Total No. of printed pages = 4

63/2 (SEM-3) MAT 305

2021

(held in 2022)

MATHEMATICS

Paper Code : MAT-305

(Special Theory of Relativity)

Full Marks – 80

Time – Three hours

The figures in the margin indicate full marks for the questions.

1. Answer any *five* of the following questions : 4×5=20

   (a) Show that Newton's laws of motion are invariant under Galilean transformations.

   (b) Discuss briefly the necessity of emergence of Lorentz transformations.

   (c) Give the geometrical interpretation of Lorentz transformation as a rotation.

[Turn over

(d) Two electrons are ejected in opposite directions from radioactive atoms in a sample of radioactive material at rest in the laboratory. Each electron has a speed 0.67c as measured by a laboratory observer. What is the speed of one electron as measured from the other according to the relativistic addition law of velocities ?

(e) The length of a space-ship is measured to be exactly half its proper length. Find the speed of the space-ship relative to the observer.

(f) If two events are simultaneous but separated in space in an inertial frame S, will they be simultaneous in any other inertial frame S' ? Explain.

2. (a) Establish the relation $E = mc^2$, where the symbols have their usual meanings. 6

(b) Derive the relation : 3+3=6

$$E^2 = p^2c^2 + m_0^2c^4$$

with usual meanings of the symbols. Hence show that the rest mass m_0 of a particle of momentum p and kinetic energy T is given by

$$m_0 = \frac{p^2c^2 - T^2}{2Tc^2}$$

(c) Calculate the velocity at which the mass of a particle becomes 8 times its rest mass. 3

(d) A body of rest mass m_0, travelling initially at a speed 0.6c, makes a completely inelastic collision with an identical body initially at rest. Find the speed of the resulting single body and its rest mass. 5

Or

The rest energy of an electron is 0.51 Mev. Calculate the amount of work to be done to increase the speed of electron from 0.6c to 0.8c.

3. (a) Discuss Minkowski's four dimensional space-time continuum. 10

Or

Show that the space-time interval

$$ds^2 = c^2(t_2 - t_1)^2 - (x_2 - x_1)^2 - (y_2 - y_1)^2 - (z_2 - z_1)^2$$

between two events in Minkowski's space is Lorentz invariant. Hence discuss and physically interpret the space-like and time-like intervals.

(b) Derive the expressions for four velocity, four force and four momentum and interpret them in terms of their three dimensional counterparts. 10

4. Answer any *two* of the following questions : 10×2=20

(a) Obtain the transformation equations of charge and current densities.

(b) Show that Maxwell's electromagnetic field equations are invariant under Lorentz transformations.

(c) Derive the transformation equations of electric field strength and magnetic field induction components.