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63/2 (SEM-3) MAT 302

2021

(held in 2022)

MATHEMATICS

(Theory Paper)

Paper Code : MAT-302 (New)

(Graph Theory)

Full Marks – 80

Time – Three hours

The figures in the margin indicate full marks
for the questions

1. Answer any *four* from the following questions :

4×5=20

(a) What is graph ? Give example. Define the degree of vertex. Define subgraph and induced subgraph of a graph ?

2+1+2=5

(b) Define complete graph, regular graph and complete bipartite graph. Give examples. Is there any difference between complete graph and regular graph ?

4+1=5

[Turn over

- (c) Define intersection graph. Give example. What is the main difference between a trail and a path of a graph ? $3+2=5$
- (d) Show that a graph is bipartite if and only if its cycles are even. 5
- (e) What is the relation between a complete graph and a totally disconnected graph ? Explain with appropriate example. $3+2=5$
2. Answer any *four* from the following questions : $4 \times 5 = 20$
- (a) Define connected graph, give example. Draw the following graphs : $3+2=5$
- (i) A graph that contains only one cut vertex without a bridge.
- (ii) A graph that contains a bridge.
- (iii) A block.
- (b) Show that a graph H is the block graph of some graph if and only if every block of H is complete. 5
- (c) Define tree and forest. Give examples. Show a graph G is a tree if every two points of G are joined by a unique path. $2+3=5$

- (d) Define spanning tree. Find out the spanning trees of the graph K_4 . $1+4=5$
- (e) Define connectivity and line-connectivity of a graph. Give examples. For any graph G , show $\kappa(G) \leq \lambda(G)$. $2+3=5$

3. Answer any *four* from the following questions : $4 \times 5 = 20$

- (a) Define Eulerian trail and Eulerian graph. Find out the Eulerian trails of the graph K_5 . $2+3=5$
- (b) Define Hamiltonian graph. Is there any difference between Eulerian graph and Hamiltonian graph ? Explain with example. $1+4=5$
- (c) For any nontrivial connected graph G , prove $\alpha_0 + \beta_0 = p = \alpha_1 + \beta_1$, where p is the number of vertices of G . 5
- (d) If G is bipartite graph, then show that the number of lines in a maximum matching equals the point covering number. 5
- (e) Define critical points and lines. Give examples. Show that a point v is critical in a graph if and only if some minimum point cover contains v . $3+2=5$

4. Answer any *four* from the following questions :

$$4 \times 5 = 20$$

- (a) Define n -factor and n -factorization of a graph. Show that the graph K_{2n} is 1-factorable.

$$2 + 3 = 5$$

- (b) Draw the graph K_7 . Find its all edge disjoint spanning cycles. Is K_7 2-factorable ?

$$1 + 3 + 1 = 5$$

- (c) Define embedding of a graph in a surface. What are planar and non-planar graph ? Give example. State Euler Polyhedron Formula.

$$1 + 3 + 1 = 5$$

- (d) Define genus, thickness, coarseness and crossing number of a graph. What is the genus of the graph K_6 ?

$$4 + 1 = 5$$

- (e) If G is a (p, q) plane map in which every face

is an n -cycle, then show $q = \frac{n(p-2)}{(n-2)}$. Also

show that K_5 and $K_{3,3}$ are nonplanar.

$$3 + 2 = 5$$