2017

MATHEMATICS

PAPER: MTC 404

GROUP A

FLUID DYNAMICS (Optional)

FULL MARKS: 80

TIME: 3 HOURS

The figures in the margin indicate full marks for the questions

(All questions are compulsory)

1. (a) Define compressible fluid. The velocity components for a two dimensional fluid system in Eulerian system are $u = 2x + 2y + 3t$, $v = x + y + t/2$. Find the displacement of a fluid particle in the Lagrangian system. 1+7=8

(b) Define laminar flow. For a two dimensional flow the velocities at a point in a fluid may be expressed in the Eulerian coordinates by $u = x + y + 2t$ and $v = 2y + t$. Determine the Lagrange coordinates as functions of the initial positions x_0 and y_0 and the time t. 1+7=8

Or,

Prove that at all points of the field of flow the equipotentials are cut orthogonally by the streamlines. The velocity potential of a two dimensional flow is $\phi = cxy$. Find the streamlines.

2. **Answer any two :** 2×8=16

(a) Prove that the stress tensors are symmetric.

(b) Prove that for each state of stress at a point, there exists at least one set of three mutually perpendicular principal directions. Mention the three assumptions of Stoke's law of viscosity.

(c) Define principal axes of stress tensor. Prove that $\frac{\partial u_i}{\partial t} + u_j u_{i,j} = X_i + \frac{1}{\rho}\sigma_{ji,j}$.

3. (a) Prove that the mean value of a ϕ over any spherical surface S drawn in the fluid throughout whose interior $\nabla^2\phi = 0$, is equal to the value of ϕ at the centre of the sphere. Also prove that in an irrotational motion the maximum value of the fluid velocity occurs at the boundary. 8

(b) A velocity field is given by $\vec{q} = (-\hat{i}y + \hat{j}x)/(x^2 + y^2)$. Determine whether the flow is irrotational. Calculate the circulation around a square with its corners at (1,0), (2,0), (2,1), (1,1) and a unit circle with centre at the origin. 8

Or,

Define circulation. State and prove Kelvin's circulation theorem.

4. (a) State and prove Buckingham's pie theorem. 8

(b) State and prove Bernoulli's theorem. Define Beltrami vector. 8

Or,

Define the terms vortex motion, vortex lines and vortex filaments. State and prove Milne's circle theorem.

5. (a) Discuss the average and extreme values of velocity in Coutte flow. 8

Or,

Discuss the steady flow through tube of uniform circular cross-section.

(b) A liquid occupying the space between two co-axial circular cylinders is acted upon by a force c/r per unit mass, where r is the distance from the axis, the lines of force being circles around the axis. Prove that in the steady motion the velocity at any point is given by the formula

$$\frac{c}{2v}\left\{\frac{b^2}{r}\frac{r^2-a^2}{b^2-a^2}\log\frac{b}{a} - r\log\left(\frac{r}{a}\right)\right\},$$

where a, b are the radii and v is the coefficient of kinematic viscosity. 8

GROUP B

ADVANCE NUMBER THEORY

1. **Answer any four questions :** 5X4=20

(a) Define primitive root of an integer n. Show that if r is a primitive root of a prime p of the form 4k + 1, then –r is also a primitive root of p.

(b) For $k \geq 3$, the integer 2^k has no primitive roots. Prove it.

(c) Using a table of indices for a primitive root of 11, solve the following congruence $7x^3 \equiv 3 \pmod{11}$.

(d) If n has a primitive root r and ind a denotes the index of a relative to r, then establish the followings:

(i) ind (ab) ≡ ind a + ind b (mod φ (n)).

(ii) ind $(a^k) \equiv$ k ind a (mod φ (n)), for k >0.

(e) Show that the congruence $x^3 \equiv 4 \pmod{13}$ is not solvable but $x^3 \equiv 5 \pmod{13}$ is solvable.

2. **Answer any four questions :** 5X4=20

(a) Show that an even perfect number n ends in the digit 6 or 8.

(b) Show that the Fermat number F_5 is divisible by 641.

(c) Prove that if n>6 is an even perfect number then $n \equiv 4 \pmod 6$.

(d) If p and q = 2p +1 are primes, then show that either $q|M_p$ or $q|M_p + 2$, but not both, where M_p denotes the p^{th} Mersenne number.

(e) Show that gcd $(F_m, F_n) = 1$ where F_m and F_n are Fermat numbers with $m > n \geq 0$.

3. **Answer any two questions :** 8X2=16

a) Prove that the Diophantine equation $x^4+y^4=z^2$ has no solutions in positive integers x, y, z.

(b) Prove that all the solutions of the integers of the Pythagorean equation $x^2+y^2=z^2$ satisfying the conditions gcd (x, y, z) = 1, $2|x$, $x>0, y>0, z>0$ are given by the formulas $x=2st$, $y=s^2-t^2$, $z=s^2+t^2$, for integers $s>t>0$ such that gcd (s,t) =1 and $s \neq t \pmod 2$.

(c) Show that any prime p can be written as sum of four squares.

4. Prove that the area of Pythagorean triangle can never be equal to a perfect (integral) square. 4

5. Answer any two questions : 5X2=10

(a) If $m \geq 1$, $n \geq 1$ are two integers,then show that U_{mn} is divisible by U_m, where U_m denotes the m^{th} Fibonacci number.

(b) Using the finite continued fraction solve the following linear Diophantine equation

$172x + 20y = 1000$.

(c) Find the fundamental solutions of the following equation $x^2 - 41y^2 = 1$.

6. **Answer any five questions :** 2X5=10

(a) Encrypt the message 'RETURN HOME' by using Caesar ciper.

(b) Using the linear ciper $C \equiv 5P+11 \pmod{26}$, encrypt the message 'NUMBER THEORY IS EASY'.

(c) Decrypt the message RXQTGUHOZTKGHFJKTMMTG,which was produced using the linear ciper $C \equiv 3P+7 \pmod{26}$.

(d) Enciper the message 'HAVE A NICE DAY', using a Vigenere ciper with the Keyword MATH.

(e) Encrypt the message 'GOOD', by using Hill's ciber $C_1 \equiv 2P_1 + 3P_2 \pmod{26}$ and $C_2 \equiv 5P_1 + 8P_2 \pmod{26}$.

(f) Find the unique solution of the following superincreasing knapsack problem $118 = 4x_1 + 5x_2 + 10x_3 + 20x_4 + 41x_5 + 99x_6$.

P.T.O.

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