Chapter 5

Anisotropic LRS Bianchi type-V Cosmological Models with Bulk Viscous String within the Framework of Sáez-Ballester Theory in Five Dimensional Space time

5.1 Introduction

Bulk viscosity is crucial in cosmology because it plays a significant part in the fast expansion of the universe, often known as the inflationary phase. There are numerous scenarios in which bulk viscosity could emerge during the evolution of the universe (Ellis and Sachs, 1971). Viscosity appears when neutrinos disconnect from the cosmic fluid (Misner, 1968), during galaxies formation and during particle synthesis in the early universe.

5.2 Metric and field equations

We assume the spatially homogeneous and anisotropic Bianchi type-V space-time defined by the line element

$$ds^2 = -dt^2 + A^2dx^2 + e^{2\alpha x}B^2(dy^2 + dz^2) + e^{2\alpha x}C^2d\psi^2 \tag{5.1}$$

where *A*, *B* and *C* are functions of cosmic time *t* respectively and α is a constant.

S$\acute{a}$ez and Ballester's field equations for coupled scalar and tensor fields are as follows

$$R_{ij} - \frac{1}{2}g_{ij}R - \omega\phi^m(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,k}\phi^{,k}) = -8\pi T_{ij} \tag{5.2}$$

The scalar field ϕ also fulfils the equation

$$2\phi^m\phi^{;i}_{;i} + n\phi^{n-1}\phi_{,k}\phi^{,k} = 0 \tag{5.3}$$

and also

$$T^{ij}_{;j} = 0 \tag{5.4}$$

Here, T_{ij} is the matter's energy momentum tensor, R_{ij} is the Ricci tensor, *R* is the Ricci scalar and the comma and semicolon respectively signify partial and covariant derivatives. ω and *n* are constants.

The energy momentum tensor for a bulk viscous fluid containing one-dimensional cosmic strings is defined as follows

$$T_{ij} = (\rho + \bar{p})u_iu_j + \bar{p}g_{ij} - \lambda x_ix_j \tag{5.5}$$

$$\bar{p} = p - 3\zeta H \tag{5.6}$$

where ρ is the system's rest energy density, ζ is the coefficient of bulk viscosity, $\bar{p}$ is the coefficient of bulk viscous pressure, *H* is Hubble's parameter and λ is the string tension density.

Also u^i denotes the fluid's five velocity vector and x^i is the direction of the string which satisfies the following conditions

$$u^i u_i = -x^i x_i = -1 \; and \; u^i x_i = 0 \tag{5.7}$$

$$u^i = (0,0,0,0,1) \; and \; x^i = (\frac{1}{A}, 0,0,0,0) \tag{5.8}$$

Here we assume ρ, $\bar{p}$ and λ as functions of time *t* only.

The field equations (5.2)–(5.4) for the metric (5.1) generate the following independent field equations using co-moving coordinates and equations (5.5) - (5.8)

$$2\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + 2\frac{\dot{B}}{B}\frac{\dot{C}}{C} + \frac{\dot{B}^2}{B^2} - 3\frac{\alpha^2}{A^2} - \omega\phi^m\frac{\dot{\phi}^2}{2} = -8\pi(\bar{p} - \lambda) \tag{5.9}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} - 3\frac{\alpha^2}{A^2} - \omega\phi^m\frac{\dot{\phi}^2}{2} = -8\pi\bar{p} \tag{5.10}$$

$$\frac{\ddot{A}}{A} + 2\frac{\ddot{B}}{B} + 2\frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{B}^2}{B^2} - 3\frac{\alpha^2}{A^2} - \omega\phi^m\frac{\dot{\phi}^2}{2} = 8\pi\bar{p} \tag{5.11}$$

$$2\frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} + 2\frac{\dot{B}}{B}\frac{\dot{C}}{C} - 6\frac{\alpha^2}{A^2} + \frac{\dot{B}^2}{B^2} + \omega\phi^m\frac{\dot{\phi}^2}{2} = 8\pi\rho \tag{5.12}$$

$$3\frac{\dot{A}}{A} - 2\frac{\dot{B}}{B} - \frac{\dot{C}}{C} = 0 \tag{5.13}$$

$$\ddot{\phi} + \dot{\phi}(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B} + \frac{\dot{C}}{C}) + \frac{m}{2}\frac{\dot{\phi}^2}{2} = 0 \tag{5.14}$$

Here an overhead dot denotes differentiation with respect to time *t*.

The scale factor for the metric (5.1) and the spatial volume are defined respectively by

$$a = (AB^2C)^{\frac{1}{4}} \tag{5.15}$$

$$V^4 = AB^2C \tag{5.16}$$

The important physical quantities, the Hubble's parameter *H*, expansion scalar θ, anisotropy parameter Δ and the shear scalar σ^2 are defined by as follows

$$H = \frac{1}{4}(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B} + \frac{\dot{C}}{C}) \tag{5.17}$$

$$\theta = 4H = (\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B} + \frac{\dot{C}}{C}) \tag{5.18}$$

$$\Delta = \frac{1}{4}\sum_{i=1}^{4}(\frac{H_i - H}{H})^2 \tag{5.19}$$

$$2\sigma^2 = \sigma_{ij}\sigma^{ij} = \sum_{i=1}^{4} H_i^2 - 4H^2 = 4\Delta H^2 \tag{5.20}$$

5.3 Solution of the field equations

The field equations (5.9)–(5.14) are reduced to the independent equations as given below

$$\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} - \frac{\ddot{A}}{A} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} - \frac{\dot{A}}{A}\frac{\dot{B}}{B} - \frac{\dot{A}}{A}\frac{\dot{C}}{C} = 8\pi\lambda \tag{5.21}$$

$$\frac{\ddot{B}}{B} - \frac{\ddot{C}}{C} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} - \frac{\dot{B}}{B}\frac{\dot{C}}{C} - \frac{\dot{A}}{A}\frac{\dot{C}}{C} + \frac{\dot{B}^2}{B^2} = 0 \tag{5.22}$$

$$2\frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} + 2\frac{\dot{B}}{B}\frac{\dot{C}}{C} - 6\frac{\alpha^2}{A^2} + \frac{\dot{B}^2}{B^2} + \omega\phi^m\frac{\dot{\phi}^2}{2} = 8\pi\rho \tag{5.23}$$

$$A^3 = lB^2C \tag{5.24}$$

$$\dot{\phi}\phi^{\frac{m}{2}}AB^2C = \phi_0 \tag{5.25}$$

where *l* and ϕ_0 are integration constants. Without sacrificing generality, the constant *l* can be chosen as unity, resulting in equation (5.24),

$$A^3 = B^2C \tag{5.26}$$

Now equations (5.21) - (5.25) are a system of five independent equations with seven unknowns *A*, *B*, *C*, *p*, ρ, ϕ and λ. The equations are also extremely non-linear. As a result, we apply the following physically feasible requirements to define at a determinate solution

(i) Collins et al. (Collins, 1980) claim that the shear scalar σ^2 is proportional to expansion scalar θ

$$B = C^k \quad k \neq 0 \; is \; a \; constant \tag{5.27}$$

(ii) The combined effect of the appropriate pressure and the bulk viscous pressure on a barotropic fluid can be represented as

$$\bar{p} = p - 3\zeta H = \varepsilon \rho \tag{5.28}$$

where

$$\varepsilon = \varepsilon_0 - \beta(0 \leq \varepsilon_0 \leq 1), \; p = \varepsilon_0 \rho \tag{5.29}$$

Here ε_0 and β are constants.

From equation (5.22), (5.26) and (5.27), we define the expressions for the metric coefficients as

$$C = [\frac{4(2k+1)}{3}(C_0 t + d)]^{\frac{3}{4(2k+1)}} \tag{5.30}$$

$$B = [\frac{4(2k+1)}{3}(C_0 t + d)]^{\frac{3k}{4(2k+1)}} \tag{5.31}$$

$$A = [\frac{4(2k+1)}{3}(C_0 t + d)]^{\frac{1}{4}} \tag{5.32}$$

where C_0 and *d* are integration constants.

The metric (5.1) can be expressed as (by an appropriate choice of coordinates and constants, such as $C_0 = 1$ and $d = 0$) using equations (5.30)-(5.32).

$$ds^2 = -dt^2 + [\frac{4(2k+1)}{3}t]^{\frac{1}{2}} dx^2 + e^{2\alpha x}[\frac{4(2k+1)}{3}t]^{\frac{3k}{2(2k+1)}}(dy^2 + dz^2) + e^{2\alpha x}[\frac{4(2k+1)}{3}t]^{\frac{3}{2(2k+1)}} d\psi^2 \quad (5.33)$$

5.4 Physical properties of the model

Equation (5.33) represents the Bianchi type-V bulk viscous string cosmological model in the S$\acute{a}$ez-Ballester scalar-tensor theory of gravitation, with the physical and kinematical characteristics listed below, which are important in cosmology.

The volume is

$$V^4 = [\frac{4(2k+1)}{3}t] \quad (5.34)$$

The Hubble's parameter is given by

$$H = \frac{1}{4t} \quad (5.35)$$

expansion scalar is obtained as

$$\theta = \frac{1}{t} \quad (5.36)$$

The mean anisotropy parameter is given by

$$\Delta = \frac{3(k^2 - 2k + 1)}{2(2k+1)^2} \quad (5.37)$$

shear scalar is

$$\sigma^2 = \frac{3(k^2 - 2k + 1)}{16t^2(2k+1)^2} \quad (5.38)$$

The string tension density

$$\lambda = 0 \quad (5.39)$$

The energy density is given by

$$8\pi\rho = \frac{3(14k^2 + 20k + 3\omega\phi_0^2 + 2)}{32t^2(2k+1)^2} - 6\alpha^2[\frac{4(2k+1)}{3}t]^{-\frac{1}{2}} \quad (5.40)$$

pressure is obtained as

$$8\pi p = \varepsilon_0 [\frac{3(14k^2 + 20k + 3\omega\phi_0^2 + 2)}{32t^2(2k+1)^2} - 6\alpha^2 [\frac{4(2k+1)}{3} t]^{-\frac{1}{2}}] \quad (5.41)$$

coefficient of bulk viscosity is

$$8\pi\zeta = \frac{t}{3}(\varepsilon_0 - \varepsilon)[\frac{3(14k^2 + 20k + 3\omega\phi_0^2 + 2)}{32t^2(2k+1)^2} - 6\alpha^2 [\frac{4(2k+1)}{3} t]^{-\frac{1}{2}}] \quad (5.42)$$

The scalar field is given by

$$\phi = [\frac{3\phi_0(m+2)}{8(2k+1)} \log \frac{t}{t_0}]^{\frac{2}{m+2}} \quad (5.43)$$

Here t_0 is the integration constant.

The deceleration parameter is given by

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = 3 \quad (5.44)$$

Also, from equation (5.36) and (5.38), we obtained,

$$\lim_{t\to\infty} \frac{\sigma^2}{\theta^2} = \frac{3(k^2 - 2k + 1)}{16(2k+1)^2} \neq 0 \quad (5.45)$$

The following graphs have been drawn by choosing, $k = \varepsilon = .5, \omega = 500, \phi_0 = \alpha = \varepsilon_0 = m = t_0 = 1$.

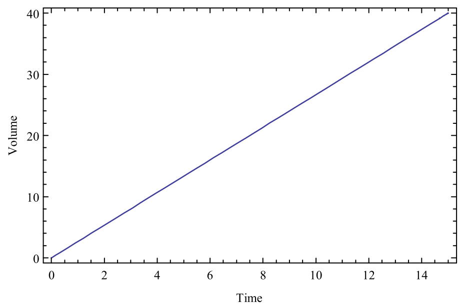

Figure 5.1: *V* vs. *t* (billion years)

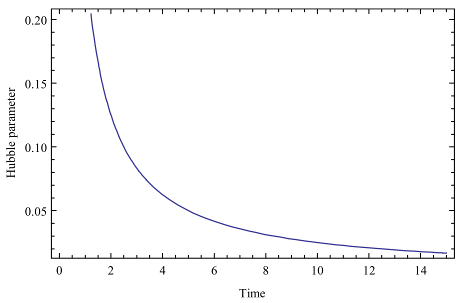

Figure 5.2: *H* vs. *t* (billion years)

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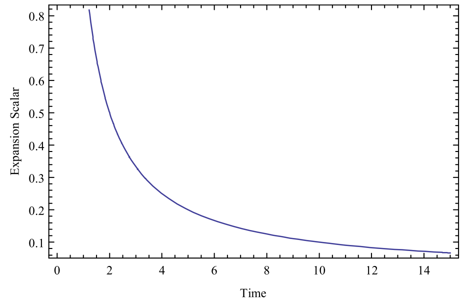

Figure 5.3: θ vs. t (billion years)

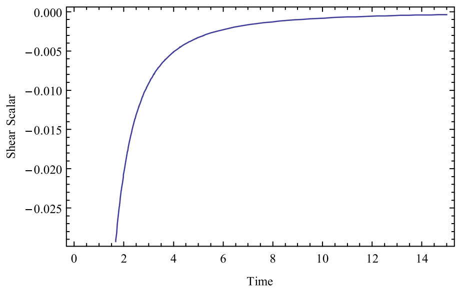

Figure 5.4: σ^2 vs. t (billion years)

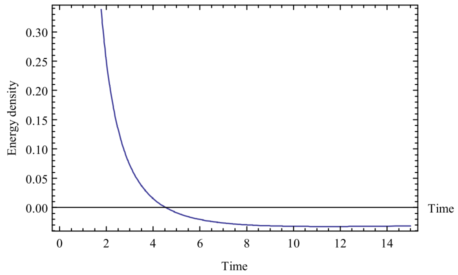

Figure 5.5: ρ vs. t (billion years)

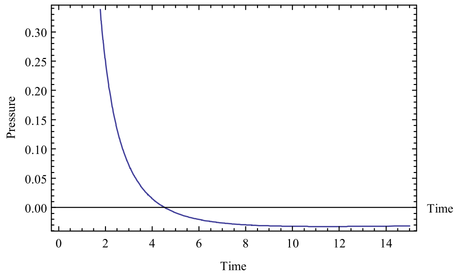

Figure 5.6: p vs. t (billion years)

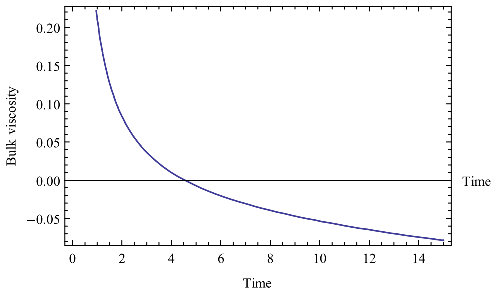

Figure 5.7: ζ vs. t (billion years)

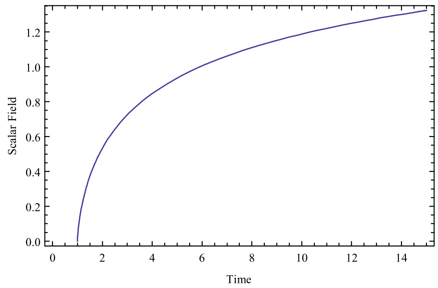

Figure 5.8: ϕ vs. t (billion years)

5.5 Physical and geometrical discussion

The above findings are useful in debating the behaviour of the S$\acute{a}$ez-Ballester cosmological model, which is provided by equation (5.33). From equation (5.34) we get that, at the initial singularity, the appropriate volume V approaches to zero. In the limit as $t \to \infty$, the universe approaches towards an infinitely large volume as time passes [as shown in fig-5.1]. As a result, the model increases over time. The model defined in this chapter is free from initial singularity. It is interesting to note from equation (5.39) that strings in this model do not survive. Moreover it is also obtained that the Hubble's parameter H, expansion scalar θ, shear scalar σ^2, energy density ρ, pressure p and the coefficient of bulk viscosity ζ decreases when time is increases. At $t = 0$, they are all becomes infinitely large as shown in fig-5.2, fig-5.3, fig-4, fig-5.5, fig-5.6 and fig-5.7. The scalar field ϕ on the other hand, approaches zero as $t \to \infty$ as shown in fig-5.8. The behaviour of bulk viscosity ζ follows the well-known trend of decreasing with time, resulting in an inflationary model (Padmanabhan and Chitre, 1987). Furthermore, from equation (5.45) we get the model is anisotropic throughout, which helps in a better understanding of the universe's early stages of evolution.