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63/2 (SEM-1) PHY 101

2021

(held in 2022)

PHYSICS

(Theory Paper)

Paper Code : PHY-101

(Mathematical Physics–I)

Full Marks – 80

Time – Three hours

The figures in the margin indicate full marks for the questions.

1. Answer *all* the following questions : $1\times5=5$

(a) The value of

$$\int_{-1}^{+1} p_3^2(x)\,dx$$

(i) 4/7 (ii) 3/7

(iii) 2/7 (iv) 1/7

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(b) The value of $\Gamma(1/4)\ \Gamma(3/4)$ is

(i) $2\sqrt{\pi}$ (ii) $\sqrt{2}\pi$

(iii) $\sqrt{2\pi}$ (iv) $\sqrt{\pi}$

(c) Choose the correct relation

(i) $P_2(x) = 3x\,P_1(x) + \frac{1}{2}P_0(x)$

(ii) $P_2(x) = \frac{3}{2}x\,P_1(x) - \frac{1}{2}P_0(x)$

(iii) $P_2(x) = 3x\,P_1(x) - \frac{1}{2}P_0(x)$

(iv) $P_2(x) = \frac{1}{2}x\,P_1(x) + \frac{3}{2}P_0(x)$

(d) The C-R equation in Polar form is given by

(i) $\frac{du}{dr} = \frac{1}{r}\frac{dv}{d\theta},\ \frac{dv}{d\theta} = -\frac{1}{r}\frac{du}{dr}$

(ii) $\frac{du}{d\theta} = \frac{1}{r}\frac{dv}{dr},\ \frac{du}{dr} = r\frac{dv}{d\theta}$

(iii) $\frac{du}{d\theta} = r\frac{dv}{dr},\ \frac{du}{dr} = \frac{1}{r}\frac{dv}{d\theta}$

(iv) $\frac{du}{dr} = r\frac{dv}{d\theta},\ \frac{du}{d\theta} = -r\frac{dv}{dr}$

(e) If u and v be any two vectors in an inner product space V, then $\|u + v\|$

(i) $< \|u\| + \|v\|$ (ii) $= \|u\| + \|v\|$

(iii) $\leq \|u\| + \|v\|$ (iv) $= \|u\|\ \|{<}v\|$

2. Answer *all* the following questions : 2×5=10

(a) Express the following function in terms of Legendre polynomial

$$f(x) = 1 + 2x - 3x^2 + 4x^3$$

(b) Show that

$$\Gamma(n)\,\Gamma(1-n) = \frac{\pi}{\sin n\pi}$$

(c) Prove that

$$J_{-n}(x) = (-1)^n\ J_n(x)$$

(d) Check the differentiability of the function $f(z) = \overline{z}$ at origin.

(e) Examine whether the following sets of function are linearly independent or dependent on the real x – axis

(i) $f(x) = x,\ g(x) = x^2,\ h(x) = x^3$

(ii) $f(x) = x,\ g(x) = 5x,\ h(x) = x^2$

3. Answer any *five* of the following questions : 5×5=25

(a) If $J_n(x)$ is the Bessel function of first kind of order n, then show that 5

$$e^{\frac{x}{2}\left(t-\frac{1}{t}\right)} = \sum_{n=-\infty}^{\infty} J_n(x)t^n$$

(b) Using generating function of Legendre polynomials, prove that 5

$$\int_{-1}^{+1} [P_n(x)]^2 \, dx = \frac{2}{2n+1}$$

(c) Establish the following relation 5

$$\beta(m,n) = \frac{\Gamma(m), \Gamma(n)}{\Gamma(m+n)}$$

(d) Evaluate the following integral 5

$$\oint \frac{z^2 - z + 1}{z - 1} dz$$

at (i) $|z| = 1$ and (ii) $|z| = \frac{1}{2}$

(e) Define harmonic function. Find the harmonic conjugate of the function : 1+3+1=5

$$u(x, y) = 2x(1 - y)$$

When the variables u and v are called conjugate harmonics ?

(f) Define inner product of a pair of vectors. What is 'norm' of a vector ? If u and v be any two vectors in an inner product space V over the field F, then show that

$$|(u,v)| \leq \|u\| \, \|v\|$$

Where $|(u,v)|$ denotes the modulus of the numbers (u, v) real or complex.

4. Answer any *four* of the following questions : 10×4=40

(a) Solve the following differential equation using power series method : 10

$$9x(1-x)\frac{d^2y}{dx^2} - 12\frac{dy}{dx} + 4y = 0.$$

(b) (i) Deduce the following Rodrigues formula from Legendre's differential equation : 8+2=10

$$P_n(x) = \frac{1}{2^n n!} \frac{d^n(x^2-1)^n}{dx^n}$$

(ii) Find the Legendre polynomials for $n = 1$ and 2.

(c) Obtain the Legendre duplication formula

8+2=10

$$\Gamma(m)\,\Gamma\left(m+\frac{1}{2}\right)=\frac{\sqrt{\pi}\,\Gamma(2m)}{2^{2m-1}}$$

and show that

$$\beta(m,m)=2^{2m-1}\beta\left(m,\frac{1}{2}\right)$$

(d) Derive the Cauchy-Riemann equation in the polar form. Show that the function $f(z) = e^x(\cos y - i\sin y)$ is analytic and evaluate $f'(z)$. 5+5=10

(e) (i) Determine the value of α, β, γ

when $\begin{bmatrix} 0 & 2\beta & \gamma \\ \alpha & \beta & -\gamma \\ \alpha & -\beta & \gamma \end{bmatrix}$ is orthogonal.

(ii) Find the eigenvalues and eigenvectors of the matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & -1 \\ 0 & -1 & 3 \end{bmatrix}$$

5+5=10