Total No. of printed pages = 7

63/2 (SEM-1) MCA 1.3

2021

(held in 2022)

MCA

(Theory Paper)

Paper Code : MCA-1.3

(Mathematical Foundation of Computer Sc.)

Full Marks – 75

Time – Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : $1\times5=5$

(a) Consider the following statements :

S1: There exist infinite sets A, B & C such that $A\cap(B\cup C)$ is finite.

S2: There exist two irrational numbers x and y such that (x+y) is rational.

[Turn over

Which of the folloiwng is true about S1 & S2

(i) Only S1 is correct.

(ii) Only S2 is correct.

(iii) Both S1 and S2 are correct.

(iv) None of S1 and S2 are correct.

(b) Let R be non-empty relation on a collection of sets defined by A R B if and only if $A \cap B = \phi$. Then

(i) R is reflexive and transitive.

(ii) R is symmetric and not transitive.

(iii) R is an equivalence relation.

(iv) R is not reflexive and not symmetric.

(c) If the binary operations $*$ is defined on a set of ordered pairs of real numbers as.

$(a, b)*(c, d) = (ad + bc, bd)$ as is associative, then $(1, 2)*(3, 5)*(3, 4) = ?$

(i) (74, 40) (ii) (32, 40)

(iii) (23, 11) (iv) (7, 11)

(d) Which of the following proposition is tautology ?

(i) $(P \vee Q) \rightarrow P$ (ii) $P \vee (Q \rightarrow P)$

(iii) $(P \vee (P \rightarrow Q))$ (iv) $P \rightarrow (Q \rightarrow P)$

(e) $(P \rightarrow Q) \wedge (R \rightarrow Q)$ is equivalent to

(i) $(P \vee R) \rightarrow Q$ (ii) $P \vee (R \rightarrow P)$

(iii) $P \vee (R \rightarrow Q)$ (iv) $P \rightarrow (Q \rightarrow R)$

2. Answer any *five* of the following questions : 2×5=10

(a) Verify associativity for the following three mappings :

$f : R \rightarrow R$ such that $f(x) = 2x$

$g : R \rightarrow R$ such that $g(x) = \frac{1}{2-x}$

$h : R \rightarrow R$ such that $h(x) = x^2$

(b) Use truth tables to verify the following equivalence :

$\neg(P \rightleftarrows Q) \equiv (P \vee Q) \wedge \neg(P \wedge Q)$

Turn over

(c) Prove that the order of any integral power of an element a cannot exceed the power of a.

(d) Show that the multiplication of permutations is not in general commutative.

(e) If H is a sub-group of group G and let $a \in G$. Then show that $a \in Ha$ and $a \in aH$

(f) Show that the orders of ab and ba are equal, where a and b are any elements of a group G.

3. Answer any *six* of the following questions : 6×5=30

(a) let Z be the set of integers. Let m>1 be any fixed integer. Then we say a is congruent to another integer b modules if (a–b) is divisible by m.

Equivalently we can say if there exists an integer k such that $(a-b) = km$ and it is symbolically written as, $a \cong b \pmod{m}$.

Show that this defines an equivalence relation.

(b) Show that the set Q–{1} of rational numbers other than 1 is an abelian group under the composition $*$ defined as,

$x * y = x + y - xy.$

(c) Prove that a group G is abelian if and only if, $(ab)^{-1} = a^{-1}b^{-1}, \forall a, b \in G.$

(d) Let G be a group and

$H = \{x \in G : xg = gx, \forall g \in G\}$

Show that H is normal subgroup of G.

(e) Test the validity of the following argument: If milk is black then every cow is white. If every cow is white then it has four legs. If every cow has four legs then every buffalo is white and brish The milk is black.

(f) Show that :

$(\neg P \wedge (\neg Q \wedge R)) V (Q \wedge R) V (P \wedge R) \equiv R.$

(g) Discuss the validity of the following argument : All educated persons are well behaved. Ram is educated.

No well-behaved person is quarrelsome therefore, Ram is not quarrelsome.

Answer any *three* questions from the question numbers 4, 5, 6, 7 & 8.

4. For an equivalence relation R on a set A and also a, $b \in A$, Show that : 10

   (i) $a \in [a]$

   (ii) $b \in [a] \Rightarrow [b] = [a]$

   (iii) $[a] = [b] \Leftrightarrow (a, b) \in R$

   (iv) Any two equivalence classes are either identical or disjoint.

5. Answer the following questions : 10

   (i) If a, b are two elements of a group G, then show that $(ab)^2 = a^2b^2$ if and only if G is abelian.

   (ii) If H is a subgroup of G such that $x^2 \in H$, $\forall x \in G$, then prove that H is a normal subgroup of G.

6. Answer the following questions : 10

   (i) Obtain the principal disjunctive normal form of, $P \rightarrow (P \rightarrow Q) \wedge \neg(\neg Q \vee \neg P)$

   (ii) Obtain the principal conjunctive normal form of the formula, $(\neg P \rightarrow R) \wedge (Q \rightleftharpoons P)$

7. Answer the following questions : 10

   (i) Show that the order of the elements a and $x^{-1}ax$ are the same where a, x are any two elements of a group G.

   (ii) If $f: A \rightarrow B$, $g: B \rightarrow C$ and $h: C \rightarrow D$ are three functions such that $h_0(g_0 f)$ and $(h_0 g)_0 f$. Show that $h_0(g_0 f) = (h_0 g)_0 f$.

8. Give the defination of walk, path, circuits and components with example. 10

   Show that a simple graph with n-vertices are k-components can have at most $\frac{(n-k)(n-k+1)}{2}$ edges.