2017

MATHEMATICS

PAPER : MAT 205

CONTINUUM MECHANICS AND LATTICE THEORY

Full Marks: 80

Time: 3 hours

The figures in the margin indicate full marks for the questions

(Group A- Continuum Mechanics)

1. **Attempt any *Seven* from the following questions** 5X7=35

(a) Explain what is mean by "Stress quadric of Cauchy".

(b) Explain Deviator and Spherical stress.

(c) The stress matrix at a point P in a material is given as follows

$$(\sigma_{ij}) = \begin{pmatrix} 3 & 1 & 4 \\ 1 & 2 & -5 \\ 4 & -5 & 0 \end{pmatrix}$$

Determine the stress vector on the plane at P whose unit normal is

$$\hat{n} = \frac{2}{\sqrt{6}}\hat{e}_1 + \frac{1}{\sqrt{6}}\hat{e}_2 - \frac{1}{\sqrt{6}}\hat{e}_3$$

And also determine the magnitude of $t_i^{(\hat{n})}$.

(d) A continuum body undergoes the deformation

$$x_1 = X_1, x_2 = X_2 + AX_3, x_3 = X_3 + AX_2,$$

where A is a constant. Determine the Lagrangian finite strain tensor.

(e) Derive the equations of compability.

(f) Explain about material derivatives.

(g) Show that the stress tensor is symmetric.

(h) Establish equilibrium equations of an arbitrary volume V of a Continuum subject to a system of surface forces and body forces.

(i) Derive the Lagrangian differential form of the continuity equation.

Group-B (Lattice Theory)

2. Attempt any *Seven* from the following **5X7=35**

(a) What is partially ordered set? Show that in a poset a< a for no a and

$a < b, b < c \Rightarrow a < c.$

(b) What is Hasse diagram of poset? Represent the Hasse diagram of the poset {2,3,4,6,8,12,15}.

(c) What is a lattice? Draw the diagram of lattice of factors of 20, under divisibility and show that it is same as that of the product of two chains with three and two elements.

(d) Show that a poset $(L, \leq)$ is a lattice iff every non-empty finite subset of L has Sup and Inf.

(e) What is a chain? If G be a cyclic group of prime power order. Then show that the lattice of all subgroups of G is a chain.

(f) Show that dual of a lattice is a lattice.

(g) If $(P, \leq)$ is a poset with greatest element u such that every non empty subset S of P has inf then P is a complete lattice

(h) Show that two Lattice L and M are modular if LXM is modular.

(i) Show that Homomorphic image of relatively complemented lattice is relatively complemented.

3. Define the following (*any five*) **2X5=10**

(a) Realizer (b) Linear orders

(c) Complete lattice (d) Continuum

(e) fluid (f) path line

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