2016

MATHEMATICS

MTC 401

PARTIAL DIFFERENTIAL EQUATIONS

Full Marks : 80

Time : 3 Hrs

Figures in the right hand margin indicate full marks for the question

1. Answer any two questions: $2 \times 10 = 20$

a) Solve $(D^2 - 3DD' + 2D'^2)z = e^{2x-y} + e^{x+y} + \cos(x + 2y)$.

b) Solve $(D^2 - DD' + D' - 1)z = \cos(x + 2y) + e^y$.

c) Reduce $\frac{\partial^2 z}{\partial x^2} = x^2 \frac{\partial^2 z}{\partial y^2}$ to canonical form.

2. Answer any two questions: $2 \times 10 = 20$

a) Show that in a cylindrical coordinates Laplace's equation has solution of the form $R(\rho)e^{\pm mz \pm in\varphi}$ where $R(\rho)$ is a solution of Bessel's equation.

b) Discuss the Neumann problem in a rectangle.

c) Evaluate the steady temperature in a rectangular plate of length a and width b, the sides of which are kept at temperature zero, the lower end is kept at temperature $f(x)$ and the upper edge is kept insulated.

3. Answer any two questions: $2 \times 10 = 20$

a) Solve the wave equation $\frac{\partial^2 u}{\partial r^2} + \frac{2}{r}\frac{\partial u}{\partial r} + \frac{1}{r}\frac{\partial^2 u}{\partial \theta^2} + \frac{\cot\theta}{r^2}\frac{\partial u}{\partial \theta} + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 u}{\partial \phi^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$ if there is axial symmetry and solutions remain finite for $\theta = 0$.

b) Discuss the solution of transverse vibrations of a this membrane bounded by a circle of radius a in xy −plane described by the function $z(x, y, t)$ satisfying the wave equation, $\frac{\partial^2 z}{\partial r^2} + \frac{1}{r}\frac{\partial z}{\partial r} + \frac{1}{r^2}\frac{\partial^2 z}{\partial \theta^2} = \frac{1}{c^2}\frac{\partial^2 z}{\partial t^2}$ with conditions $z = 0$ on $r = a$ and $z = f(r), \frac{\partial z}{\partial t} = 0$ at $t = 0$.

c) Discuss D'Alemberts solution of one dimensional wave equation.

4. Answer any two questions: $2 \times 10 = 20$

a) Determine the temperature in a sphere of radius a when its surface is maintained at zero temperature and its initial temperature is $f(r, \theta)$.

b) Obtain the solution of the two-dimensional diffusion equation $\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} = \frac{1}{k}\frac{\partial \theta}{\partial t}$.

c) Determine the temperature distribution in the infinite cylinder the initial temperature is θ and the surface is maintained at zero temperature.

****