Total No. of printed pages = 4

63/2 (SEM-3) MAT 302

2022

(Held in 2023)

MATHEMATICS

(Theory Paper)

Paper Code : MAT 302

(Graph Theory)

Full Marks – 80

Pass Marks – 32

Time – Three hours

The figures in the margin indicate full marks for the questions.

1. Answer any *four* from the following questions : $5\times4=20$

(a) What is a graph ? Define degree of a vertex in a simple graph. Show that the maximum number of edges in a simple graph with n vertices is $\frac{n(n-1)}{2}$. 2+3=5

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(b) Define bipartite and complete bipartite graph. Draw a 4-regular graph, $K_{3,3}$ and a multigraph. 2+3=5

(c) Define walk, trail and path of a graph. Give examples. Is a path trail ? Justify your answer. 3+2=5

(d) Define weighted graph and directed graph, give examples. Define adjacency matrix of a labelled graph. Write the adjacency matrix of K_5. 3+1+1=5

(e) What is component of a graph ? Show that for any graph G with six points, G or $\bar{G}$ contains a triangle. 1+4=5

2. Answer any *four* from the following questions : 5×4=20

(a) Define acyclic graph, give example. Show that if G is a (p, q) tree then, G is connected and $p=q+1$. 2+3=5

(b) Define a connected graph, give example. Is a non-separable graph connected ? Draw the following graphs : 1+1+3=5

(i) A graph that contain only one cut vertex without a bridge.

(ii) A graph that contain a bridge.

(iii) A block.

(c) Define connectivity of a graph G. For any graph G, show $K(G) \leq \lambda(G)$. 1+4=5

(d) Show that a graph H is the block graph of some graph if and only if every block of H is complete. 5

(e) Show that every tree has a center consisting of either one point or two adjacent points. 5

3. Answer any *four* from the following questions : 5×4=20

(a) Define Eulerian graph, give example. Show that a connected graph G is an Euler graph if and only if it can be decomposed into circuits. 1+4=5

(b) Define Hamiltonian cycle of a graph. Find all spanning cycles in the complete bigraphs $K_{4,3}$. 1+4=5

(c) Define point independence number and line independence number. Show that for any (p, q) graph G, $\alpha_1+\beta_1=p$. 2+3=5

(d) Define critical points and lines. Show that a point v is critical in a graph G if and only if some minimum points cover contains v. 2+3=5

(e) Write about Travelling salesman problem. 5

4. Answer any *four* from the following questions : 5×4=20

(a) When a graph is said to be n-factorable. Show that the graph K_{2n} is 1-factorable. 1+4=5

(b) Find out all the 2-factors of K_7. Is every regular graph 2-factorable ? Justify your answer. 3+2=5

(c) Define planar graph, give example. What are the faces of a plane graph ? Define them and give example. Is the complete graph K_6 planar ? 2+2+1=5

(d) Show that if G is any (p, q) planar graph in which every face is n-cycle, then

$$q = \frac{n(p-2)}{(n-2)}.$$ 5

(e) Define thickness and crossing number of a graph. Find the genus, thickness and crossing number of K_5 when it was embedded on plane. 2+3=5