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63/2 (SEM-1) PHY 102

2021

(held in 2022)

PHYSICS

(Theory Paper)

Paper Code : PHY-102

(Classical Mechanics)

Full Marks – 80

Time – Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : $1\times5=5$

   (a) In Lagrange's equation, virtual displacement does not involve

   (i) Time

   (ii) Space

   (iii) N number of particles

   (iv) None of the above

[Turn over

(b) If the Lagrangian does not depend on time explicitly

(i) the kinetic energy is constant

(ii) the potential energy is constant

(iii) the Hamiltonian is constant

(iv) the Hamiltonian cannot be constant

(c) In variational principle the line integral of some function between two end points is

(i) Zero (ii) One

(iii) Extremum (iv) Infinite

(d) If the Poisson bracket of a function with the Hamiltonian vanishes

(i) The function depends upon time

(ii) The function is constant of motion

(iii) The function is not the constant of motion

(iv) None of the above

(e) Which of the following is true ?

(i) The Jacobian determinant of univalent canonical transformation is unity.

(ii) The volume element of in the phase space remain invariant under canonical transformation.

(iii) The Poisson bracket operation is used in phase space

(iv) All of the above

2. Answer the following questions : 2×5=10

(a) Show that the generalized momentum conjugate to a cyclic coordinate is conserved.

(b) Describe Hamiltonian and Hamilton's equation for an ideal spring mass arrangement.

(c) If the rigid body with one point fixed rotates about principle axis of the body, then show that kinetic energy of the body is constant throughout the motion.

(d) Show that the transformation

$Q = \sqrt{2q}\, e^{a} \cos p$ and $P = \sqrt{2q}\, e^{-a} \sin p$ is canonical.

(e) Show that the Poisson bracket $[q_k, p_k] = 1$.

3. Answer any *five* of the following questions : 5×5=25

(a) What is Hamilton's principle? Obtain Lagrange's equation of motion from this principle. 1+4=5

(b) Construct the Lagrangian of Atwood machine and derive its equation of motion. 5

(c) Deduce Lagrange equation of motion of a compound pendulum in a vertical plane about a fixed horizontal axis. 5

(d) A canonical transformation $(p, q) \to (P, Q)$ is performed on the Hamiltonian $H = \frac{p^2}{2m} + \frac{1}{2}mw^2q^2$ via a generating function $F = \frac{1}{2}mwq^2 \cot Q$. Plot the Q(t) vs. t graph. 5

(e) What is stable equilibrium? Obtain the Lagrangian of small oscillation and deduce the equation of motion. 1+4=5

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(f) If the potential 5

$$V(x) = -\frac{1}{2}x^2 + \frac{1}{4}x^4$$

Find the

(i) Equilibrium position

(ii) Identify the stability

(iii) Draw the V(x) vs. x graph.

4. Answer any *four* of the following questions : 10×4=40

(a) (i) Define generalized momentum. Derive Hamilton's canonical equation of motion. 1+4=5

(ii) Obtain Hamilton's equation of motion in terms of spherical coordinates. 5

(b) (i) Define Euler's angles and obtain an expression for the complete transformation matrix. 1+6=7

(ii) Deduce Euler's equations of motion for a rotating rigid body with a fixed point in Newtonian method. 3

(c) Find the canonical transformation equations corresponding to the generating function $F_1(q,Q,t)$ and $F_3(q,Q,t)$ and $F_4(q, P, t)$. 10

(d) Obtain the Hamilton-Jacobi differential equation for Hamilton's principal function and solve the harmonic oscillator problem by Hamilton-Jacobi method. 10

(e) Discuss the significance of canonical transformation. Show that the volume element in the phase space remains invariant under canonical transformation. 2+8=10