2015

MATHEMATICS

Paper : 101 (Old Course)

ALEGEBRA

Full Marks : 80

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer any three of the followings: $3 \times 5 = 15$

   (a) If G is an abelian group with N is any subgroup of G, prove that G/N is also abelian.

   (b) Prove that an infinite cyclic group is isomorphic to the additive group of integers, (Z,+).

   (c) If G is a cyclic group of order 30 with a generator a, then find all the distinct element of the subgroup gen erated by a^5. Also find all the generator of the group G.

   (d) By Using the Sylow's Theorem, show that a group of order 35 is not a simple group.

2. Answer the followings: $5 \times 2 = 10$

   (a) Prove that if G is a finite p-group, then G is solvable.

   (b) If G is an abelian group having a composition series

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then show that G is finite.

3. If R is a commutative ring with unity and M is an ideal of R then M is a maximal ideal of R if and only if R/M is a field. 7

4. Answer any two of the followings: 4 × 2 =8

(a) Prove that for any two ideals A and B of a ring R, $A \cup B$ is an ideal of R if and only if either $A \subseteq B$ or $B \subseteq A$.

(b) Let R be a ring and R_1 be defined by the set $R_1 = \{\begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix} : a \in R\}$. Then show that there exists an isomor phism from R to R_1.

(c) Let I be an ideal of a commutative ring with unity, then R = I if 1 ∈ I.

5. Answer any three questions 5 × 3 = 15

(a) Show that every Euclidean Domain is a Principal Integral Domain.

(b) Show that the ring $Z[\sqrt{-5}]$ is not a UFD.

(c) Show that the ring of Gaussian integers, Z[i] is an Eu clidean domain.

(d) In Q[x], the set of all polynomial over the field of rational Q,then show that the ideal < x + 2 > is maximal.

6. Answer the followings: 5 × 3 = 15

(a) Let F be a field and a and b be members of a field con taining F. Suppose m and n are relative are algebraic of degree and over F respectively. Suppose m and n are relatively prime. Show that [F(a,b):F] =mn.

(b) Show that an element a of an extension field K is alge braic over a field F if and only if [F (a) : F] is finite.

(c) Show that is algebraic over Q of degree 6.

7. Answer the followings :

(a) Let W be a subspace of a finite dimensional vector V_F. Then establish that dim(V/W) = dim(V) − dim(W). 6

(b) Show that the vectors (α_1, α_2) and (β_1, β_2) are linearly dependent if and only if $\alpha_1 \beta_2 = \alpha_1 \beta_2$. 4

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