2021

(held in 2022)

MATHEMATICS

(Theory Paper)

Paper Code : MAT-302 (New)

(Graph Theory)

Full Marks — 80

Time — Three hours

The figures in the margin indicate full marks
for the questions

1. Answer any *four* from the following questions :
   4×5=20

   (a) What is graph ? Give example. Define the degree of vertex. Define subgraph and induced subgraph of a graph ?   2+1+2=5

   (b) Define complete graph, regular graph and complete bigraph. Give examples. Is there any difference between complete graph and regular graph ?   4+1=5

(c) Define intersection graph. Give example. What is the main difference between a trail and a path of a graph ? 3+2=5

(d) Show that a graph is bipartite if and only if its cycles are even. 5

(e) What is the relation between a complete graph and a totally disconnected graph ? Explain with appropriate example. 3+2=5

2. Answer any *four* from the following questions : 4×5=20

(a) Define connected graph, give example. Draw the following graphs : 3+2=5

(i) A graph that contains only one cut vertex without a bridge.

(ii) A graph that contains a bridge.

(iii) A block.

(b) Show that a graph H is the block graph of some graph if and only if every block of H is complete. 5

(c) Define tree and forest. Give examples. Show a graph G is a tree if every two points of G are joined by a unique path. 2+3=5

(d) Define spanning tree. Find out the spanning trees of the graph K_4. 1+4=5

(e) Define connectivity and line-connectivity of a graph. Give examples. For any graph G, show $\kappa(G) \le \lambda(G)$. 2+3=5

3. Answer any *four* from the following questions : 4×5=20

(a) Define Eulerian trail and Eulerian graph. Find out the Eulerian trails of the graph K_5. 2+3=5

(b) Define Hamiltonian graph. Is there any difference between Eulerian graph and Hamiltonian graph ? Explain with example. 1+4=5

(c) For any nontrivial connected graph G, prove $\alpha_0 + \beta_0 = p = \alpha_1 + \beta_1$, where p is the number of vertices of G. 5

(d) If G is bipartite graph, then show that the number of lines in a maximum matching equals the point covering number. 5

(e) Define critical points and lines. Give examples. Show that a point v is critical in a graph if and only if some minimum point cover contains v. 3+2=5

4. Answer any *four* from the following questions :

$$4 \times 5 = 20$$

(a) Define n-factor and n-factorization of a graph. Show that the graph K_{2n} is 1-factorable.

$$2+3=5$$

(b) Draw the graph K_7. Find its all edge disjoint spanning cycles. Is K_7 2-factorable ?

$$1+3+1=5$$

(c) Define embedding of a graph in a surface. What are planar and non-planar graph ? Give example. State Euler Polyhedron Formula.

$$1+3+1=5$$

(d) Define genus, thickness, coarseness and crossing number of a graph. What is the genus of the graph K_6 ?

$$4+1=5$$

(e) If G is a (p, q) plane map in which every face is an n-cycle, then show $q = \dfrac{n(p-2)}{(n-2)}$. Also show that K_5 and $K_{3,3}$ are nonplanar.

$$3+2=5$$