Total No. of printed pages = 4

63/2 (SEM-1) MAT 105

2021

(held in 2022)

MATHEMATICS

(Theory Paper)

Paper Code : MAT-105

(Tensor Analysis)

Full Marks – 80

Time – Three hours

The figures in the margin indicate full marks for the questions.

1. Answer any *four* of the following : 5×4=20

(a) Establish the transformation law of contravariant tensor $A'^{P} = \frac{\partial x'^{P}}{\partial x^{\alpha}} A^{\alpha}$.

(b) Defining contraction in a tensor, show that every contraction reduces the rank of a tensor by two.

[Turn over

(c) If B^α is a contravariant tensor and $B^\alpha A_\alpha$ is invariant, prove that A_α is a covariant tensor of rank one.

(d) Explain how to distinguish covariant and contravariant tensors of rank one.

(e) Show that the number of independent components of a symmetric tensor A_{ij} is $\frac{1}{2}n(n+1)$, i, j = 1, 2,......n.

(f) Show that tensor law of transformation possesses the group property.

2. Answer any *two* of the following questions : 10×2=20

(a) Derive the transformation law of the Christoffel symbol of the first kind $\Gamma_{ij,k}$ (or $[ij,k]'$) as

$$\Gamma'_{ij,k} = \Gamma_{ab,c} \frac{\partial x^a}{\partial x'^i} \frac{\partial x^b}{\partial x'^j} \frac{\partial x^c}{\partial x'^k} + g_{ab} \frac{\partial^2 x^a}{\partial x'^i \partial x'^j} \frac{\partial x^b}{\partial x'^k}$$

(b) Determine the covariant derivative of the second order covariant tensor namely $A_{ij,k}$ with respect to x^k.

Hence write down the result of $A^{ij}_{qr,k}$ 8+2=10

(c) (i) Determine the non-vanishing Christoffel symbols Γ^i_{jk} (or $\{^{\ i}_{j\ k}\}$) of the second kind for the metric $ds^2 = a^2(d\theta^2 + \sin^2\theta \, d\phi^2)$, where a is a constant. 5

(ii) If A_{ij} is a curl of a covariant vector B_i prove that 5

$A_{ij,k} + A_{jk,i} + A_{ki,j} = 0$.

3. Answer any *two* of the following questions : 10×2=20

(a) Write down the equations of geodesics in a Riemannian space V_n. Determine the differential equations of geodesics for

$$ds^2 = f(x)dx^2 + dy^2 + dz^2 + \frac{1}{f(x)}dt^2.$$

What is the geodesic in case of spherical surface in 3-dimensions ? 9+1=10

(b) What do you mean by geodesic coordinates? Hence establish the Bianchi identity

$$R^a_{ijk,l} + R^a_{ikl,j} + R^a_{ilj,k} = 0.$$

Write down another form of Bianchi identity. 2+7+1=10

(c) Derive the expression of the curvature tensor of the second kind as

$$R^a_{ijk} = \frac{\partial}{\partial x^j}\left(\Gamma^a_{ik}\right) - \frac{\partial}{\partial x^k}\left(\Gamma^a_{ij}\right) + \Gamma^a_{\alpha j}\Gamma^\alpha_{ik} - \Gamma^a_{\alpha k}\Gamma^\alpha_{ij} \;;$$

Γ^c_{ab} $\left(\text{or}\left\{ {\alpha \atop ij} \right\}\right)$ is the Christoffel symbol of the second kind.

Why it is named as Curvature tensor? Explain. 8+2=10

4. Answer any *two* of the following questions : 10×2=20

(a) (i) Define intrinsic derivative of a vector. Show that a vector of constant magnitude is orthogonal to its intrinsic derivative. 1+4=5

(ii) Show that a geodesic is an autoparallel curve. 5

(b) Prove that the differential equation $A_{i,j} = 0$ is integrable if $R^a_{ijk} = 0$.

(c) Derive the expression of tensor derivative of a tensor $A^\alpha_{\beta\gamma}$ with usual notation.