Total No. of printed pages = 5

63/2 (SEM-3) PHY 301

2022

(Held in 2023)

PHYSICS

(Theory Paper)

Paper Code : PHY 301

(Mathematical Physics-II)

Full Marks – 80

Pass Marks – 32

Time – Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : $1\times5=5$

(a) Value of $L(3t^2 + 5\cos 4t)$ is

(i) $\frac{6}{s^3}+\frac{5s}{s^2+16}$ (ii) $\frac{18}{s^3}+\frac{20}{s^2+16}$

(iii) $\frac{18}{s^3}+\frac{20}{s^2-16}$ (iv) $\frac{6}{s^3}+\frac{5s}{s^2-16}$

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(b) Find the inverse Laplace transform for $\frac{s}{s^2a^2+b^2}$

(i) $\frac{1}{a^2}\cos\left(\frac{a}{b}\right)t$ (ii) $\frac{1}{a^2}\cos\left(\frac{b}{a}\right)t$

(iii) $\frac{1}{a^2}\sin\left(\frac{a}{b}\right)t$ (iv) $\frac{1}{a^2}\sin\left(\frac{b}{a}\right)t$

(c) If λ belongs to the eigenvalue spectrum of K (kernel), what happens to the solutions of homogeneous and inhomogeneous Fredholm integral equations ?

(d) Write down the expression for divergence of a contravariant tensor A^p.

(e) Write the following using the Einstein summation convention,

$$d\phi = \frac{\partial\phi}{\partial x^1}dx^1 + \frac{\partial\phi}{\partial x^2}dx^2 + \ldots + \frac{\partial\phi}{\partial x^N}dx^N.$$

2. Answer the following questions : 2×5=10

(a) Find the Laplace transform $e^{\lambda t}$.

(b) Find inverse Laplace transform of $\frac{2s-5}{9s^2-25}$.

(c) Write down the expressions for the following partial differential equations :

(i) Schrodinger equation

(ii) Heat flow equation in three dimension

(iii) Poisson's equation.

(d) Write down the wave equation in polar coordinate which represents vibration of a circular membrane.

(e) Show that :

$$\left\{ {s \atop pq} \right\} = \left\{ {s \atop qp} \right\}.$$

3. Answer any five of the following questions : 5×5=25

(a) Find the inverse Laplace transform of

$$\frac{14s+10}{49s^2+28s+13}.$$

(b) Develop 1–Dimensional wave equation from the transverse vibration of a stretched string.

(c) Solve the integral equation :

$f(x) = 1 + \lambda\int_a^b dy\, K(x,y)\, f(y)$ for the Kernel $K(x,y) = x + y$.

(d) Find the eigenvalues of the homogeneous Fredhlom integral equation

$$y(x) = \lambda \int_0^{\pi} \sin(x+z)\, y(z)\, dz.$$

(e) Determine the conjugate metric tensor in cylindrical coordinates.

(f) Express the Laplacian $\phi, \nabla^2\phi$ in spherical polar coordinates.

4. Answer any *four* of the following questions : 10×4=40

(a) (i) Find the Laplace transform of

$$F(t) = \begin{cases} 1 & 0 \le t \le 1 \\ t & 1 \le t \le 2 \\ t^2 & 2 \le t \le \infty \end{cases}$$

(ii) If $L\{f(t)\} = F(s)$ then show that

$$L\{f(at)\} = \frac{1}{a} F\left(\frac{s}{a}\right).$$ 5+5=10

(b) Develop one dimensional heat flow equation and solve it by using method of separation of variables. 10

(c) Solve the time dependent heat flow equation in one dimension for an infinite bar which is insulated laterally given by

$$\frac{\partial\psi}{\partial t} = h^2 \frac{\partial^2\psi}{\partial x^2}.$$

Where ψ is the temperature. The initial temperature $\psi(x, 0)$ of the bar $\phi(x)$ is a known function of x for $-\alpha < x < \alpha$. 10

(d) Solve the Laplace's equation (2-D) for steady state heat flow on a rectangular plate bounded by x=0, x=1 and y=0 and y=α (infinity). Temperature of the plate at the edge y=0 is constant i.e. $\psi = f(x)$ at y=0. Note that initial condition has passed and temperature is not varying with time. 10

(e) Define covariant derivative of a contravariant tensor. Show that covariant derivative of a tensor is a tensor. 1+9=10

(f) With the aid of the resolent Kernel, find the solution of the integral equation : 10

$$f(x) = \varphi(x) + \lambda \int_0^x dt\, e^{x-t} f(t).$$