Total No. of printed pages = 4

63/2(SEM-2)MAT 201

2022

MATHEMATICS

(Theory Paper)

Paper Code : MAT 201

(Complex Analysis)

Full Marks – 80

Time – Three hours

The figures in the margin indicate full marks for the questions.

1. Answer any *four* from the following questions : 5×4=20

   (a) Prove that $u = e^{-x}(x\sin y - y\cos y)$ is harmonic. Find v such that $f(z) = u + iv$ is analytic.

   (b) Prove that if a function $f(z) = u + iv$ is analytic in region R then u and v satisfies the Cauchy Riemann equations, provided that the partial derivatives u_x, u_y, v_x and v_y exist.

[Turn over

(c) Show that the function $f(z)=\sqrt{|xy|}$ is not regular at the origin, although Cauchy-Riemann equations are satisfy at 0.

(d) Show that an analytic function with constant modulus is constant.

(e) Prove that if $u=x^2-y^2, v=-\frac{y}{(x^2+y^2)}$ both u and v satisfy Laplace equations, but u + iv is not analytic function of z.

2. Answer any *four* from the following questions : 5×4=20

(a) State and prove maximum modulus theorem.

(b) State and prove argument theorem.

(c) Evaluate $\oint_C \frac{e^{2z}}{(z+1)^4}dz$, where C is the circle $|z|=3$.

(d) Show that if $f(z)=\frac{1}{1-z}$, then $f^n(z)=\frac{n!}{(1-z)^{n+1}}$.

(e) Find the value of $\oint_C \frac{\sin^6 z}{\left(z-\frac{\pi}{6}\right)^3}dz$ if C is the circle $|z|=1$.

3. Answer any *four* from the following questions : 5×4=20

(a) State and prove Taylor's theorem.

(b) Expand $\frac{1}{(z+1)(z+3)}$ in a Laurent's series valid for $1<|z|<3$.

(c) State and prove Cauchy residue theorem.

(d) Show that the function $f(z)=e^z$ has an isolated essential singularity at $z=\infty$.

(e) Evaluate $\oint \frac{2z^2+5}{(z+2)^3(z^2+4)z^2}dz$ where C is the circle $|z-2i|=6$.

4. Answer any *four* from the following questions : 5×4=20

(a) Prove that if $a > 0$ then

$$\int_0^\infty \frac{dx}{x^4 + a^4} = \frac{\pi}{2\sqrt{2}\, a^3}$$

(b) Prove that if $a > 0$, $m > 0$ then

$$\int_0^\infty \frac{\cos mx}{a^2 + x^2} dx = \frac{\pi}{2a} e^{-ma}$$

(c) Find a bilinear transformation that maps points z_1, z_2, z_3 of the z-plane into the points w_1, w_2, w_3 of the w-plane respectively.

(d) Define translation and rotation. Let R be the rectangular region in the z-plane bounded by $x = 0$, $y = 0$, $x = 2$, $y = 1$. Determine the region R' of the w-plane into which R is mapped under the transformation $w = z + (1 - 2i)$.

(e) Let $w = f(z)$ be a transformation such that $f(z)$ is analytic at z_0 and $f(z_0) \neq 0$. Prove that under this transformation, the tangent at z_0 at ant curve C in the z-plane passing through z_0 rotated through the angle $\alpha = \arg f'(z_0)$.