63/2 (SEM-1) MAT 103

2021

(held in 2022)

MATHEMATICS

(Theory Paper)

Paper Code : MAT-103

(Mechanics)

Full Marks – 80

Time – Three hours

The figures in the margin indicate full marks
for the questions.

1. Choose the correct option : 1×5=5

(i) The path that gives the shortest distance between two points on a sphere is a

(a) Straight line

(b) Circle

(c) Great Circle

(d) Sphere

[Turn over

(ii) The action integral must be a _____ value for actual path.

(a) real (b) stationary

(c) maximum (d) minimum

(iii) Poisson's bracket is useful to find the

(a) Equation of motion

(b) Lagrange's function

(c) Hamilton's function

(d) Integrals of motion

(iv) Which property holds good

(a) $\{q_i, -q_j\} = 1$ (b) $\{q_i, p_j\} = \delta_{qp}$

(c) $\{q_i, q_j\} = 1$ (d) $\{q_i, q_j\} = 0$

(v) The δ-variation considered refers to variation

(a) quantity (b) time

(c) coordinate (d) integration.

2. What do you mean by geodesics ? How do you find the extremal of a function ? 2

3. State Hamilton's principle. What is the solid figure of revolution for which, for a given surface area, has maximum volume ? 2

4. What is the Lagrangian of a free particle ? Under which condition total energy T+V is conserved ?

2

5. What do you mean by configuration space ?

2

6. Define generating function. What is the value of K.E. for a simple pendulum ? 2

7. Answer any *five* of the following : 5×5=25

(i) Discuss the Brachistochrone problem.

(ii) Deduce Euler-Lagrange equation from Hamilton's principle.

(iii) Discuss about the lagrangian of compound pendulum.

(iv) Show that the transformation

$$Q = \sqrt{(2q)}\, e^{\alpha} \cos p$$

$$P = \sqrt{(2q)}\, e^{-\alpha} \sin p$$

is a canonical transformation.

(v) Establish the relation

$$\sum_{l-1}^{2n}\{u_l, u_i\}[u_l, u_j] = \delta_{ij}$$

for set of 2n independent functions u_l of coordinates $(q_1, q_2, ..., q_n)$ and $(p_1, p_2, ..., p_n)$.

(vi) What do you mean by Poisson's bracket? Mention all the properties of Poisson's bracket.

(vii) Derive the Hamilton-Jacobi equation.

8. Discuss about the Hamilton-Jacobi equation for Hamilton's characteristic function.   6

9. Explain about different types of constraints. 6

10. Prove that $[aF+bG, w] = a[F, w] + b[G, w]$. 8

Discuss the conditions for a transformation to be canonical.

11. Answer any *two* of the following: $10 \times 2 = 20$

(i) Discuss briefly about generalized coordinates. Derive the Lagrange's equation of first kind.

(ii) Deduce the equations of generalized velocity and generalized acceleration. Prove that the geodesics of a right circular cylinder is circular helix.

(iii) Discuss about all the four forms of generating function.