A STUDY OF SOME COSMOLOGICAL MODELS IN SCALAR-TENSOR THEORY OF GRAVITATION

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Chapter 7

Conclusions

The cosmological models of the universe are derived in this research using scalar theories of gravitation, specifically Sáez-Ballester theory of gravitation with time dependent deceleration parameter.

In chapter-2, the interaction of an anisotropic Bianchi type-I string cosmological model universe with an electromagnetic field is examined in Sáez-Ballester theory. In this chapter we obtained the model, where $H > 0$ and $q < 0$, which shows that the universe expands after the big bang according to a power law-expansion and start with a finite volume at cosmic time $t = 0$ and expanding with an acceleration. For the model defined in this chapter, the relationship between Hubble's parameter and average scale factor assumed in equation (2.25) yields a constant negative value of deceleration parameter. At time $t = 0$, a point type (MacCallum, 1971) singularity exists in the derived model. The particles density and the string tension density are comparable, but tension density vanishes faster than particle density, indicating that particles dominate the universe as time increases. According to our hypothesis, the universe has a probability of becoming anisotropic at any time during its existence, from the beginning to the end. According to current study, there is a disparity in measuring microwave intensities emitted from various directions of the sky. This motivated us to examine the universe using the anisotropic Bianchi type-I metric to better describe our universe. Several CMB (de Bernardis and et al., 2000; Hanany and et al., 2000) anomalies, such as the inconsistency of temperature anisotropies in the CMB with the exact homogeneous and isotropic FRW model measured by COBE/WMAP (Bennett and et al., 2003) satellites, foregrounds/systematics, and exotic topologies, are also evidence that we live in a globally anisotropic universe. Shear decreases during inflation, eventually leading to an isotropic phase with no shear. Hence, in order to produce any significant amount of shear in recent times, one must induce anisotropy in space-time.

Chapter-3 deals using the S$\acute{a}$ez-Ballester theory of gravitation, cosmological models corresponding to perfect fluid dispersion with trace free matter source were created. The models obtained in five dimensions illustrate closed, open and flat FRW radiating perfect fluid types. At the initial epoch, all physical quantities diverges and vanishes for infinitely large values of cosmic time. At $t = 0$, the spatial volume is zero. It demonstrates that the evolution of our universe begins with the big bang scenario.The results obtained in chapter-3 will help us to understand spatially homogenous and isotropic five-dimensional universes.

In chapter-4, we investigated a five dimensional spatially homogenous and anisotropic Bianchi type-I in the context of S$\acute{a}$ez-Ballester's scalar-tensor theory of gravitation (Saez and Ballester, 1986). The field equations were precisely solved by employing a special law of variation of Hubble's parameter, which yields a constant value of the deceleration parameter. In case(I) and case(II), two accurate and physically valid Bianchi type-I models were obtained. A model recently obtained by Reddy et al. (Reddy et al., 2006b) is found to generalise the model gained in case (I). For both models, expressions for certain crucial cosmological parameters have been found, and the physical behaviour of the models has been thoroughly explained. All matter and radiation are concentrated at the big bang epoch for $n \neq 0$, and the cosmic expansion is powered by the big bang impulse. The scale factors and volume vanishes at the initial epoch, resulting in a point singularity in the model. After the big bang impulse, the universe has a solitary origin and exhibits power-law expansion. As $t \to \infty$, the rate of expansion slows and eventually drops to zero. As $t \to \infty$, the pressure, energy density and scalar field become negligible, and the scale factors and spatial volume become infinitely large, resulting in an essentially empty universe. The model has no real singularity for $n = 0$, because the density is finite. As a result, the universe had a non-singular origin, and cosmic expansion is fueled by matter particle formation. The universe expands at an exponential rate and uniformly. The universe is dominated by vacuum energy at late times in this model, which is thought to be responsible for cosmic expansion.

For large values of time, both models reflect a shearing, non-rotating, expanding universe that approaches isotropy. In both models, the scalar field decreases to zero as $t \to \infty$. If we set $h = 0$, the solutions simplify to general relativity solutions (Kumar and Singh, 2007), and S$\acute{a}$ez-Ballester's scalar-tensor theory of gravitation approaches the standard general theory of relativity in every way. As mentioned in case (I) and case (II), the solutions defined in the models are shown to be consistent with recent observations of type Ia supernovae. The results defined in this chapter may aid in a better understanding of the evolution of the universe in Bianchi type-I space-time using S$\acute{a}$ez-Ballester's theory of gravitation.

In chapter-5, we explored the spatially homogenous and anisotropic five dimensional Bianchi type-V cosmological model in the presence of bulk viscous fluid with one dimensional cosmic strings in S$\acute{a}$ez-Ballester's scalar-tensor theory of gravitation. It is discovered that the resultant model is singularity free, shearing, non-spinning and remains anisotropic throughout the universe's evolution. The physical parameters are infinite at $t = 0$ and tend to zero as $t \to \infty$. As *t* becomes infinitely large, the scalar field approaches zero. In this situation, the model decelerates in the standard way. With cosmic time, bulk viscosity falls, leading to an inflationary model. In this universe, cosmic strings do not survive. Considering the fact that scalar fields and bulk viscosity have a vital role in describing the early universe. In the context of the S$\acute{a}$ez-Ballester scalar-tensor theory of gravitation, the model presented here will help in a better understanding of the universe's evolution in five dimensional Bianchi type-V space-time.

In chapter-6, we have obtained cosmological models with the help of S$\acute{a}$ez-Ballester theory by using five dimensional FRW space-time in source of bulk viscous fluid which can be assumed as analogous radiating models in closed, open and flat space-times. For all the case [i.e. Case (i), Case (ii) and Case (iii)] we have obtained the models [i.e. equations (6.19), (6.26) and (6.30)] which is expanding and free from initial singularity. Here, at $t = 0$, the spatial volume $V = 0$ and thereafter increases continuously with the increase of cosmic time *t*, which represents the circumstances of the Big Bang. From the results obtained in this chapter will help us to understand S$\acute{a}$ez-Ballester theory in five dimension.

In this thesis we have investigated some cosmological models with electromagnetic field, perfect fluid, viscous fluid and string in S$\acute{a}$ez-Ballester theory of gravity. We obtained the model which is a shearing, non-rotating, expanding universe with constant acceleration that approaches isotropy at late time. The model is free from initial singularity. At initial time $t = 0$, the spatial volume is zero. It demonstrates that the evolution of our universe begins with the Big Bang scenario. We also obtained the model which is anisotropic at initial epoch but approaches isotropy after the evolution. The consideration of time-dependent deceleration parameters reflects models that induce a universe transition from the early decelerated phase to the recent accelerated phase. However string tension density vanishes quicker than particle density in chapter-2. This demonstrates that our model depicts a matter-dominated universe at late time period, which matches current observational data. Finally, the thesis solutions could be one of the probable candidates for describing the observed universe.

7.1 Future scope of the work

From the motivation of the above works and the findings of the recent work on S$\acute{a}$ez-Ballester theory, future scopes regarding the following works may be expected. The scopes are

1. Further study may explore different cosmological models.
2. Apart from S$\acute{a}$ez-Ballester theory one can study different scalar-tensor theory in higher dimension.
3. One can study different geometric techniques to analyse the exact solution in higher dimensional scalar-Tensor theory.
4. The aspects of the physical structure and the current accelerated expansion may be investigated with different scalar-tensor theories of gravitation using electromagnetic field.