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63/2 (SEM-2) CSIT 2.4

2022

COMPUTER SCIENCE AND TECHNOLOGY

(Theory Paper)

Paper Code : CSIT 2.4

(Mathematical Foundation of Computer Science)

Full Marks – 80

Time – Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : $1\times5=5$

(a) Let U = {1, 2, ..., 10}, A = {1, 2, 3} and B = {4, 5} then $A\oplus B$ is,

(i) {1, 2, 3, 4}

(ii) {2, 3, 4}

(iii) {1, 2, 3, 4, 5}

(iv) {1, 2, 3}

[Turn over

(b) The binary relation S = ϕ (empty set) on set A = { 1, 2, 3} is

(i) Neither reflexive nor symmetric

(ii) Symmetric and reflexive

(iii) Transitive and reflexive

(iv) Transitive and symmetric

(c) Let R denote the set of real numbers. Let $f: R\times R \to R\times R$ be a bijective function defined by, $f(x,y) = (x+y, x-y)$. The inverse function of f is given by,

(i) $f^{-1}(x,y) = \left(\frac{1}{x+y}, \frac{1}{x-y}\right)$

(ii) $f^{-1}(x,y) = (x-y, x+y)$

(iii) $f^{-1}(x,y) = \left(\frac{x+y}{2}, \frac{x-y}{2}\right)$

(iv) $f^{1}(x,y) = (2(x-y), 2(x+y))$

(d) $(P\to Q) \times (R\to Q)$ is equivalent to

(i) $(P \vee R) \to Q$ (ii) $P \vee (R\to P)$

(iii) $P \vee (R\to Q)$ (iv) $P\to(Q\to R)$

(e) In any undirected graph, the sum of degrees of all the vertices

(i) must be even

(ii) is twice the number of edges

(iii) must be odd

(iv) Both (i) and (ii).

2. Answer the following questions : 2×5=10

(a) Given A = {1, 2, 3, 4} and B = {x, y, z}. Let R be the following relation from A to B, R = {(1, y), (1, z), (3, y), (4, x), (4, z)}

(i) Find the inverse relation R^{-1} of R

(ii) Determine the domain and range of R.

(b) Find $g_o f$ and $f_o g$, when $f : R\to R$ and $g : R\to R$ defined as, $f(x) = 2x+3$ and $g(x) = x^2+5$.

(c) Show that the order of any integral power of an element a cannot exceed the power of a.

(d) Let a, b, c be any elements in a Boolean algebra B. Then show that, a+a = a and a*a = a.

(e) Construct the truth table for, $(A\vee B) \rightleftharpoons ((\neg A \wedge C) \to (B \wedge C))$.

3. Answer any *seven* of the following questions : 5×7=35

(a) Consider the set of ordered pair of natural numbers $N\times N$ defined by,

$$(a,b)\ R\ (c,d) \Leftrightarrow a+d = b+c.$$

Prove that, R is a equivalence relation.

(b) If $f: Q\to Q$ such that $f(x)=2x$ and $g: Q\to Q$ such that $g(x) = x+2$, be two functions, then varify that,

$(g.f)^{-1} = f^{-1}.g^{-1}$, where Q is a set of rational numbers.

(c) Prove that a group G is abelian if and only if, $(ab)^{-1} = a^{-1}\, b^{-1},\ \forall a, b \in G$

(d) Let 'a' be an element of a group G. Then show that the function $f_a : G\to G$ given by,

$f_a(x) = axa^{-1},\ \forall x \in G$ is an automorphism of G.

(e) Let a, b, c be any elements in a Boolean algebra B. Show that,

$$(a * b) * c = a * (b * c).$$

(f) Check the validity of the following argument :

If Ram has completed B.E or MBA, then he is assured of a good job. If Ram is assured of a good job, he is happy. Ram is not happy. So Ram has not completed MBA.

(g) Find conjunctive normal form and disjunctive normal form for

$$P \rightleftharpoons (\neg P \vee \neg Q)$$

(h) Show that $(S\vee R)$ is tautologically implied by $(P\vee Q)\ (P\to R)\wedge(Q\to S)$.

4. Answer any *three* of the following questions : 10×3=30

(a) (i) Show that (with the help of logical identities)

$$(P\vee Q)\wedge\neg(\neg P\wedge(\neg Q\vee\neg R))$$
$$\vee(\neg P\wedge\neg Q)\wedge(\neg P\vee\neg R)$$ is a tautology.

(ii) Then the validity of the following arguments :

All integers are irrational numbers. Some integers are powers of 2. Therefore, some irrational number is a power of 2.

(b) Let a, b be any element in a Boolean algebra B. Show that,

(i) $(a+b)' = a' * b'$ and $(a * b)' = a' + b'$

(ii) $a + (a * b) = a$ and $a * (a + b) = a$

(c) In a group G, if a^{-1} is the inverse of $a \in G$. Then show that $(a^{-1})^{-1} = a$. And also show that, $(ab)^{-1} = b^{-1}a^{-1}$, $\forall$ a, b $\in$ G.

(d) Prove that the orders of the elements a and $x^{-1}a\,x$ are the same where "a" and "x" are any two elements of a group G. Also find the solution of the equation,

$abxax = cbx$, $\forall$ a, b, c $\in$ G.

(e) Let R be an equivalence relation defined on a set A. Let "a" and 'b' be arbitrary elements in A. Then show that,

(i) $[a] = [b] \Leftrightarrow (a,b) \in R$.

(ii) Any two equivalence classes are either, identical or disjoint.