Total No. of printed pages = 12

63/2 (SEM-1) PHY 103

2021

(held in 2022)

PHYSICS

(Theory Paper)

Paper Code : PHY-103 (New)

(Quantum Mechanics-I)

Full Marks – 80

Time – Three hours

The figures in the margin indicate full marks for the questions.

1. Choose the correct answer from the following : $1\times5=5$

(a) Perturbation theory is a systematic procedure for obtaining

(i) Exact solutions to the unperturbed problem

(ii) Exact solutions to the perturbed problem

(iii) Approximate solutions to the perturbed problem

(iv) None of the above

[Turn over

(b) The commutation relation between x and P_x^n is

(i) $ni\hbar P_x^{n-1}$ (ii) $ni\hbar P_x^N$

(iii) $n\hbar P_x^{n-1}$ (iv) $n\hbar P_x^N$

(c) The wave function $\psi(\bar{r},t)$ is said to be normalized if

(i) $\int |\Psi(\bar{r},t)| d\tau = 1$

(ii) $\int |\Psi(\bar{r},t)|^2 d\tau = 1/N^2$

(iii) $\int |\Psi(\bar{r},t)|^2 d\tau = 1$

(iv) $\int |\Psi^*(\bar{r},t)| d\tau = 1$

(d) Stationary states are those for which the probability density ρ is

(i) time-dependent (ii) time-independent

(iii) space-dependent (iv) space-independent

(e) If R and T be the reflection and transmission probabilities of a particle through a potential step, then

(i) $R - T = \text{constant}$ (ii) $R + T = 1$

(iii) $R + T = 0$ (iv) $R + T = \infty$

2. Answer the following questions : 2×5=10

(a) Give Born's interpretation of wave function.

(b) Write two properties of ket vectors.

(c) Define projection operator. Show that the operator $|\phi_i\rangle\langle\phi_i|$ is a projection operator when $|\phi_i\rangle$ is normalized.

(d) Explain the significance of zero-point energy of a particle in a one-dimensional box.

(e) Why the prescription of non-degenerate perturbation theory is not applicable for degenerate energy levels ?

3. Answer any *five* of the following : 5×5=25

(a) Define expectation value. A particle constrained to move along X-axis in the domain $0 \le x \le L$ has a wave function $\Psi(x) = \sin\left(\frac{n\pi x}{L}\right)$. Normalize the wave function and find the expectation value of its momentum. 1+2+2=5

(b) Define dimension and basis of a vector space. Show that every vector in a finite dimensional vector space V over the field F can be uniquely expressed as a linear combination of the vectors of its basis. 2+3=5

(c) State the admissibility conditions of wave functions. A wave function of a particle is $\Psi(x) = Ae^{-ax^2}$ defined in the domain $-\infty < x < \infty$. Find the probability of finding the particle in the region $0 < x < \infty$. 2+3=5

(d) What is density of state ? Show that the density of states per unit length is proportional to $E^{-\frac{1}{2}}$. 2+3=5=10

(e) Find the first order correction and hence the ground state energy for a harmonic oscillator, when a perturbation γx^4 is applied. 5

(f) Suppose we put a delta function bump i.e. $H' = \alpha\delta\left(x - \frac{a}{2}\right)$ in the center of the infinite square well. Find the 2nd order correction to the energies. 5

(g) Discuss briefly the implication of natural unit. Show that 2+3=5

$1\text{m} \approx 5.07 \times 10^{15}\, \hbar c/\text{GeV}$.

4. Answer any *four* of the following questions : 10×4 = 40

(a) A particle traveling with energy E along X-axis has in its path a rectangular potential barrier of height $V_0 > E$ and width *a*.

(i) Show that there is a finite probability of transmission even if $E < V_0$

(ii) Apply the result to explain the phenomenon of α-decay in nuclei. 5+5=10

(b) Using operator algebra, derive the following expression for uncertainty relation between two operators :

$$\Delta A \Delta B \geq \frac{1}{2} |\langle [\hat{A}, \hat{B}] \rangle|$$

Hence derive the expression for Heisenberg uncertainty relation between the components of position and momentum operators. 8+2=10

(c) Write a few important features of Klein-Gordon equation. Derive the Klein-Gordon equation in covariant form. Show that Klein-Gordon equation can not predict the spin i.e it only describes spin-zero particles. 2+3+5 =10

(d) Discuss briefly the variational method. Using the principle of variational method estimate the ground state energy of the hydrogen atom. Use the trial wave function as $\Psi(r,\theta,\phi) = e^{-r/\alpha}$. Calculate the dimension of the scale parameter α. 3+6+1 =10

(e) Using first-order perturbation theory, estimate the energy of the first excited state for a cubical well : 10

$$V(x,y,z) = \begin{cases} 0, & \text{if } 0<x<L,\ 0<y<L,\ 0<z<L \\ \infty & \text{otherwise} \end{cases}$$

if the following perturbation is applied,

$$H' = V_0 L^3 \delta\left(x - \frac{L}{4}\right)\delta\left(y - \frac{3L}{4}\right)\delta\left(z - \frac{L}{4}\right).$$

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(b) The commutation relation between x and P_x^n is

(i) $ni\hbar P_x^{n-1}$ (ii) $ni\hbar P_x^n$

(iii) $n\hbar P_x^{n-1}$ (iv) $n\hbar P_x^n$

(c) The momentum operator in one dimension is

(i) $-i\hbar\frac{\partial}{\partial x}$ (ii) $i\hbar\frac{\partial}{\partial x}$

(iii) $-i\hbar\frac{\partial}{\partial t}$ (iv) $-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}$

(d) If R and T be the reflection and transmission probabilities of a particle through a potential step, then

(i) $R-T=\text{constant}$ (ii) $R+T=1$

(iii) $R+T=0$ (iv) $R+T=\infty$

(e) Which one of the following is correct ?

(i) $a_+\Psi_n=\sqrt{n+1}\,\Psi_{n+1}$

(ii) $a_-\Psi_n=\sqrt{n-1}\,\Psi_{n-1}$

(iii) $a_+\Psi_n=\sqrt{n-1}\,\Psi_{n+1}$

(iv) $a_+\Psi_n=\sqrt{n}\,\Psi_{n+1}$

2. Answer the following questions : 2×5=10

(a) Give Born's interpretation of wave function.

(b) Write two properties of ket vectors.

(c) Why the prescription of non-degenerate perturbation theory is not applicable for degenerate energy levels ?

(d) Show that the quantity $|\Psi\rangle\langle\Psi|$ is a projection operator when $|\Psi\rangle$ is normalized.

(e) Consider the ket: $|\Psi\rangle=\begin{pmatrix}5i\\2\\-i\end{pmatrix}$. Is $|\Psi\rangle$ normalized ?

If not, normalize it.

3. Answer any *five* of the following : 5×5=25

(a) Define expectation value. A particle constrained to move along X-axis in the domain $0\le x\le L$ has a wave function $\Psi(x)=\sin\left(\frac{n\pi x}{L}\right)$. Normalize the wave function and find the expectation value of its momentum. 1+2+2=5

(b) Find the constant β so that the states $|\Psi\rangle = \beta|\phi_1\rangle + 3|\phi_2\rangle$ and $|x\rangle = 4\beta|\phi_2\rangle - 9|\phi_2\rangle$ are orthogonal. Given, the states $|\phi_1\rangle$ and $|\phi_2\rangle$ are orthonormal.

(c) What does commutation relation in quantum mechanics signify ? If $A = \frac{d}{dx} + x$ and $A^\dagger = -\frac{d}{dx} + x$, evaluate $[A, A^\dagger]$. 1+4=5

(d) Define Hermitian and skew-Hermitian operator. Show that $(\hat{A} + \hat{A}^\dagger)$ is Hermitian while $i(\hat{A} + \hat{A}^\dagger)$ is skew-Hermitian. 1+1+1½+1½=5

(e) What do you mean by projection operator ? Show that the product of two commuting projection operators is also a projection operator. 1+4=5

(f) Suppose we put a delta function bump i.e. $H' = \alpha\delta\left(x - \frac{a}{2}\right)$ in the center of the infinite square well. Find the 2nd order correction to the energies. 5

4. Answer any *four* of the following questions : 10×4=40

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$$\Delta A \Delta B \geq \frac{1}{2} |\langle [\hat{A}, \hat{B}] \rangle|.$$

Hence derive the expression for Heisenberg uncertainty relation between the components of position and momentum operators. 8+2=10

(b) Calculate the ground state energy and normalized ground state wave function for a linear harmonic oscillator with the help of operator algebra. 10

(c) Using first-order perturbation theory, estimate the energy of the first excited state for a cubical well : 10

$$V(x, y, z) = \begin{cases} 0, & \text{if } 0<x<L,\ 0<y<L,\ 0<z<L \\ \infty & \text{otherwise} \end{cases}$$

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