Total No. of printed pages = 4

63/2 (SEM-3) MAT 303

2021

(held in 2022)

MATHEMATICS

(Theory Paper)

Paper Code : MAT-303

(Number Theory)

Full Marks – 80

Time – Three hours

The figures in the margin indicate full marks for the questions.

1. Answer any *four* of the following questions : $5\times4=20$

(a) Explain and verify Euclid's Algorithm.

(b) "If a and b are two integers not both zero then there exists integers x and y such that $g\,Cd(a,b) = ax + by$." Prove it.

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(c) State and prove fundamental theorem of Arithmetic.

(d) Solve the system of linear congruence :

$$x \equiv 3 \pmod 5; x \equiv 2 \pmod 3; x \equiv 2 \pmod 7.$$

(e) Find the general solution of $39x - 56y = 11$.

(f) If a_n is the n^{th} term of the Fibonacci sequence and $\alpha = \frac{1+\sqrt{5}}{2}$ then show that $a_n > \alpha^{n-1}, \forall n > 1$.

2. Answer any *four* of the following questions : 5×4=20

(a) Every even perfect number is of the form $2^{k-1}(2^k - 1)$; where $(2^k - 1)$ is prime.

(b) Every even perfect number $n = 2^{k-1}[2^k - 1]$ is the sum of first $2^{(k-1)/2}$ odd cubes.

(c) State and prove Fermat's Little theorem.

(d) If p and q are two distinct primes such that $a^p \equiv a \pmod q$ and $a^q \equiv a \pmod p$ then show that $a^{pq} \equiv a \pmod{pq}$. Also, verify it.

(e) "Fermat numbers are relatively prime", establish it.

(f) Prove that : if $p>2$ is a prime then any prime divisor of Mersene prime, Mp must be of the form $2kp+1$.

3. Answer any *four* of the following questions : 5×4=20

(a) Define Mobius function. Show that it is multiplicative.

(b) If n has k distinct prime factors and its divisors are arranged in an accending order as $d_1 < d_2 < \ldots < d_m$ then show that :

$$|\mu(d_1)| + |\mu(d_2)| + \ldots + |\mu(d_m)| = 2^k.$$

(c) Prove that : $\sum_{d|n} \phi(d) = n$.

(d) Explain complete residue system with example.

(e) Show that the set of integers $\{1,5,7,11\}$ is a RRS (mod 12) · (RRS : Reduced Residue System)

(f) Prove that $\sum_{d|n}(-1)^d\phi(d) = n - 2m$, m is the largest odd integer.

4. Answer any *four* of the following questions : 5×4=20

(a) State and prove Euler's Criterion.

(b) Solve the congruence : $5x^2 - 6x + 2 \equiv 0 \pmod{13}$.

(c) State and prove Gauss Lemma.

(d) Show that the congruence, $x^2 \equiv 105 \pmod{199}$ has no solution.

(e) Show that Jacobi symbol, $\left(\frac{2}{P}\right) = (-1)^{(P^2-1)/8}$

(f) Prove that, there is exactly $\frac{P-1}{2}$ quadratic residue and $\frac{P-1}{2}$ quadratic non residue mod P.