

2017
MATHEMATICS
 PAPER: MAT 402

NUMERICAL ANALYSIS

FULL MARKS: 80
 TIME: 3 HOURS

The figures in the margin indicate full marks for the questions

1. Answer any two: 2 × 10 = 20

(i) Solve, by Gauss elimination method, the equations

$$2x + y + z = 10, 3x + 2y + 3z = 18, x + 4y + 9z = 16.$$

(ii) Using Escaletor method, find the inverse of

$$A = \begin{bmatrix} 13 & 14 & 6 & 4 \\ 8 & -1 & 13 & 9 \\ 6 & 7 & 3 & 2 \\ 9 & 5 & 16 & 11 \end{bmatrix}.$$

(iii) Write in detail about Cholesky's decomposition method.

2. Answer any two: 2 × 10 = 20

(i) Write in detail about Jacobi's method. Using Jacobi's method, find all the eigen values and the eigen vectors of the matrix

$$\begin{bmatrix} 1 & \sqrt{2} & 2 \\ \sqrt{2} & 3 & \sqrt{2} \\ 2 & \sqrt{2} & 1 \end{bmatrix}$$

(ii) Using Given's method, reduce the following matrix to the tri-diagonal form:

$$A = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 4 & 2 \\ 3 & 2 & 3 \end{bmatrix}$$

- (iii) Using House-holder method, reduce the following matrix to the tri-diagonal form

$$A = \begin{bmatrix} 1 & 4 & 3 \\ 4 & 1 & 2 \\ 3 & 2 & 1 \end{bmatrix}$$

3. Answer any two:

2 × 10 = 20

- (i) Write in detail about Modified Euler's method. Using modified Euler's method, find an approximate value of y when $x = 0.3$, given that $\frac{dy}{dx} = x + y$ and $y=1$ when $x = 0$.
- (ii) Write in detail about Runge-Kutta method. Using Runge-Kutta method of fourth order, solve $\frac{dy}{dx} = \frac{y^2-x^2}{y^2+x^2}$ with $y(0) = 1$ at $x = 0.2, 0.4$.
- (iii) Write in detail about Finite difference method. Determine values of y at the pivotal points of the interval $(0,1)$ if y satisfies the boundary value problem $y^{iv} + 81y = 81x^2$, $y(0) = y(1) = y''(0) = y''(1) = 0$.

4. Answer any two:

2 × 10 = 20

- (i) Write about the polynomial approximation by use of orthogonal polynomials.
- (ii) By the method of least squares, find the straight line that best fits the following data:

$$\begin{array}{cccccc} x: & 1 & 2 & 3 & 4 & 5 \\ y: & 14 & 27 & 40 & 55 & 68 \end{array}$$

- (iii) Prove that

$$\frac{1-t^2}{1-2tx-t^2} = T_0(x) + 2 \sum_{n=1}^{\infty} T_n(x)t^n$$

(Symbols have their usual meaning)
