Total No. of printed pages = 8

63/2 (SEM-1) MCA 1·5

2021

(held in 2022)

MCA

(Theory Paper)

Paper Code : MCA-1·5

(Probability & Statistics)

Full Marks – 75

Time – Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : 1×5=5

   (a) In random experiment, observation of random variable are classified as

   (i) Events (ii) Composition

   (iii) Trials (iv) Functions

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(b) The sum of values divided by their number is called

(i) Median (ii) Harmonic mean

(iii) Mean (iv) Mode

(c) In binomial distribution, formula of calculating standard deviation is

(i) Square root of p

(ii) Square root of pq

(iii) Square root of npq

(iv) Square root of np

(d) The collection of one or more outcomes from an experiment is called

(i) Probability (ii) Event

(iii) Random variable (iv) Z-value

(e) A listing of the possible outcomes of an experiment and their corresponding probability is called

(i) Random variable

(ii) Contingency table

(iii) Bayesia table

(iv) Probability distribution.

2. Answer the following questions : 2×5=10

(a) A pair of dice is tossed and the two numbers appearing on the top are recorded. Find the number of elements in each of the following events :

(i) A = {Two numbers are equal}

(ii) B = {Sum is 10 or more}

(iii) C = {5 appears on first die}

(iv) D = {5 appears on at least one die}

(b) If $P(A) = P_1$, $P(B) = P_2$, $P(A \cap B) = P_3$, find $P(A \cup B)$, $P(A^c \cap B)$ and $P(A^c \cup B)$.

(c) Show tht for any event A, $0 \leq P(A) \leq 1$.

(d) Suppose a student is selected at random from 80 students where 30 are taking mathematics, 20 are taking chemistry and 10 are taking mathematics and chemistry. Find the probability p that the student is taking mathematics or chemistry.

(e) Let A and B be events with $P(A) = \frac{3}{8}$ and $P(B) = \frac{5}{8}$ and $P(A \cup B) = \frac{3}{4}$. Find $P(A|B)$ and $P(B|A)$.

3. Answer any *six* of the following questions : 5×6=30

(a) Three men A, B, C fire at a target. Suppose $P(A) = \frac{1}{6}$, $P(B) = \frac{1}{4}$, $P(C) = \frac{1}{3}$ denote their probabilities of hitting the target (we assume that the events that A, B, C hit the target are independent).

(i) Find the probability P that they all hit the target.

(ii) Find the probability P that they all miss the target.

(iii) Find the probability P that at least one of them hits the target.

(b) Show that,

$$\mathrm{Cov}(X,Y) = \sum_{i,j} x_i y_j f(x_i, y_j) - ve_x ve_y$$

(c) A fair die is tossed yielding the equiprobable space, S = {1, 2, 3, 4, 5, 6} Let Y be 1 or 3 accordingly as an odd or even number appears. Find the distribution g, expectation ve_y, Variance σ_y^2 standard deviation σ_y of Y.

(d) Let $X_1, X_2, \ldots, X_n$ be independent random variables on S. Then show that, $\mathrm{Var}(X_1+X_2+\ldots+X_n) = \mathrm{Var}(X_1) + \mathrm{Var}(X_2) + \ldots + \mathrm{Var}(X_n)$.

(e) The standard deviation of two sets containing n_1 and n_2 members are σ_1 and σ_2 respectively, being measured from their respective means m_1 and m_2. If the two sets are grouped together as one set of $(n_1 + n_2)$ members. Show that the standard deviation σ, of this set measured from its mean is given by.

$$\sigma^2 = \frac{n_1\sigma_1^2 + n_2\sigma_2^2}{n_1 + n_2} + \frac{n_1 n_2}{(n_1 + n_2)^2}(m_1 + m_2)^2$$

(f) Let X be the binomial random variable B (n.p). Then show that,

(i) ve = E(X) = n.p

(ii) Var(X) = npq.

(g) Find the ve = E(X), variance $\sigma^2 = Var(x)$ and standard deviation $\sigma = \sigma_x$ of each distribution

(i)

x_i	−5	−4	1	2
p_i	1/4	1/8	1/2	1/8

(ii)

x_i	1	3	5	7
p_i	0·3	0·1	0·4	0·2

(h) Let X and Y be independent random variable on S. Then show that

(i) E (XY) = E (X) E (Y)

(ii) Var (X+Y) = Var (X) + Var (Y)

Answer any *three* questions from the question number 4, 5, 6 and 7 :

4. Calculate the correlation co-efficient and also find the equation of the lines of regression from the following table giving the ages of 100 husbands and their wives in years. 10

Age of husbands

Age of wives	20-30	30-40	40-50	50-60	60-70	Total
15 - 25	5	9	3	–	–	17
25 - 35	–	10	25	2	–	37
35 - 45	–	1	12	2	–	15
45 - 55	–	–	4	16	5	25
55 - 65	–	–	–	4	2	6
Total	5	20	44	24	7	100

5. Let X and Y be random variable with joint distribution as : 10

X	−3	2	4	Sum
1	0·1	0·2	0·2	0·5
3	0·3	0·1	0·1	0·5
Sum	0·4	0·3	0·3	1·0

(i) Find the distribution of X and Y.

(ii) Find Cov (X, Y), the covariance of X and Y.

(iii) Find ρ (X, Y), the correlation of X & Y.

(iv) Are X and Y independent random variables ?

6. Let X and Y be random variables on S and let K be a real number and also a and b be any constants. Then show that : 10

(i) $E(kX) = k\,E(X)$

(ii) $E(X + k) = E(X) + k$

(iii) $E(X + Y) = E(X) + E(Y)$

(iv) $Var(aX+b) = a^2\,Var(X)$.

7. The measurements of the diameters (in cms) of the plates prepared in a factory are given below :

Diameter (cm)	21-24	25-28	29-32	33-36	37-40	41-44
Number of Plates	15	18	20	16	8	7

Find its median, upper quartiles, 8th decile, 5th per centile, standard deviation and co-efficient of variation. 10