Total No. of printed pages = 9

63/2 (SEM-2) MAT 205(N/O)

2022

MATHEMATICS

(Theory Paper)

Paper Code : MAT 205(New)

(Mathematical Method and Computer Application)

Full Marks – 60

Time – Three hours

The figures in the margin indicate full marks for the questions.

1. Answer any *three* from the following questions : 5×3=15

(a) Show that the function $y(x) = (1+x^2)^{-3/2}$ is a solution of the Volterra integral equation

$$y(x) = \frac{1}{1+x^2} - \int_0^x \frac{t}{1+x^2}\, y(t)dt.$$

(b) Convert the following differential equation into integral equation $y'' + y = 0$ when $y(0) = y'(0) = 0$.

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(c) Find the eigen values and eigen function of the integral equation

$$y(x) = \lambda \int_0^1 (2xt - 4x^2)\ y(t)\,dt.$$

(d) Determine the resolvent kernel for the Fredholm integral equation having kernel $K(x,t) = e^{x+t}$; $a = 0,\ b = 1$.

2. Answer any *three* from the following questions : 5×3=15

(a) Find $L\left\{\sin\sqrt{t}\right\}$.

(b) Show that $L\left\{\frac{\cos\sqrt{t}}{\sqrt{t}}\right\} = \sqrt{\left(\frac{\pi}{s}\right)}\ e^{-1/4s}]$.

(c) If $L\left\{2\sqrt{\left(\frac{t}{\pi}\right)}\right\} = \frac{1}{s^{3/2}}$, show that

$$\frac{1}{s^{1/2}} = L\left\{\frac{1}{\sqrt{(\pi t)}}\right\}.$$

(d) If $L\left\{\sin\sqrt{t}\right\} = \frac{\sqrt{\pi}}{2s^{3/2}} e^{-1/4s}$, Show that

$$L\left\{\frac{\cos\sqrt{t}}{\sqrt{t}}\right\} = \sqrt{\left(\frac{\pi}{s}\right)}\ e^{-1/4s}.$$

(e) Find :

(i) $L^{-1}\left\{\frac{2s^2 - 6s + 5}{s^3 - 6s^2 + 11s - 6}\right\}$

(ii) $L^{-1}\left\{\frac{6s^2 + 22s + 18}{s^3 + 6s^2 + 11s + 6}\right\}$.

3. Answer any *three* from the following questions : 5×3=15

(a) Show that

$$^{-x}\cos x = \frac{2}{\pi}\int_0^\infty \frac{(u^2+2)\cos ux}{u^2+4}\,du,\ x > 0.$$

(b) Express $f(x) = \begin{cases} 1 & \text{for } 0 \le x \le \pi \\ 0 & \text{for } x > \pi \end{cases}$

as a Fourier sine integral and hence evaluate

$$\int_0^\infty \frac{1 - \cos\pi\lambda}{\lambda} \sin x\lambda\ d\lambda.$$

(c) Find the Fourier sine and cosine transform of f(x),

$$\text{if } f(x) = \begin{cases} x, & 0 < x < 1 \\ 2 - x, & 1 < x < 2 \\ 0, & x > 2 \end{cases}$$

(d) Find the Fourier cosine transform of e^{-x^2}.

(e) Solve $\partial u/\partial t = \partial^2 u/\partial x^2$, $x > 0$, $t > 0$, subject to conditions $u(0, t) = 0$.

$$u(x, 0) = \begin{cases} 1, & 0 < x < 1 \\ 0, & x > 1 \end{cases}, \quad u(x, t) \text{ is bounded.}$$

Give a possible physical interpretation of the problem.

4. Answer any *three* from the following questions : 5×3=15

(a) Prove the Euler's equation

$$\frac{d}{dx}\left(F - y'\frac{\partial F}{\partial y'}\right) - \frac{\partial F}{\partial x} = 0.$$

(b) Show that the Euler's equation for the functional $\int_{x_1}^{x_2} \{a(x)y'^2 + 2b(x)yy' + c(x)y^2\}dx$ is a second order linear differential equation.

(c) Find the extremals of the functional $\int_0^{\pi} (4y\cos x + y'^2 - y^2)\, dx$ that satisfy the given boundary conditions $y(0) = y(\pi) = 0$.

(d) Show that a necessary condition for

$I = \int_{x_1}^{x_2} F(x, y, y', y'')\, dx$ to be extremum is

$$\frac{\partial F}{\partial y} - \frac{d}{dx}\left(\frac{\partial F}{\partial y'}\right) + \frac{d^2}{dx^2}\left(\frac{\partial F}{\partial y''}\right) = 0.$$

(e) Find the extremal of the functional

$I[y(x)] = \int_0^1 (1 + y''^2)dx$, $y(0) = 0$, $y'(0) = 1$, $y(1) = 1$, $y'(1) = 1$.

(Theory Paper)

Paper Code : MAT 205(Old)

(Mathematical Method)

Full Marks – 80

Time – Three hours

The figures in the margin indicate full marks for the questions.

1. Answer any *two* of the following questions : $10\times2=20$

(i) Solve by using the method of successive approximations, the integral equation

$$y(x)=1+\lambda\int_0^1 xty(t)dt.$$

(ii) Define linear and non linear integral equation. Define first, second and third kind of an integral equation. Define symmetric and degenerate karnel with example. Show that the function $y(x)=xe^x$ is a solution of the integral equation

$$Y(x)=\sin x+2\int_0^x \cos(x-t)y(t)dt.$$

(iii) Solve the homogeneous integral equation of second kind $Y(x)=\lambda\int_0^{2\pi}\sin(x+t)\,y(t)dt$.

2. Answer any *two* of the following : $10\times2=20$

(i) Find $L\left\{\frac{1}{\sqrt{\pi t}}\right\}$. If $L\left\{2\sqrt{\left(\frac{t}{\pi}\right)}\right\}=\frac{1}{s^{3/2}}$ then prove that $\frac{1}{s^{1/2}}=L\left\{\frac{1}{\sqrt{\pi t}}\right\}$.

(ii) Show that

(a) $L^{-1}\left\{\frac{1}{s}\sin\frac{1}{s}\right\}=t-\frac{t^3}{(3!)^2}+\frac{t^5}{(5!)^2}-\frac{t^7}{(7!)^2}+\ldots\ldots$

(b) $L^{-1}\left\{\frac{1}{s^3+1}\right\}=\frac{t^2}{2!}-\frac{t^5}{5!}+\frac{t^8}{8!}-\frac{t^{11}}{11!}+\ldots\ldots$.

(iii) Using the method of Laplace transform solve the equation $\frac{d^2y}{dx^2}+4\frac{dy}{dx}+5y=(\cos t-\sin t)e^{-2t}$ subject to the boundary conditions $y(0)=1$, $y(0)=-3$.

3. Answer any *two* of the following : 10×2=20

(i) Find the sine transform of $\frac{x}{1+x^2}$.

(ii) Solve $\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}$ if $\frac{\partial u(0,t)}{\partial x}=0$,

$$u(x,0)=\begin{cases} x, & 0\le x<1 \\ 0 & x>1 \end{cases}.$$

And u(x, t) is bounded where $x>0, t>0$.

(iii) The temperature u in a semi-infinite rod $0\le x<\infty$ determined by $\frac{\partial u}{\partial t}=k\frac{\partial^2 u}{\partial x^2}$ with the conditions

(a) $u = 0$ when $t = 0$, $x > 0$,

(b) $\frac{\partial u}{\partial x}=\mu$ when $x = 0$,

(c) Partial derivatives of u tend to zero as $x\to\infty$.

Determine the temperature formula.

4. Answer any *two* of the following : 10×2=20

(i) Find the curve passing through the points (x_1, y_1) and (x_2, y_2) which when rotated about the x-axis gives a minimum surface area.

(ii) Show that the functional

$$\int_0^{\frac{\pi}{2}}\left\{2xy+\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2\right\}dt \text{ such that}$$

$$x(0)=0,\ x\left(\frac{\pi}{2}\right)=-1,\quad y(0)=0,\ y\left(\frac{\pi}{2}\right)=1$$

is stationary for $x = -\sin t$, $y = \sin t$.

(iii) Find the extremals of the functional

$I[y(x)]=\int_0^1 \frac{1+y^2}{y}dx$, through the origin and the point (1, 1).