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63/2 (SEM-3) MAT 301

2022

(Held in 2023)

MATHEMATICS

(Theory Paper)

Paper Code : MAT-301

(Fuzzy Set Theory)

Full Marks – 80

Pass Marks – 32

Time – Three hours

The figures in the margin indicate full marks for the questions.

1. Choose the correct options of the following : $2\times5=10$

(a) Who was the inventor of Fuzzy Logic ?

(i) Doug cutting (ii) John McCarthy

(iii) Lotfi Zadeh (iv) John cutting

(b) In Membership function, graph x-axis represents

(i) universe of discourse

(ii) degrees of membership in the [0, 1] interval

(iii) degrees of discourse

(iv) universe of membership

(c) What is the form of Fuzzy logic ?

(i) Two-valued logic

(ii) Crisp set logic

(iii) Many-valued logic

(iv) Binary set logic

(d) The truth values of traditional set theory is ____ and that of fuzzy set is ____.

(i) either 0 or 1, between 0 & 1

(ii) between 0 & 1, either 0 or 1

(iii) between 0 & 1, between 0 & 1

(iv) either 0 or 1, either 0 or 1

(e) The temperature is hot. Here the hot (use of linguistic variable is used) can be represented by ____.

(i) Fuzzy Set

(ii) Crisp Set

(iii) Fuzzy and Crisp Set

(iv) None of the mentioned above.

2. Answer any *two* from the following questions : 5×2=10

(a) What is Fuzzy set ? Write the differences between General set and Fuzzy set.

(b) What is Convex fuzzy set ? Show that a fuzzy set A on IR is convex iff

$$A(\lambda x_1 + (1-\lambda)x_2) \geq \min\{A(x_1), A(x_2)\},$$

$$\forall x_1, x_2 \in \mathbb{R} \text{ and } \lambda \in]0, 1[$$

(c) Let A, B and C be fuzzy sets defined on the interval [0, 10] of real numbers by the membership grade functions

$$A(x) = \frac{x}{x+2}, B(x) = 2^{-x}, C(x) = \frac{1}{1+10(x-2)^2}$$

Then find A^C, B^C and C^C.

(d) What is α–cut and strong α–cut ? Find α–cut and strong α–cut for the following fuzzy set :

$$A = \left\{\frac{0.1}{1}, \frac{0.2}{2}, \frac{0.3}{3}, \frac{0.4}{4}, \frac{0.6}{5}, \frac{0.8}{6}, \frac{1}{7}, \frac{0.8}{8}, \frac{0.6}{9}\right\},$$

when $\alpha = 0.5$.

3. Answer any *four* from the following questions : 5×4=20

(a) Show that the belief function is monotonically increasing.

(b) Write the difference between probability distribution and basic probability assignment.

(c) Let a given finite body of evidence (F, m) be nested. Then for $A, B \in \wp(X)$ show that

(i) $Bel(A \cap B) = \min[Bel(A), Bel(B)]$

(ii) $Pl(A \cup B) = \max[Bel(A), Bel(B)]$.

(d) Show that every possibility measures Pos on a finite power set $\wp(X)$ is uniquely determined by a possibility distribution function $r : X \rightarrow [0, 1]$ via the formula $Pos(A) = \max_{x \in A} r(x)$.

(e) Show that a belief measure Bel on a finite power set $\wp(X)$ is a probability measure if and only if the associated basic probability assignment function m is given by $m(\{x\}) = Bel(\{x\})$ and $m(A) = 0$ for all subsets of X that are not singletons.

4. Answer the following questions :

(a) Define fuzzy relation on a classical set. Let X and Y be two classical sets having order m and n respectively. Find the cardinality of the collection of fuzzy relations from X to Y. 5

(b) Let R be a reflexive and symmetric fuzzy relation on a non-empty set X. Show that $R \circ R = R$. 5

Or

Let R and S be two fuzzy equivalence relation on a non-empty set X. Find the smallest fuzzy equivalence relation on X that containing RUS. 5

5. Answer any *two* from the following questions : 5×2=10

(a) Let $P^{\circ i}Q = R$, where $Q = \begin{bmatrix} .1 \\ .2 \\ .3 \end{bmatrix}$ and $R = \begin{bmatrix} .12 \\ .18 \\ .27 \end{bmatrix}$ such that $S(Q, R) \neq Q$ then find the greatest member of S(Q, R).

(b) Let $P^{\omega i}Q = R$, where $P = \begin{bmatrix} .5 & .9 \\ .4 & .9 \end{bmatrix}$ and

$R = \begin{bmatrix} .2 & 1 \\ .25 & 1 \end{bmatrix}$ such that $S(Q, R) \neq Q$ then find the smallest member of $S(Q, R)$.

(c) What is Max-Min composition? Let

$$P = \begin{pmatrix} 0.3 & 0.5 & 0.8 \\ 0 & 0.7 & 1 \\ 0.4 & 0.6 & 0.5 \end{pmatrix} \text{ and}$$

$$Q = \begin{pmatrix} 0.9 & 0.5 & 0.7 & 0.7 \\ 0.3 & 0.2 & 0 & 0.9 \\ 1 & 0 & 0.5 & 0.5 \end{pmatrix}, \text{ find } P \circ Q.$$

6. Answer any *four* from the following questions : 5×4=20

(a) Let A_i be a fuzzy set on X for $i \in I$, where I is an index set. Prove that

$\cup_{i \in I} \alpha_{A_i} \subseteq \alpha[\cup_{i \in I} A_i]$. Prove with example that equality does not hold good.

(b) Let $f : X \to Y$ be an arbitrary crisp function, then for any $A \in f(X)$, prove that ${}^{\alpha}[f(A)] \subseteq f({}^{\alpha}A)$. Prove with example that equality does not hold good.

(c) Prove that for all a, $b \in [0, 1]$, $i_{min}(a, b) \leq i(a, b) \leq \min(a, b)$, where i_{min} denotes the drastic intersection.

(d) Let A and B be two fuzzy sets numbers defined by

$$A(x) = \begin{cases} \frac{x+2}{2}, & -2 < x \leq 0 \\ \frac{2-x}{2}, & 0 < x < 2 \\ 0, & \text{otherwise} \end{cases}$$

And $$B(x) = \begin{cases} \frac{x-2}{2}, & -2 < x \leq 0 \\ \frac{6-x}{2}, & 0 < x \leq 6 \\ 0, & \text{otherwise} \end{cases}$$

Find the solution of the equation $A + X = B$.

(e) What is equilibrium of a fuzzy complement? Show that every fuzzy complement has at most one equilibrium.