, \mu_1, \mu_2, \rho_2),$
$(\mathscr{X}, \mu_1, \mu_3, \rho_1), (\mathscr{X}, \mu_1, \mu_3, \rho_2), (\mathscr{X}, \mu_2, \mu_2, \rho_1), (\mathscr{X}, \mu_2, \mu_2, \rho_2),$
$(\mathscr{X}, \mu_2, \mu_3, \rho_1), (\mathscr{X}, \mu_2, \mu_3, \rho_2), (\mathscr{X}, \mu_3, \mu_3, \rho_1), (\mathscr{X}, \mu_3, \mu_3, \rho_2).$

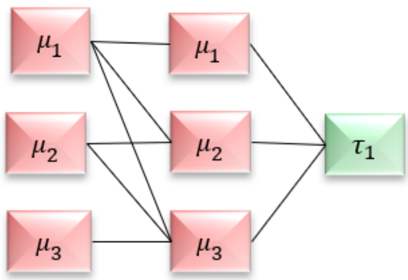

Figure 4.25: NCLTRSs having $(\mathfrak{m}, \mathfrak{m}, k)$-open sets for example 4.6.5

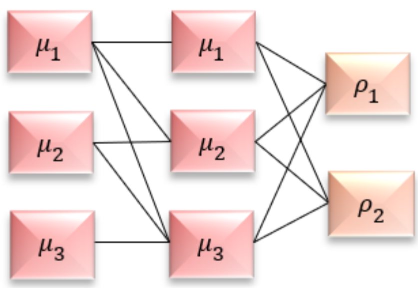

Figure 4.26: NCLTRSs having $(\mathfrak{m}, \mathfrak{m}, l)$-open sets for example 4.6.5

The NCLTRSs having $(k, l, \mathfrak{m})$-open sets are

$$(\mathscr{X}, \tau_1, \rho_1, \mu_1), (\mathscr{X}, \tau_1, \rho_1, \mu_2), (\mathscr{X}, \tau_1, \rho_1, \mu_3),$$
$$(\mathscr{X}, \tau_1, \rho_2, \mu_1), (\mathscr{X}, \tau_1, \rho_2, \mu_2), (\mathscr{X}, \tau_1, \rho_2, \mu_3).$$

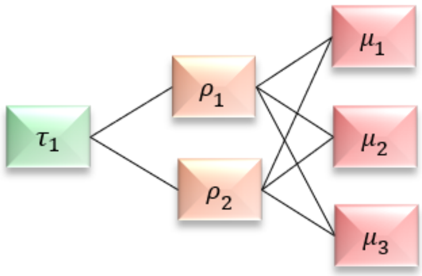

Figure 4.27: NCLTRSs having $(k, l, \mathfrak{m})$-open sets for example 4.6.5

Therefore, the total number of NCLTRSs having k, l *&* $\mathfrak{m}$*-open sets with repetition* $= 56 = \frac{1}{6}[(1+2+3)^3 + 3(1+2+3)^2 + 2(1+2+3)]$,
and the total number of NCLTRSs having k, l *&* $\mathfrak{m}$*-open sets without repetition* $= 20 = \frac{1}{6}[(1+2+3)^3 - 3(1+2+3)^2 + 2(1+2+3)]$.