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63/2 (SEM-1) MAT 102

2021

(held in 2022)

MATHEMATICS

(Theory Paper)

Paper Code : MAT-102 (New)

(Differential Equations and Computer Applications)

Full Marks – 60

Time – Three hours

The figures in the margin indicate full marks for the questions.

I. Answer the following questions : 1×10=10

1. The number of arbitary constants a general solution of first order equation contains :

   (a) 0 (b) 1

   (c) 2 (d) 3

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2. The n^{th} order homogeneous linear differential equation $y^{(n)} + p_0 y^{(n-1)} + ... + p_n y = 0$ has general solution if the coefficients $p_0(x)$, ..., $p_n(x)$ on some interval 1 are

(a) Continuous

(b) Discontinuous

(c) Discontinuous and differentiable

(d) None of these

3. Any equation contains n-arbitary constants, then the order of differential equation derived from it is

(a) n (b) n – 1

(c) 2 (d) n + 1

4. A first order differential equation M(x,y)dx + N(x,y)dy = 0 is exact if

(a) $\frac{\partial M}{\partial x} = \frac{\partial N}{\partial y}$ (b) $\frac{\partial M}{\partial x} \neq \frac{\partial N}{\partial y}$

(c) $\frac{\partial M}{\partial x} \pm \frac{\partial N}{\partial y}$ (d) $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

5. Solving by variation of parameter for the equation $y'' + y = \tan x$, the value of Wronskian is

(a) 1 (b) 2

(c) 3 (d) 4

6. Solution of $(mz-ny)p + (nx-lz)q = ly-mx$ is

(a) $f(lx + my + nz,\ x^2 - y^2 - z^2) = 0$

(b) $f(lx - my - nz,\ x^2 + y^2 + z^2) = 0$

(c) $f(lx + my + nz,\ x^2 + y^2 + z^2) = 0$

(d) $f(lx - my - nz,\ x^2 - y^2 - z^2) = 0$

7. Consider a general partial differential equation of second order of the form $Rr + Ss + Tt + f = 0$, where, R, S, T are continuous function of x and y, then

(a) Parabolic, if $S^2 - 4RT = 0$

(b) Hyperbolic, if $S^2 - 4RT < 0$

(c) Elliptical, if $S^2 - 4RT > 0$

(d) None of these

8. Particular integral of $(D^2 - D'^2)z = \cos(x + y)$

(a) $x \cos(x+y)$ (b) $\frac{x}{2}\cos(x+y)$

(c) $x \sin(x+y)$ (d) $\frac{x}{2}\sin(x+y)$

9. D' Alembert's equation of the wave equation is

(a) $u(x,t) = \phi(x+ct) + \psi(x-ct)$

(b) $u(x,t) = \phi(x+ct) + \psi(x-ct)$

(c) $u(x,t) = \phi'(x+ct) - \psi'(x-ct)$

(d) $u(x,t) = \phi(x-ct) + \psi(x+ct)$

10. The partial differential equation

$$x^2\frac{\partial^2 z}{\partial x^2} - (y^2-1)x\frac{\partial^2 z}{\partial x \partial y} + y(y-1)^2\frac{\partial^2 z}{\partial y^2} + x\frac{\partial z}{\partial x}$$

$+y\frac{\partial z}{\partial y} = 0$ is Hyperbolic in a region in a XY plane, if

(a) $x = 0$ and $y = 1$

(b) $x \neq 0$ and $y = 1$

(c) $x = 0$ and $y \neq 1$

(d) $x \neq 0$ and $y \neq 1$.

II. Answer any *eight* of the following questions : 5×8=40

1. Using the Picard's method of successive approximations, find the third approximation of the solution of the equation : $\frac{dy}{dx} = x + y^2$ where $y = 0$ when $x = 0$.

2. For the initial value problem $\frac{dy}{dx} = e^y$, $y(0) = 0$. Find the largest interval $|x| \leq a$ in which the Picard's theorem holds.

3. Reduce the one-dimensional wave equation $\frac{\partial^2 z}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 z}{\partial t^2}$ to canonical form.

4. Solve : $x^2y'' + xy' - y = x^2e^x$.

5. Solve : $x^2y'' + y = 3x^2$.

6. Find the surface which intersects the surfaces of the system $z(x+y) = c(3z+1)$ orthogonally and which passes through the circle $x^2 + y^2 = 1$, $z = 1$.

7. Prove that : for the equation $\frac{\partial^2 z}{\partial x \partial y} + \frac{z}{4} = 0$, the Green's function is

$$w(x, y; \xi, \eta) = J_0(\sqrt{(x-\xi)(y-\eta)})$$

where $J_0(z)$ denotes Bessel's function of the first kind of order zero.

8. Find the eigen values and eigen functions of Strum-Liouville problem :

$$X'' + \lambda X = 0; \; X'(0) = 0; \; X'(L) = 0.$$

9. Find the complete integral of $pxy + pq + qy = yz$.

10. Find general solution of one-dimensional heat equation :

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{k}\frac{\partial u}{\partial t}.$$

III. Answer any *one* of the following questions : 10×1=10

1. Show that the general solution of Legendre's equation $(1-x^2)y'' - 2xy' + n(n+1)y = 0$, where $n \in Z^+$ is $y = AP_n(x) + BQ_n(x)$ where $P_n(x)$ and $Q_n(x)$ have their usual meaning.

2. Solve in series : $x^2y'' + xy' + (x^2 - n^2)y = 0$.

3. Find general solution of two-dimensional Laplace's equation : $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$.

(Theory Paper)

Paper Code : MAT-102 (Old)

(Differential Equations)

Full Marks – 80

Time – Three hours

The figures in the margin indicate full marks for the questions.

1. Choose the correct option : 1×10=10

(i) Any equation contains n-arbitrary constants, then the order of differential equation derived from it is

(a) n (b) n – 1

(c) 2 (d) n +1

(ii) The n^{th} order homogeneous linear differential equation have

(a) n singular solution

(b) No singular solution

(c) One singular solution

(d) None of these

(iii) The Wronskian value of the equation : $y'' + 4y = \tan 2x$ is

(a) 1 (b) 2

(c) 3 (d) 4

(iv) Which of the following is false, when find the general solution of Riccati's equation ?

(a) $\frac{dy}{dx} = P + Qy + Ry^2$ is the form of Riccati's equation.

(b) $y = -\frac{du}{dx}(Ru)^{-1}$

(c) $U = Af(x) + Bf(x)$

(d) $R\frac{d^2u}{dx^2} + \left(QR + \frac{dR}{dx}\right)\frac{du}{dx} + PR^2u = 0$.

(v) Which of the following is not related to Strum-Liouville problem :

(a) $[r(x)y']' + [q(x) + \lambda p(x)]y = 0$

(b) $a_1y(a) + a_2y'(a) = 0$

(c) $b_1y(b) + b_2y'(b) = 0$

(d) $P(x) < 0$ on $a \le x \le b$

(vi) Solution of $xp + yq = z$ is

(a) $\phi\left(\frac{x}{y}, \frac{y}{z}\right) = 0$

(b) $\phi\left(\frac{x}{z}\right) = \frac{y}{z}$

(c) $\phi\left(\frac{y}{x}, \frac{z}{y}\right) = 0$

(d) All of these

(vii) The complete integral of $z = px + qy - \sin pq$ is

(a) $z = ax + qy - \sin aq$

(b) $z = ax - by + \sin ab$

(c) $z = bx + ay - \sin ab$

(d) None of these

(viii) Particular integral of $(2D^2 - 3DD' + D'^2)z = e^{x+2y}$ is

(a) $\frac{1}{2}e^{x+2y}$ (b) $-\frac{x}{2}e^{x+2y}$

(c) xe^{x+2y} (d) x^2e^{x+2y}

(ix) D'Alembert's solution of the wave equation is

(a) $u(x,t) = \phi(x+ct) - \psi(x-ct)$

(b) $u(x,t) = \phi(x+ct) + \psi(x-ct)$

(c) $u(x,t) = \phi'(x+ct) - \psi'(x-ct)$

(d) $u(x,t) = \phi(x+ct) + \psi(x+ct)$

(x) The general 2nd order partial differential equation of the form

$Rr + Ss + Tt + f = 0$ where R, S, T are continuous function of x and y then

(a) Parabolic, it $S^2 - 4RT = 0$

(b) Hyperbolic, if $S^2 - 4RT < 0$

(c) Elliptical, if $S^2 - 4RT > 0$

(d) None of these.

2. Answer any *six* of the following questions : $5\times6=30$

(a) Solve : $y'' + a^2y = \tan ax$ (Method of Variation of Parameter).

(b) Show that $f(x,y) = xy^2$ satisfies the Lipschitz condition on the rectangle $R : |x| \le 1, |y| \le 1$ but does not satisfy Lipschitz condition on the strip $S : |x| \le 1, |y| < \infty$.

(c) Show that the set of functions $\{\cos nx\}$, $n=0,1,2,3,....$ is orthogonal on the interval $-\pi \le x \le \pi$, and find the corresponding orthonormal set of functions.

(d) Solve : $(y+z)p+(x+z)q=(x+y)$ where $p=\frac{\partial z}{\partial x}$ and $q=\frac{\partial z}{\partial y}$.

(e) Find complete integral, general integral and singular integral of $z=px+qy=pq$.

(f) Reduce the one-dimensional wave equation, $\frac{\partial^2 z}{\partial x^2}=\frac{1}{c^2}\frac{\partial^2 z}{\partial t^2}$ to canonical form.

(g) Using Picard's method of successive approximations, find the third approximation of the solution of the equation :

$$\frac{dy}{dx}=x+y^2 \text{ where } y=0 \text{ when } x=0.$$

(h) Solve : $x^2\frac{d^2z}{dx^2}+x\frac{dz}{dx}-4z=0$.

(i) Solve : $(D^2+2DD'+D'^2)z=e^{2x+3y}$ where $D\equiv\frac{\partial}{\partial x}$ and $D'\equiv\frac{\partial}{\partial y}$.

3. Answer any *five* of the following questions : 8×5=40

(a) Solve : $\frac{d^2y}{dx^2}-\cot x\frac{dy}{dx}-(1-\cot x)y=e^x \sin x$.

(b) Find the integral surface of Linear partial differential equation, $x(y^2+z)p-y(x^2+z)q=z(x^2-y^2)$; which contains a straight line $x+y=0$ and $z=1$.

(c) Find the general solution of Bessel's equation

$$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+(x^2-x^2)y=0.$$

(d) Find all the eigenvalues and eigenfunctions of the Strum-Liouville problem $X''+\lambda X=0$ with $X'(0)=0$; $X'(L)=0$.

(e) Find general solution of heat flow equation :

$$\frac{\partial^2 u}{\partial x^2}=\frac{1}{k}\left(\frac{\partial u}{\partial t}\right),\ k>0.$$

(f) Find general solution of two-dimensional Laplace's equation :

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0.$$

(g) Solve in series : $(1-x^2)\frac{d^2y}{dx^2} = 2x\frac{dy}{dx} + n(n+1)$

$y = 0$; $n \in z^+$.

(h) Find the complete integral of :

$pxy + pq + qy = yz$.